

**INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS**

FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

January 2026

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$z(96.5\%) = 1.8119$	B1	AWRT 1.81
	$22.6 \pm 1.8119 \sqrt{\frac{1.3}{300}}$	M1	Applies correct formula with their z value to find the upper or lower limit of the confidence interval Condone 1.3 instead of $\sqrt{1.3}$
	$= (22.48, 22.72)$	A1	AWRT 22.48 and 22.72
		3	

Q	Answer	Marks	Comments
1(b)	22.8 is outside the confidence interval	M1	Correct comparison with their confidence interval
	The confidence interval does not support the student's claim	A1ft	Correct conclusion following correct comparison Must refer to the claim ft their confidence interval
		2	

Q	Answer	Marks	Comments
1(c)	$0.1 = 2 \times 1.8119 \times \sqrt{\frac{1.3}{n}}$	M1	Sets up equation with their z value to find n oe Condone missing 2, use of 1.3 for $\sqrt{1.3}$
	$n = 1707.15$	m1	PI Solves their equation correctly to find a value for n
	$n = 1707$	A1	Obtains $n = 1707$ or 1708
		3	

	Question 1 Total	8	
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Q	Answer	Marks	Comments
2	$H_0 : \sigma_M^2 = \sigma_E^2$ $H_1 : \sigma_M^2 \neq \sigma_E^2$ $\frac{6.2}{3.1}$ $= 2$ $F_{11,9}(99\%) = 5.178$ $[0.2158 <] 2 < 5.178$ <p>Do not reject H_0</p> <p>Insufficient evidence to suggest that the population variances of Mork's and Ellen's measurement errors are not equal</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1ft</p>	<p>Both hypotheses, oe</p> <p>PI Condone squaring 6.2 and 3.1 May divide the other way round If divides the other way round should obtain 0.5</p> <p>Finds correct critical value, AWRT 5.2 or correct probability, AWRT 0.15 If divides the other way round PI by AWRT 0.193</p> <p>Correctly compares their F test statistic and their critical value or their probability and 0.01 and makes the correct conclusion as to whether to reject the null hypothesis Condone incorrect critical values for the other tail</p> <p>Gives a conclusion in context, by referring to population variances of measurement errors or Mork's claim, based on a comparison of their test statistic and their critical value Conclusion must not be definite, eg use of "suggest" or "support"</p>
		6	
	Question 2 Total	6	

Q	Answer	Marks	Comments
3(a)	$z(95\%) = 1.6449$	B1	AWRT 1.64 or 1.645
	$\frac{22 - 18 - k}{\sqrt{\frac{8.2}{14} + \frac{4.7}{21}}}$	M1	Find test statistic in terms of k
	$1.6449 < \frac{22 - 18 - k}{\sqrt{\frac{8.2}{14} + \frac{4.7}{21}}}$	A1	Forms correct equation or inequality
	$k < 2.52$	m1	Solves their equation or inequality correctly to find an upper limit for k
	$k = 1$ or 2	A1	Finds the possible values of k
		5	

Q	Answer	Marks	Comments
3(b)	$z(99\%) = 2.3263$	B1	Finds critical value AWRT 2.33 or p -value AWRT 0.013
	$z = \frac{22 - 18 - 2}{\sqrt{\frac{8.2}{14} + \frac{4.7}{21}}}$	M1	Find test statistic
	$z = 2.22$	A1	AWRT 2.22
	$2.22 < 2.3263$	A1	Correct comparison and conclusion
	Null hypothesis is not rejected		
		4	

	Question 3 Total	9	
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Q	Answer	Marks	Comments	
4(a)	Expected Frequencies			
	City	Number of households with a pet		Number of households without a pet
	<i>A</i>	37.5		22.5
	<i>B</i>	56.25		33.75
	<i>C</i>	6.25	3.75	
		2		

Q	Answer	Marks	Comments
4(b)	One of the city <i>C</i> expected frequencies is less than 5	B1ft	oe ft their expected frequencies provided at least one is less than 5
		1	

Q	Answer	Marks	Comments																		
4(c)	<p>H_0: There is no association between the city a household is in and whether the household has a pet</p> <p>H_1: There is an association between the city a household is in and whether the household has a pet</p> <table border="1" data-bbox="236 589 866 786"> <thead> <tr> <th>E</th> <th>Households with pets</th> <th>Households without pets</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>37.5</td> <td>22.5</td> </tr> <tr> <td>B or C</td> <td>62.5</td> <td>37.5</td> </tr> </tbody> </table> <table border="1" data-bbox="236 808 866 1005"> <thead> <tr> <th>O</th> <th>Households with pets</th> <th>Households without pets</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>31</td> <td>29</td> </tr> <tr> <td>B or C</td> <td>69</td> <td>31</td> </tr> </tbody> </table> $\sum \frac{(O - E - 0.5)^2}{E} = \frac{(31 - 37.5 - 0.5)^2}{37.5} + \frac{(29 - 22.5 - 0.5)^2}{22.5} + \frac{(69 - 62.5 - 0.5)^2}{62.5} + \frac{(31 - 37.5 - 0.5)^2}{37.5}$ <p>= 4.096</p> <p>$\chi^2_1(95\%) = 3.841$</p> <p>4.096 > 3.841</p> <p>Reject H_0</p> <p>Sufficient evidence to suggest that there is an association between the city a household is in and whether the household has a pet</p>	E	Households with pets	Households without pets	A	37.5	22.5	B or C	62.5	37.5	O	Households with pets	Households without pets	A	31	29	B or C	69	31	<p>B1</p> <p>B1ft</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1</p>	<p>Both hypotheses oe Variables need to be mentioned in at least the null hypothesis</p> <p>Correctly merged expected frequencies using their expected frequencies</p> <p>Correctly merged observed frequencies</p> <p>Attempts to calculate test statistic</p> <p>Condone use of $\sum \frac{(O - E)^2}{E}$</p> <p>AWRT 4.1</p> <p>Obtains correct critical value, AWRT 3.8 or correct probability, AWRT 0.043</p> <p>Correctly compares their χ^2 test statistic and their critical value and rejects null hypothesis and makes the correct conclusion as to whether to reject the null hypothesis</p> <p>Gives a conclusion in context, by referring to association between city and households owning pets or the government's claim, based on a comparison of the correct test statistic and the correct critical value oe Condone definite conclusion</p>
E	Households with pets	Households without pets																			
A	37.5	22.5																			
B or C	62.5	37.5																			
O	Households with pets	Households without pets																			
A	31	29																			
B or C	69	31																			
		8																			

	Question 4 Total	11	
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Q	Answer	Marks	Comments
5	$H_0: \mu_J = \mu_K$ $H_1: \mu_J > \mu_K$ <p>Assume that the population variances are equal</p> $s_p^2 = \frac{(13-1) \times 9.2 + (15-1) \times 9.1}{13+15-2}$ $s_p^2 = 9.1[4615\dots]$ $t = \frac{95.2 - 93.3}{\sqrt{9.14\dots \left(\frac{1}{13} + \frac{1}{15}\right)}}$ $= 1.66$ $t_{26}(95\%) = 1.706$ $1.66 < 1.706$ <p>Do not reject H_0</p> <p>Insufficient evidence to suggest that the population mean job happiness score is greater for company J than company K</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1 M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1</p>	<p>Both hypotheses</p> <p>If different subscripts are used they must be defined</p> <p>Allow $H_0: \mu_J \leq \mu_K$</p> <p>oe</p> <p>Applies formula</p> <p>AWRT 9.1</p> <p>oe</p> <p>M1: correct numerator</p> <p>M1: correct denominator</p> <p>AWRT 1.66</p> <p>Obtains correct critical value, AWRT 1.71 or correct probability, AWRT 0.054</p> <p>Correctly compares their z or t test statistic and their critical value or their probability and 0.05 and makes the correct conclusion as to whether to reject the null hypothesis</p> <p>At least one method mark needs to be scored for the calculation of the test statistic</p> <p>Gives a conclusion in context, by referring to mean job happiness score or company J's claim, based on a comparison of the correct test statistic and the correct critical value oe</p> <p>Condone definite conclusion</p>
		10	
	Question 5 Total	10	

Q	Answer	Marks	Comments
6(a)	$E(T) = 3\mu - 5a\mu$ $3\mu - 5a\mu = \mu$ $3 - 5a = 1$ $a = \frac{2}{5}$	<p>M1</p> <p>A1</p>	<p>Finds $E(T)$ in terms of a and μ oe</p> <p>AG Must be convincingly shown Must see $3\mu - 5a\mu = \mu$ and an intermediate line with μ cancelled out or factorised or like terms collected</p>
		2	

Q	Answer	Marks	Comments
6(b)	$\text{Var}(T) = \frac{9\sigma^2}{n} + \frac{25b\sigma^2}{n}$ As $n \rightarrow \infty$, $\text{Var}(T) \rightarrow 0$ so estimator is consistent	<p>M1</p> <p>A1</p>	<p>Finds $\text{Var}(T)$ in terms of b, n and σ^2 oe</p> <p>Must see conclusion and justification</p>
		2	

Q	Answer	Marks	Comments
6(c)(i)	\bar{X} is more efficient as $0.05 < 1$	B1	<p>oe</p> <p>A comparison of variances must reach $\text{Var}(T) > \text{Var}(\bar{X})$</p>
		1	

Q	Answer	Marks	Comments
6(c)(ii)	$\frac{\frac{\sigma^2}{n}}{(9+25b)\frac{\sigma^2}{n}} = 0.05$ $\frac{1}{9+25b} = 0.05$ $b = 0.44$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Forms correct equation</p> <p> Cancels σ^2 from their equation of the form $\frac{P\sigma^2}{(Q \pm Rb)\sigma^2} = 0.05$ or $\frac{1}{0.05}$ oe</p> <p>oe eg $\frac{11}{25}$</p>
		3	
	Question 6 Total	8	

Q	Answer	Marks	Comments										
7	<p>(0.2, 0.2), (0.2, 0.5), (0.2, 1), (0.5, 0.5), (0.5, 1), (1, 1)</p> <p>0, 0.3, 0.5, 0.8</p> <p>(0.2, 0.2) Probability $\left(\frac{20}{100}\right)^2$</p> <p>(0.5, 0.5) Probability $\left(\frac{30}{100}\right)^2$</p> <p>(1, 1) Probability $\left(\frac{50}{100}\right)^2$</p> <p>(0.2, 0.5) Probability $2 \times \frac{20}{100} \times \frac{30}{100}$</p> <p>(0.2, 1) Probability $2 \times \frac{20}{100} \times \frac{50}{100}$</p> <p>(0.5, 1) Probability $2 \times \frac{30}{100} \times \frac{50}{100}$</p> <table border="1" data-bbox="240 1312 777 1469"> <tr> <td>d</td> <td>0</td> <td>0.3</td> <td>0.5</td> <td>0.8</td> </tr> <tr> <td>$P(D = d)$</td> <td>$\frac{19}{50}$</td> <td>$\frac{3}{25}$</td> <td>$\frac{3}{10}$</td> <td>$\frac{1}{5}$</td> </tr> </table>	d	0	0.3	0.5	0.8	$P(D = d)$	$\frac{19}{50}$	$\frac{3}{25}$	$\frac{3}{10}$	$\frac{1}{5}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p>	<p>Identifies all possible combinations of coins (ignoring order selected)</p> <p>Allow 20 for 0.2, 50 for 0.5 and 100 for 1</p> <p>Identifies all possible values of D in either cents or dollars with no incorrect values seen</p> <p>Correct calculation of probability of obtaining (0.2, 0.2), (0.5, 0.5) or (1, 1) oe PI</p> <p>Correct calculation of probability of obtaining (0.2, 0.5), (0.2, 1) or (0.5, 1) including different permutations oe PI</p> <p>Correct calculation of all the probabilities oe PI</p> <p>M1: Probability distribution table with their differences and probabilities A1: Fully correct, oe</p>
d	0	0.3	0.5	0.8									
$P(D = d)$	$\frac{19}{50}$	$\frac{3}{25}$	$\frac{3}{10}$	$\frac{1}{5}$									
		7											

	Question 7 Total	7	
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Q	Answer	Marks	Comments
8(a)	$M_{X_i}(t) = (1-p)e^0 + pe^t$ $= 1-p + pe^t$	B1	AG Must be convincingly shown e^0 or e^{0t} must be seen
		1	

Q	Answer	Marks	Comments
8(b)(i)	$M_X(t) = M_{X_1}(t) \times M_{X_2}(t) \times \dots \times M_{X_n}(t)$ $M_X(t) = (pe^t + 1 - p) \times (pe^t + 1 - p) \times \dots \times (pe^t + 1 - p)$ $M_X(t) = (pe^t + 1 - p)^n$ <p>X and Y have the same moment generating function</p>	M1 A1 A1	Identifies $M_X(t)$ as equal to the mgfs multiplied together Finds correct unsimplified $M_X(t)$ or $M_X(t) = (M_{X_i}(t))^n$ Must be convincingly proved Must see conclusion oe
		3	

Q	Answer	Marks	Comments
8(b)(ii)	$M'_X(t) = np e^t (p e^t + 1 - p)^{n-1}$ $M'_X(0) = np$ $M''_X(t) = np e^t (p e^t + 1 - p)^{n-1} + n(n-1) p^2 e^{2t} (p e^t + 1 - p)^{n-2}$ $M''_X(0) = np + n(n-1) p^2$ $\text{Var}(X) = np + n(n-1) p^2 - (np)^2$ $\text{Var}(X) = np - np^2 = np(1-p)$	<p>M1 A1</p> <p>A1</p> <p>m1 A1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>M1: $M'_X(t) = \dots (p e^t + 1 - p)^{n-1}$ A1: Fully correct</p> <p>Finds correct $M'_X(0)$</p> <p>m1: $M''_X(t) = \dots (p e^t + 1 - p)^{n-1} + \dots (p e^t + 1 - p)^{n-2}$ A1: Fully correct</p> <p>Finds correct $M''_X(0)$</p> <p>Applies formula to find the variance Dependent on first M mark</p> <p>AG Must be convincingly shown</p>
		8	

Q	Answer	Marks	Comments
8(b)(iii)	$M_{2-3X}(t) = e^{2t} (p e^{-3t} + 1 - p)^n$	B1	oe
		1	

	Question 8 Total	13	
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Q	Answer	Marks	Comments
9(a)	$X \sim \text{Po}(7.5)$ $P(X \leq 1) = 0.0047$ $P(X \leq 2) = 0.0203$ $P(X \geq 15) = 1 - 0.9897 = 0.0103$ $P(X \geq 16) = 1 - 0.9954 = 0.0046$ Rejection region $X \leq 1$ or $X \geq 16$ $Y \sim \text{Po}(8)$ $P(Y \leq 1) + P(Y \geq 16)$ $= P(Y \leq 1) + 1 - P(Y \leq 15)$ $= 0.0030 + 1 - 0.9918$ $= 0.0112$ $= 0.011$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	Finds the value of $P(X \leq 1)$ or $P(X \leq 2)$ Condone incorrect labelling Finds the value of $P(X \geq 15)$, $P(X \geq 16)$, $P(X \leq 14)$ or $P(X \leq 15)$ oe Condone incorrect labelling Identifies rejection or acceptance region Correct calculation to find the probability of their rejection or acceptance region for $Y \sim \text{Po}(8)$ AG Must be convincingly shown Must see $P(X \leq 1)$, $P(X \leq 2)$, $P(X \geq 15)$ and $P(X \geq 16)$ oe with correct labelling and value for power of test AWRT 0.0112 or 0.01125 before final answer
		5	

Q	Answer	Marks	Comments
9(b)	The probability of the lower tail would decrease The probability of the upper tail would increase Overall would increase [as probability of lower tail is already very small]	<p>M1</p> <p>M1</p> <p>A1</p>	oe or considers parameter greater than 8 eg finds $P(T \leq 15) = \text{AWRT } 0.978$ or $P(T \leq 1) = \text{AWRT } 0.001$ for $T \sim \text{Po}(9)$ oe or calculates power of test for parameter greater than 8 eg for $T \sim \text{Po}(9)$ finds power = AWRT 0.023 Allow “could increase, decrease or stay the same as one tail fell and one tail increased”
		3	

	Question 9 Total	8	
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