OXFORD AQA

INTERNATIONAL QUALIFICATIONS

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I declare this is my own work.

# INTERNATIONAL A-LEVEL
# FURTHER MATHEMATICS

## (9665/FM04) Unit FS2 Statistics

Monday 12 January 2026 07:00 UK Time Time allowed: 1 hour 30 minutes

**Materials**
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| **TOTAL** | |

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Answer **all** questions in the spaces provided.

**1** A student is studying the maximum daily temperature recorded in a town in the month of April.

A random sample of 300 maximum daily temperatures is taken from data collected by an organisation that studies the weather.

The sample mean is 22.6 °C

The population variance of the maximum daily temperatures is known to be 1.3 °C

**1 (a)** Construct a 93% confidence interval for the population mean maximum daily temperature.

Give your values to two decimal places.

**[3 marks]**

Answer _____

**1 (b)** The student claims that the population mean maximum daily temperature is 22.8 °C

Explain whether the confidence interval found in **part (a)** supports the student's claim.

**[2 marks]**

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**1 (c)** The student collects their own random sample of $n$ temperatures.

A 93% confidence interval is constructed using their sample.

The width of this confidence interval is 0.1

Find the value of $n$

**[3 marks]**

$n =$ _____

8

**Turn over for the next question**

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**2** Mork and Ellen are taking measurements in a physics experiment.

The measurement errors that Mork makes and the measurement errors that Ellen makes are known to be independent and have normal distributions.

A random sample is taken from Mork's measurement errors and a random sample is taken from Ellen's measurement errors.

Unbiased estimates of the population variances are calculated using the samples.

The results are given in **Figure 1**

**Figure 1**

| Sample | Sample size | Unbiased estimate of population variance |
|---|---|---|
| Mork | 12 | 6.2 |
| Ellen | 10 | 3.1 |

Mork claims that the population variances of his and Ellen's measurement errors are equal.

Test Mork's claim using the 2% level of significance.

**[6 marks]**

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6

**Turn over for the next question**

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**3** A population $X$ has a normal distribution with mean $\mu_X$ and variance 8.2

A population $Y$ has a normal distribution with mean $\mu_Y$ and variance 4.7

Populations $X$ and $Y$ are independent.

A random sample of size 14 is taken from population $X$ and has sample mean 18

A random sample of size 21 is taken from population $Y$ and has sample mean 22

A hypothesis test is carried out at the 5% level of significance with the hypotheses

$$H_0 : \mu_Y - \mu_X = k$$
$$H_1 : \mu_Y - \mu_X > k$$

where $k$ is a positive integer.

The result of this hypothesis test is to reject the null hypothesis.

**3 (a)** Find the possible values of $k$

**[5 marks]**

$k =$ _____

*Do not write outside the box*

**3 (b)** It is given that $k = 2$

A second hypothesis test is carried out using the same random samples and hypotheses at the 1% level of significance.

Determine whether this hypothesis test will reject the null hypothesis.

**[4 marks]**

9

**Turn over for the next question**

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**4** A government is studying households that own a pet in its three main cities: *A*, *B* and *C*

They take random samples of households in each of the three cities.

The results are shown in **Figure 2**

**Figure 2**

| | **Observed Frequencies** | |
|---|---|---|
| **City** | **Number of households with a pet** | **Number of households without a pet** |
| *A* | 31 | 29 |
| *B* | 60 | 30 |
| *C* | 9 | 1 |

The government claims that there is an association between the city that a household is in and whether the household has a pet.

**4 (a)** Calculate the expected frequencies to complete the contingency table in **Figure 3**

**[2 marks]**

**Figure 3**

| | **Expected Frequencies** | |
|---|---|---|
| **City** | **Number of households with a pet** | **Number of households without a pet** |
| *A* | | |
| *B* | | |
| *C* | | |

**4 (b)** Explain why the data for two of the cities must be combined.

**[1 mark]**

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**4 (c)** Test the government's claim, using the 5% level of significance, by combining the data for city *B* and city *C*

**[8 marks]**

11

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**5** Job happiness scores at company $J$ and company $K$ are known to have independent normal distributions.

A random sample of 13 employees is taken from company $J$ and a random sample of 15 employees is taken from company $K$ and their job happiness scores are recorded.

For each sample, the sample mean $\overline{x}$ and the unbiased estimate of the population variance $s^2$ are calculated.

The results are given in **Figure 4**

**Figure 4**

| Company | Sample size | $\overline{x}$ | $s^2$ |
|---|---|---|---|
| $J$ | 13 | 95.2 | 9.2 |
| $K$ | 15 | 93.3 | 9.1 |

Company $J$ claims that their population mean job happiness score is greater than company $K$'s population mean job happiness score.

Test company $J$'s claim using the 5% level of significance.

State any assumptions that you make.

**[10 marks]**

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10

**Turn over for the next question**

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**6** A population has mean $\mu$ and variance $\sigma^2$

A random sample of size $n$ is taken from the population and has mean $\bar{X}$

A second population has mean $a\mu$ and variance $b\sigma^2$ where $a$ and $b$ are positive constants.

A random sample also of size $n$ is taken from this second population and has mean $\bar{Y}$

The two populations are independent.

The pooled estimator $T$ is given by

$$T = 3\bar{X} - 5\bar{Y}$$

$T$ is an unbiased estimator of $\mu$

**6 (a)** Show that $a = \frac{2}{5}$

**[2 marks]**

**6 (b)** Determine whether $T$ is a consistent estimator of $\mu$

**[2 marks]**

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**6 (c)** The efficiency of $T$ relative to $\bar{X}$ is equal to 0.05

**6 (c) (i)** Explain which of $T$ or $\bar{X}$ is the more efficient estimator of $\mu$

**[1 mark]**

**6 (c) (ii)** Find the value of $b$

**[3 marks]**

$b =$ _____

8

**Turn over ►**

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**7** A bag contains coins with values as shown in **Figure 5**

**Figure 5**

| Value of coin | Frequency |
|---|---|
| 1 dollar | 50 |
| 50 cents | 30 |
| 20 cents | 20 |

There are 100 cents in 1 dollar.

A coin is selected at random from the bag.

The value of the coin, $X_1$ dollars, is recorded and the coin is replaced.

A second coin is selected at random from the bag and its value, $X_2$ dollars, is recorded.

Find the sampling distribution of $D$ dollars where $D = |X_1 - X_2|$

**[7 marks]**

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7

Answer _____

**Turn over for the next question**

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**8** The random variable $X_i$ has a Bernoulli distribution with parameter $p$

**8 (a)** Show that $\mathrm{M}_{X_i}(t)$, the moment generating function of $X_i$, is given by

$$\mathrm{M}_{X_i}(t) = 1 - p + p\mathrm{e}^t$$

**[1 mark]**

**8 (b)** The random variables $X_1$, $X_2$, …, $X_n$ are independent Bernoulli distributions, each with parameter $p$

The random variable $X = X_1 + X_2 + ... + X_n$

The random variable $Y$ has moment generating function $\mathrm{M}_Y(t) = \left(p\mathrm{e}^t + 1 - p\right)^n$

**8 (b) (i)** Prove that the random variables $X$ and $Y$ have the same moment generating function.

**[3 marks]**

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**8 (b) (ii)** Use differentiation to show that the variance of $X$ is given by $np(1-p)$

**[8 marks]**

**8 (b) (iii)** Find $M_{2-3X}(t)$, the moment generating function of the random variable $2-3X$

**[1 mark]**

Answer _____

13

**Turn over ►**

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**9** A company uses a machine to produce boxes.

When the machine was new, the number of faulty boxes produced per hour by the machine was known to follow a Poisson distribution with mean 7.5

The company claims that the mean number of faulty boxes produced per hour by the machine has changed as the machine has aged.

The company carries out a hypothesis test at the 1% level of significance to test its claim.

It is given that the mean number of faulty boxes produced per hour by the machine has changed to 8

**9 (a)** Show that the power of the hypothesis test is 0.011 correct to three decimal places.

**[5 marks]**

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**9 (b)** Explain how the power of the hypothesis test would change if the mean number of faulty boxes produced per hour was greater than 8

**[3 marks]**

**8**

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