OXFORD AQA
INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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Candidate signature ______________________
I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Tuesday 6 January 2026   07:00 UK Time   Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
TOTAL	

Answer **all** questions in the spaces provided.

1 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 5 \quad \text{and} \quad u_{n+1} = u_n + 4n + 7$$

Prove by induction that for all integers $n \geq 1$

$$u_n = 2n^2 + 5n - 2$$

[4 marks]

4

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2 (a) Explain why $\int_0^1 \left(1+x^2\right) \ln x \, \mathrm{d}x$ is an improper integral.

[1 mark]

2 (b) Evaluate $\int_0^1 \left(1+x^2\right) \ln x \, \mathrm{d}x$ showing the limiting process used.

Give your answer in an exact form.

[5 marks]

Answer

6

Turn over ►

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3 (a) Use the exponential definitions of hyperbolic functions to prove that

$$1-\tanh^2 x = \operatorname{sech}^2 x$$

[2 marks]

3 (b) Use your answer to **part (a)** to solve the equation

$$2\operatorname{sech}^2 x = 1-\tanh x$$

Give any solutions in an exact logarithmic form.

[5 marks]

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Answer ____________________

7

Turn over for the next question

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4 (a) Show that

$$r^3(r+1)^3 - (r-1)^3 r^3 = ar^5 + br^3$$

where a and b are integers.

[2 marks]

4 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} r^5 = \frac{1}{12}n^2(n+1)^2(2n^2 + pn + q)$$

where p and q are integers.

[5 marks]

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7

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5 A curve C is defined for $x \geq 1$ by $y = \cosh^{-1} x$

5 (a) Show that $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{x}{\sqrt{x^2 - 1}}$

[3 marks]

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5 (b) The arc formed by the graph of C from $x = 1$ to $x = 3$ is rotated through 2π radians about the x-axis to generate a surface with area S

Find the exact value of S

Give your answer in the form $4\pi\left(p \ln(q) + r\right)$ where p, q and r are real numbers.

[5 marks]

Answer ______________________

8

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6 The points A, B and C have coordinates $A(5,4,-2)$, $B(2,-1,-1)$ and $C(-3,2,1)$

The points A, B and C lie in the plane Π_1

6 (a) Find a Cartesian equation of the plane Π_1

[5 marks]

Answer ____

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6 (b) The plane Π_2 has equation $\mathbf{r} \bullet \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} = 4$

Show that the acute angle between Π_1 and Π_2 is 85° correct to the nearest degree.

[3 marks]

8

Turn over ►

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7 The general solution of the differential equation

$$\frac{dy}{dx} - \frac{x}{x^2+1}y = x+1$$

can be written in the form $y = f(x)$

7 (a) Use an integrating factor to find $f(x)$

[7 marks]

$f(x) =$ ______

Do not write outside the box

7 (b) Find the particular solution of the differential equation where $\mathrm{f}(0) = 4$

[2 marks]

$y =$ ____________________

9

Turn over for the next question

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8 The non-singular matrix **M** is defined as

$$\mathbf{M} = \begin{bmatrix} 1 & 4 & -1 \\ 2 & -1 & 2k+3 \\ -1 & 2 & 3k+2 \end{bmatrix}$$

where k is a real number.

8 (a) (i) Find any restrictions on the value of k

[2 marks]

Answer

8 (a) (ii) Find $\mathbf{M}^{-1}$ in terms of k

[5 marks]

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$\mathbf{M}^{-1} =$ ______

8 (b) Use your answer to **part (a)(ii)** to solve

$$\begin{aligned} x + 4y - z &= 0 \\ 2x - y + (2k+3)z &= 2 \\ -x + 2y + (3k+2)z &= 3 \end{aligned}$$

Give your answer in its simplest form in terms of k

[3 marks]

$x =$ ______ $y =$ ______ $z =$ ______

10

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9 The equation $2x^2 + 7x + 4 = 0$ has roots α and β

9 (a) (i) Write down the value of $\alpha + \beta$

[1 mark]

$\alpha + \beta =$ ______

9 (a) (ii) Write down the value of $\alpha\beta$

[1 mark]

$\alpha\beta =$ ______

9 (b) (i) Use your answers to **part (a)** to show that $\alpha^2 + \beta^2 = \dfrac{33}{4}$

[2 marks]

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9 (b) (ii) Use your answers to **part (a)** to find the value of $\alpha^3 + \beta^3$

[3 marks]

$\alpha^3 + \beta^3 =$ ______

Question 9 continues on the next page

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9 (c) Use your answers to **part (a)** and **part (b)** to find a quadratic equation with integer coefficients which has roots

$$\alpha^3 + 2\beta \quad \text{and} \quad \beta^3 + 2\alpha$$

[9 marks]

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Answer

16

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10 It is given that $y = \mathrm{f}(x)$ satisfies the differential equation

$$2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = 4\mathrm{e}^{-x} + 7\mathrm{e}^{2x} + 26\cos x$$

It is also given that when $x = 0$ both $y = \frac{1}{3}$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{3}$

Find $\mathrm{f}(x)$

[11 marks]

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$f(x) =$ ______________________

11

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11 A line L_1 has direction cosines $\left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$ and goes through the point $P(1, 2, -2)$

A line L_2 has equation

$$\mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

A plane Π has equation

$$3x - 4y + z = 3$$

The line L_1 intersects the plane Π at Q

The line L_2 intersects the plane Π at R

Show that the area of the triangle PQR is $5\sqrt{3}$

[9 marks]

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9

Turn over ►

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12 (a) Use de Moivre's theorem to show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$$

[3 marks]

12 (b) Hence show that $x = \cos\left(\frac{\pi}{8}\right)$ satisfies the equation

$$8x^4 - 8x^2 + 1 = 0$$

and find the other **three** roots of this equation in the form $x = \cos\theta$

[3 marks]

Answer

12 (c) (i) Use your answer to **part (b)** to show that

$$\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{3\pi}{8}\right) = \frac{1}{8}$$

[3 marks]

12 (c) (ii) Use your answer to **part (b)** to show that

$$\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{2+\sqrt{2}}{4}}$$

[3 marks]

12

Turn over ►

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13 (a) It is given that $y = \text{sech}\, x$

Show that $\dfrac{dy}{dx} = -\,\text{sech}\, x \tanh x$

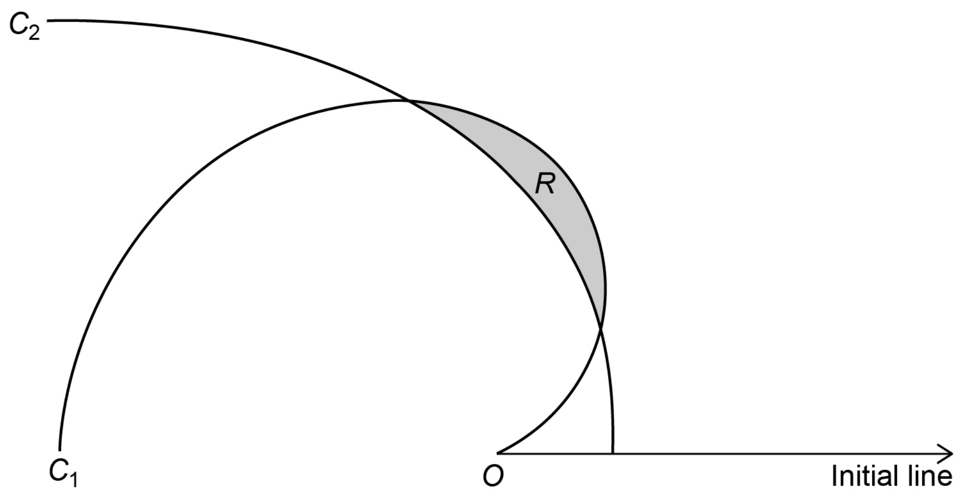
[2 marks]

13 (b) The curve C_1 has polar equation $r = \dfrac{4\sinh\theta - 2}{\cosh\theta}$ for $0.5 \leq \theta \leq \pi$

The curve C_2 has polar equation $r = \cosh\theta$ for $0 \leq \theta \leq \pi$

The finite region R is enclosed by the curves C_1 and C_2 as shown in **Figure 1**

Figure 1



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Show that the area of R is

$$\frac{31}{4}\ln\left(\frac{3+\sqrt{10}}{1+\sqrt{2}}\right)-\frac{7\sqrt{10}}{4}-\frac{3\sqrt{2}}{4}$$

[11 marks]

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13

END OF QUESTIONS

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