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FM03

(9665/FM03) Unit FP2 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	When $n=1$ $u_1 = 2\times1^2+5\times1-2=5$ [So formula is true $n=1$]	B1	Correct work to show formula is true for $n=1$
	Assume formula is true for $n=k$ **(*)** So $u_{k+1}=\left(2k^2+5k-2\right)+4k+7$	M1	Assumes formula is true for $n=k$ and considers $u_{k+1}=\left(2k^2+5k-2\right)+4k+7$
	$u_{k+1}=2\left(k^2+2k+1\right)+5\left(k+1\right)-2$		Must see an intermediate line of working.
	$u_{k+1}=2\left(k+1\right)^2+5\left(k+1\right)-2$	A1	Must be convincingly shown
	As formula is true for $n=1$ **(**)** And if true $n=k$ then true for $n=k+1$ **(***)** Hence by **induction** $u_n=2n^2+5n-2$ true for all integers $n\geq1$ **(****)**	E1	Must have **(*)**, **(**)**, **(***)** present, previous 3 marks awarded and a final statement **(****)**. Clearly indicating that it relates to positive integers $n\geq1$ by induction Must have stated "$u_n=2n^2+5n-2$" at least once in the conclusion.

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	The integrand or $\left(1+x^2\right)\ln x$ is not defined at $x=0$	E1	**oe**
		1	

Q	Answer	Marks	Comments
2(b)	$\int_0^1\left(1+x^2\right)\ln x\,dx$ $=\lim\limits_{a\to 0}\int_a^1\left(1+x^2\right)\ln x\,dx$	E1	Evidence of limit 0 having been replaced by a (**oe**, but NOT x) at any stage and $\lim\limits_{a\to 0}$ seen or taken
	$=\lim\limits_{a\to 0}\left\{\left[\left(x+\frac{1}{3}x^3\right)\ln x\right]_a^1-\int_a^1\left(1+\frac{1}{3}x^2\right)dx\right\}$	M1	Integration by parts
	$=\lim\limits_{a\to 0}\left\{-\left(a+\frac{1}{3}a^3\right)\ln a-\left[x+\frac{1}{9}x^3\right]_a^1\right\}$		
	$=\lim\limits_{a\to 0}\left\{-a\ln a-\frac{1}{3}a^3\ln a-\frac{10}{9}+a+\frac{1}{9}a^3\right\}$	A1	Fully integrated correctly with limits correctly substituted, lower limit not = 0
	Now $\lim\limits_{a\to 0}\left\{a\ln a\right\}=0$ and $\lim\limits_{a\to 0}\left\{a^3\ln a\right\}=0$	E1	Standard limit results stated in general form $\lim\limits_{a\to 0}\left\{a^k\ln a\right\}=0$ or for $k=1$ and $k=3$
	Hence $\int_0^1\left(1+x^2\right)\ln x\,dx$ $=-\dfrac{10}{9}$	A1	**E0 M1 A1 E0 A1** is possible
		5	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$1-\tanh^2 x = 1-\dfrac{\left\{\frac{1}{2}\left(e^x-e^{-x}\right)\right\}^2}{\left\{\frac{1}{2}\left(e^x+e^{-x}\right)\right\}^2}$	M1	Uses correct exponential definitions of $\sinh x$ and $\cosh x$ or $\tanh x$ to write LHS in exponential form
	$1-\tanh^2 x = \dfrac{\left\{\frac{1}{2}\left(e^x+e^{-x}\right)\right\}^2-\left\{\frac{1}{2}\left(e^x-e^{-x}\right)\right\}^2}{\left\{\frac{1}{2}\left(e^x+e^{-x}\right)\right\}^2}$		
	$1-\tanh^2 x = \dfrac{\frac{1}{4}\left(e^{2x}+2+e^{-2x}\right)-\frac{1}{4}\left(e^{2x}-2+e^{-2x}\right)}{\left\{\frac{1}{2}\left(e^x+e^{-x}\right)\right\}^2}$ $= \dfrac{\frac{2}{4}+\frac{2}{4}}{\left\{\frac{1}{2}\left(e^x+e^{-x}\right)\right\}^2} = \dfrac{1}{\cosh^2 x}$ $= \operatorname{sech}^2 x$	A1	**AG** Must be convincingly shown, including clear simplification of numerator. Must include $1-\tanh^2 x$ and $\operatorname{sech}^2 x$
		2	

Q	Answer	Marks	Comments
3(b)	$2\left(1-\tanh^2 x\right)=1-\tanh x$	B1	Correct use of identity from part **(a)** **PI**
	$2\tanh^2 x-\tanh x-1=0$	M1	Forms three term quadratic equation in $\tanh x$ or $2\left(1-\tanh x\right)\left(1+\tanh x\right)=1-\tanh x$ **PI** correct solutions
	$\tanh x = 1$ or $\tanh x = -\dfrac{1}{2}$	A1	Correct values for $\tanh x$ from correct equation. Must see both.
	$\tanh x \neq 1$ as $-1<\tanh x<1$	E1ft	Clear rejection of their $\tanh x = 1$
	$x=\tanh^{-1}\left(-\dfrac{1}{2}\right)$		
	$x=-\dfrac{1}{2}\ln(3)$	A1	Correct solution **ACF** **B1 M1 A0 E0 A1** is possible
		5	

	Question 3 Total	**7**	

Q	Answer	Marks	Comments
4(a)	$r^3(r+1)^3-(r-1)^3r^3=r^3(r^3+3r^2+3r+1)$ $-(r^3-3r^2+3r-1)r^3$	M1	Correct expansion of brackets
	$=6r^5+2r^3$	A1	Correct result. Must have $r^3(r+1)^3-(r-1)^3r^3$, $6r^5+2r^3$ and at least one line of intermediate working.
		2	

Q	Answer	Marks	Comments
4(b)	$S_n=\sum_{r=1}^{n}\left\{r^3(r+1)^3-(r-1)^3r^3\right\}$ $=\cancel{1^3\times2^3}-0^3\times1^3$ $+\cancel{2^3\times3^3}-\cancel{1^3\times2^3}$ $\vdots$ $+\cancel{(n-1)^3n^3}-\cancel{(n-2)^3(n-1)^3}$ $+n^3(n+1)^3-\cancel{(n-1)^3n^3}$	M1	Uses method of differences showing the first and last lines and at least one other line so that a pair of values which cancel are seen
	$=n^3(n+1)^3$	A1	Correct answer
	$6\sum_{r=1}^{n}r^5=n^3(n+1)^3-2\times\frac{1}{4}n^2(n+1)^2$	m1	Correct use of $\sum_{r=1}^{n}r^3=\frac{1}{4}n^2(n+1)^2$ and their S_n to obtain an equation for $\sum_{r=1}^{n}r^5$
	$\sum_{r=1}^{n}r^5=\frac{1}{12}n^2(n+1)^2\left\{2n(n+1)-1\right\}$	m1	Correctly factorises out $n^2(n+1)^2$
	$\sum_{r=1}^{n}r^5=\frac{1}{12}n^2(n+1)^2(2n^2+2n-1)$	A1	Correct result from correct working
		5	

	Question 4 Total	7	

Q	Answer	Marks	Comments
5(a)	$\dfrac{dy}{dx}=\dfrac{1}{\sqrt{x^2-1}}$	B1	Correct expression for $\dfrac{dy}{dx}$
	$\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=\sqrt{1+\dfrac{1}{x^2-1}}$ $=\sqrt{\dfrac{x^2-1+1}{x^2-1}}$ $=\sqrt{\dfrac{x^2}{x^2-1}}$	M1	Squaring $\dfrac{dy}{dx}$ and simplifying $1+\left(\dfrac{dy}{dx}\right)^2$ to $\dfrac{x^2}{x^2-1}$
	$\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=\dfrac{x}{\sqrt{x^2-1}}$	A1	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
5(a) ALT	$\cosh y = x$ $\dfrac{dy}{dx}=\dfrac{1}{\sinh y}$	B1	Correct expression for $\dfrac{dy}{dx}$
	$\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=\sqrt{1+\dfrac{1}{\sinh^2 y}}$ $=\sqrt{\dfrac{\sinh^2 y+1}{\sinh^2 y}}$ $=\sqrt{\dfrac{\cosh^2 y}{\cosh^2 y-1}}$ $=\dfrac{\cosh y}{\sqrt{\cosh^2 y-1}}$	M1	Squaring $\dfrac{dy}{dx}$ and simplifying $1+\left(\dfrac{dy}{dx}\right)^2$ to $\dfrac{\cosh^2 y}{\cosh^2 y-1}$
	$\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=\dfrac{x}{\sqrt{x^2-1}}$	A1	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
5(b)	$S = 2\pi\int_1^3 \cosh^{-1} x \frac{x}{\sqrt{x^2-1}}\,dx$	B1	Correct integrand, with 2π, dx and correct limits. All seen here
	$u = \cosh^{-1} x$ $\quad \frac{dv}{dx} = \frac{x}{\sqrt{x^2-1}}$ $\frac{du}{dx} = \frac{1}{\sqrt{x^2-1}}$ $\quad v = (x^2-1)^{\frac{1}{2}}$	M1	Selects integration by parts to calculate $\int \cosh^{-1} x \frac{x}{\sqrt{x^2-1}}\,dx$
	$\int \cosh^{-1} x \frac{x}{\sqrt{x^2-1}}\,dx = (x^2-1)^{\frac{1}{2}} \cosh^{-1} x - \int \frac{(x^2-1)^{\frac{1}{2}}}{\sqrt{x^2-1}}\,dx$ $= (x^2-1)^{\frac{1}{2}} \cosh^{-1} x - x$	A1	Obtains correct result of integrating $\int \cosh^{-1} x \frac{x}{\sqrt{x^2-1}}\,dx$ by parts. No limits needed at this stage
	$S = 2\pi\left[(x^2-1)^{\frac{1}{2}} \cosh^{-1} x - x\right]_1^3$ $= 2\pi\left[(8)^{\frac{1}{2}} \cosh^{-1} 3 - 3 + 1\right]$ $= 2\pi\left[2\sqrt{2}\ln(3+2\sqrt{2}) - 2\right]$	M1	Substitutes correct limits into their expression of the form $a(x^2-1)^{\frac{1}{2}} \cosh^{-1} x + bx$
	$S = 4\pi\left[\sqrt{2}\ln(3+2\sqrt{2}) - 1\right]$	A1	**oe** exact form
		5	

	Question 5 Total	**8**	

Q	Answer	Marks	Comments
6(a)	$\pm\begin{bmatrix}-3\\-5\\1\end{bmatrix}, \pm\begin{bmatrix}-8\\-2\\3\end{bmatrix}, \pm\begin{bmatrix}-5\\3\\2\end{bmatrix}$	B1	Obtains two of the three vectors connecting *A*, *B* and *C*
	$\mathbf{n}_1 = \begin{bmatrix}-3\\-5\\1\end{bmatrix}\times\begin{bmatrix}-8\\-2\\3\end{bmatrix} = \begin{bmatrix}-13\\1\\-34\end{bmatrix}$	M1	Takes the cross product of their direction vectors.
		A1	**oe** Correct $\mathbf{n}_1$
	$d = \begin{bmatrix}-13\\1\\-34\end{bmatrix}\bullet\begin{bmatrix}5\\4\\-2\end{bmatrix} = 7$	M1	Uses a point *A*, *B* or *C* and their $\mathbf{n}_1$ to obtain d
	$-13x + y - 34z = 7$	A1	**oe** Correct Cartesian equation of plane Π_1
		5	

Q	Answer	Marks	Comments
6(b)	$\begin{bmatrix}-13\\1\\-34\end{bmatrix}\bullet\begin{bmatrix}-2\\-3\\1\end{bmatrix} = 26 - 3 - 34[= -11]$	M1	Finds a numerical expression for the scalar product of $\mathbf{n}_2$ and their $\mathbf{n}_1$ Alternative $\begin{bmatrix}13\\-1\\34\end{bmatrix}\bullet\begin{bmatrix}-2\\-3\\1\end{bmatrix} = -26 + 3 + 34[= +11]$
	$\mathbf{n}_1 = \sqrt{13^2 + 1^2 + 34^2} = \sqrt{1326}$ $\mathbf{n}_2 = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$		
	$\cos\theta = \dfrac{-11}{\sqrt{1326}\,\sqrt{14}} = -0.08073408$	A1ft	**ft** their $\mathbf{n}_1$ only. Alternative above gives $+0.08073408$
	$\theta = 94.6[3\ldots]$		Must see $94.6[3\ldots]$ or $85.3[7\ldots]$ (from Alternative) to gain last mark
	So acute angle between Π_1 and Π_2 is $= 85^\circ$ Nearest degree	A1	**AG**
		3	

	Question 6 Total	**8**	

Q	Answer	Marks	Comments
7(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{x}{x^2+1}y = x+1$		
	I.F. is $\mathrm{e}^{\int \frac{-x}{x^2+1}\mathrm{d}x} = \mathrm{e}^{\frac{-1}{2}\ln\left(x^2+1\right)}$	M1	I.F. identified and integration attempted
	$= \frac{1}{\sqrt{x^2+1}}$	A1	Correct I.F.
	$\frac{1}{\sqrt{x^2+1}}\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{x}{\left(x^2+1\right)^{\frac{3}{2}}}y = \frac{x+1}{\sqrt{x^2+1}}$		
	$\frac{y}{\sqrt{x^2+1}} = \int \frac{x+1}{\sqrt{x^2+1}}\mathrm{d}x$	m1	Multiplying both sides of the differential equation by their I.F. and integrating LHS to get $y \times$I.F.
	$\frac{x+1}{\sqrt{x^2+1}} = \frac{1}{2}\frac{2x}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+1}}$	M1	Splits RHS integrand into form $\frac{\mathrm{f}'(x)}{\mathrm{f}(x)} + \frac{A}{\mathrm{f}(x)}$
	$\frac{y}{\sqrt{x^2+1}} =$		
	$\sqrt{x^2+1}$	A1	Integrates $\frac{x}{\sqrt{x^2+1}}$ to get $\sqrt{x^2+1}$
	$+\sinh^{-1}x$	A1	Integrates $\frac{1}{\sqrt{x^2+1}}$ to get $\sinh^{-1}x$ or $\ln\left(x+\sqrt{x^2+1}\right)$ Condone lack of $+c$
	$\mathrm{f}(x) = \left(x^2+1\right) + \sqrt{x^2+1}\left(\sinh^{-1}x + c\right)$	A1	Correct GS **ACF**, must be $\mathrm{f}(x) = \ldots$ eg $\mathrm{f}(x) = \left(x^2+1\right) + \sqrt{x^2+1}\left(\ln\left(x+\sqrt{x^2+1}\right)+c\right)$
		7	

Q	Answer	Marks	Comments
7(b)	$4 = 1 + 1\left(\sinh^{-1}0 + c\right)$ $c = 3$	M1	Substituting $x = 0$ and $\mathrm{f}\left(x\right) = 4$ into their GS (with a constant of integration) to find a value of c
	$y = \left(x^2 + 1\right) + \sqrt{x^2 + 1}\left(\sinh^{-1}x + 3\right)$	A1	Correct PS **ACF**, must be $y = \ldots$ Eg. $y = \left(x^2 + 1\right) + \sqrt{x^2 + 1}\left(\ln\left(x + \sqrt{x^2 + 1}\right) + 3\right)$
		2	

	Question 7 Total	**9**	

Q	Answer	Marks	Comments
8(a)(i)	$\lvert\mathbf{M}\rvert = 1(-3k-2-4k-6)$ $-4(6k+4+2k+3)-1(4-1)$ $= -39k-39$	**M1**	Obtains a linear expression for $\lvert\mathbf{M}\rvert$
	$k \neq -1$	**A1**	Correct restriction on k
		2	

Q	Answer	Marks	Comments
8(a)(ii)	Cofactor matrix		
	$\begin{bmatrix} -7k-8 & -8k-7 & 3 \\ -12k-10 & 3k+1 & -6 \\ 8k+11 & -2k-5 & -9 \end{bmatrix}$	**M1**	At least three entries correct
		A1	At least six entries correct
		A1	All nine entries correct
	Inverse matrix $\mathbf{M}^{-1}$	**M1**	Transpose of their cofactors with no more than one further slip **and** division by their $\lvert\mathbf{M}\rvert$ in terms of k
	$\mathbf{M}^{-1} = \frac{1}{39(k+1)}\begin{bmatrix} 7k+8 & 12k+10 & -8k-11 \\ 8k+7 & -3k-1 & 2k+5 \\ -3 & 6 & 9 \end{bmatrix}$	**A1**	Correct $\mathbf{M}^{-1}$
		5	

Q	Answer	Marks	Comments
8(b)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{39(k+1)}\begin{bmatrix} 7k+8 & 12k+10 & -8k-11 \\ 8k+7 & -3k-1 & 2k+5 \\ -3 & 6 & 9 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$		
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{39(k+1)}\begin{bmatrix} 24k+20-24k-33 \\ -6k-2+6k+15 \\ 12+27 \end{bmatrix}$	**M1 A1ft**	**M1**: Multiplies their $\mathbf{M}^{-1}$ onto $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ to form a single unsimplified vector. **A1ft**: At least 2 entries correct for their $\mathbf{M}^{-1}$
	$x = -\frac{1}{3(k+1)},\ y = \frac{1}{3(k+1)},\ z = \frac{1}{k+1}$	**A1**	**ACF** Correct and fully simplified
		3	

	Question 8 Total	**10**	

Q	Answer	Marks	Comments
9(a)(i)	$[\alpha+\beta=]\ -\frac{7}{2}$	B1	Correct value
		1	

Q	Answer	Marks	Comments
9(a)(ii)	$[\alpha\beta=]\ 2$	B1	Correct value
		1	

Q	Answer	Marks	Comments
9(b)(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1	Use of $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ with their values substituted in.
	$=\left(\frac{-7}{2}\right)^2-2\times2$		
	$=\frac{33}{4}$	A1	**AG** Must be convincingly shown. Correct value from correct working. Must have $\alpha^2+\beta^2$, $\frac{33}{4}$ and at least one line of intermediate working.
		2	

Q	Answer	Marks	Comments
9(b)(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1	Writing $\alpha^3+\beta^3$ in terms of $\alpha+\beta$ and $\alpha\beta$, fully correct. Condone $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$
	$=\left(\frac{-7}{2}\right)^3-3\times2\times\left(\frac{-7}{2}\right)$	m1	Substituting their values into their equation, in terms of $\alpha+\beta$ and $\alpha\beta$.
	$=\frac{-175}{8}$	A1	Correct value from correct working.
		3	

Q	Answer	Marks	Comments
9(c)	$\frac{-b'}{a'} = \alpha^3 + \beta^3 + 2(\alpha + \beta)$ $= \frac{-175}{8} + 2\left(\frac{-7}{2}\right)$	M1	Writing $\alpha' + \beta'$ in terms of $\alpha^3 + \beta^3$ and $\alpha + \beta$, fully correct.
	$= \frac{-231}{8}$	A1	Correct value
	$\alpha^4 + \beta^4 = (\alpha + \beta)^4 - 4(\alpha\beta)(\alpha^2 + \beta^2) - 6(\alpha\beta)^2$	M1	Writing $\alpha^4 + \beta^4$ in terms of $\alpha^2 + \beta^2$, $\alpha + \beta$ and $\alpha\beta$, fully correct. Alternative $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$, fully correct
	$\alpha^4 + \beta^4 = \left(\frac{-7}{2}\right)^4 - 4(2)\left(\frac{33}{4}\right) - 6(2)^2$	m1	Substituting their values into the equation, in terms of $\alpha + \beta$ and $\alpha\beta$ Alternative $\alpha^4 + \beta^4 = \left(\frac{33}{4}\right)^2 - 2(2)^2$
	$\alpha^4 + \beta^4 = \frac{961}{16}$	A1	Correct value from correct working
	$\frac{c'}{a'} = (\alpha\beta)^3 + 2(\alpha^4 + \beta^4) + 4(\alpha\beta)$	M1	Writing $\alpha'\beta'$ in terms of $\alpha^4 + \beta^4$ and $\alpha\beta$, fully correct.
	$= (2)^3 + 2\left(\frac{961}{16}\right) + 4(2)$	m1	Substituting their values into the equation, in terms of $\alpha^4 + \beta^4$ and $\alpha\beta$
	$= \frac{1089}{8}$	A1	Correct value from correct working
	$8x^2 + 231x + 1089 = 0$	A1	Correct equation with integer coefficients
		9	

	Question 9 Total	**16**	

Q	Answer	Marks	Comments
10	Aux. equation $2m^2+5m+3=0$; $m=-1,-\frac{3}{2}$	M1	Forming and solving the correct aux. equation **PI**
	$y_{CF}=Ae^{-x}+Be^{\frac{-3x}{2}}$	A1	Correct CF
	$y_{PI}=axe^{-x}+be^{2x}$	B1	
	$+c\cos x+d\sin x$	B1	
	$y_{PI\text{-}1}=axe^{-x}+be^{2x}$ $y'_{PI\text{-}1}=ae^{-x}-axe^{-x}+2be^{2x}$ $y''_{PI\text{-}1}=-2ae^{-x}+axe^{-x}+4be^{2x}$		
	$2\left(-2ae^{-x}+axe^{-x}+4be^{2x}\right)$ $+5\left(ae^{-x}-axe^{-x}+2be^{2x}\right)$ $+3\left(axe^{-x}+be^{2x}\right)$ $=4e^{-x}+7e^{2x}$	M1	Their $y'_{PI\text{-}1}$ and $y''_{PI\text{-}1}$ or $y'_{PI\text{-}2}$ and $y''_{PI\text{-}2}$ substituted into the differential equation
	$a=4$; $21b=7\Rightarrow b=\frac{1}{3}$		
	$y_{PI\text{-}2}=\ c\cos x+d\sin x$ $y'_{PI\text{-}2}=\ -c\sin x+d\cos x$ $y''_{PI\text{-}2}=\ -c\cos x-d\sin x$		
	$2(-c\cos x-d\sin x)$ $+5(-c\sin x+d\cos x)$ $+3(+c\cos x+d\sin x)$ $=26\cos x$		
	$c+5d=26$; $-5c+d=0$	M1	Forms a pair of simultaneous equations for their $y_{PI\text{-}2}$
	$\Rightarrow c=1, d=5$	A1	At least two of a, b, c, d correct.
	$y_{PI}=4xe^{-x}+\frac{1}{3}e^{2x}+\cos x+5\sin x$	A1	Correct particular integral
	$[y_{GS}=]\,Ae^{-x}+Be^{\frac{-3x}{2}}+4xe^{-x}+\frac{1}{3}e^{2x}+\cos x+5\sin x$	B1ft	Their CF + their PI with exactly two arbitrary constants

	When $x=0$, $y=\frac{1}{3}$ $\Rightarrow A+B=-1$ When $x=0$, $\frac{dy}{dx}=\frac{2}{3}$ $\Rightarrow A+\frac{3}{2}B=9$ $\Rightarrow A=-21;\ B=20$	**A1**	Either $A=-21$ or $B=20$
	$f(x)=-21e^{-x}+20e^{\frac{-3x}{2}}+4xe^{-x}+\frac{1}{3}e^{2x}+\cos x+5\sin x$	**A1**	Must be $f(x)=\ldots$
		11	

	Question 10 Total	**11**	

Q	Answer	Marks	Comments
11	$\mathbf{r}_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{-2}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \end{bmatrix}$	B1	Correct equation for line L_1 **PI** Correct components substituted into equation of the plane
	L_1 $3\left(1+\frac{\mu}{\sqrt{6}}\right)-4\left(2-\frac{2\mu}{\sqrt{6}}\right)+\left(-2-\frac{\mu}{\sqrt{6}}\right)=3$	M1	Substituting components of their L_1 **or** L_2 into equation of the plane and attempting to solve.
	$\mu = \sqrt{6}$	A1	Correct value of μ **or** λ
	$Q = \left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} + \sqrt{6} \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{-2}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \end{bmatrix} \right\} = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$	A1ft	Correct position vector (or coordinates) of Q **ft** their μ and $\mathbf{r}_1$
	L_2 $3(1+2\lambda)-4(2+\lambda)+(-2+3\lambda)=3$	(M1)	Substituting components of their L_1 **or** L_2 into equation of the plane and attempting to solve. Can only score once.
	$\lambda = 2$	(A1)	Correct value of μ **or** λ Can only score once.
	$R = \left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \right\} = \begin{bmatrix} 5 \\ 4 \\ 4 \end{bmatrix}$	A1	**CAO**

	$\left[\overrightarrow{PQ}=\right]\begin{bmatrix}1\\-2\\-1\end{bmatrix}$, $\left[\overrightarrow{PR}=\right]\begin{bmatrix}4\\2\\6\end{bmatrix}$, $\left[\overrightarrow{QR}=\right]\begin{bmatrix}3\\4\\7\end{bmatrix}$	**B1ft**	Two correct directional vectors ft their Q & R.
	$\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{bmatrix}1\\-2\\-1\end{bmatrix}\times\begin{bmatrix}4\\2\\6\end{bmatrix}=\begin{bmatrix}-10\\-10\\10\end{bmatrix}$	**M1**	Cross product of their directional vectors. Or scalar product of their directional vectors $\cos QPR=\frac{-\sqrt{21}}{14}$
	Area $=\frac{1}{2}\sqrt{(-10)^2+(-10)^2+10^2}$	**M1**	Uses correct formula for area of a triangle with their cross product. Or $\frac{1}{2}\left\|\overrightarrow{PQ}\right\|\left\|\overrightarrow{PR}\right\|\sin QPR=\frac{1}{2}\sqrt{6}\times\sqrt{56}\times\sqrt{\frac{25}{28}}$, but $\sin QPR$ MUST be exact, $\sin QPR=\sqrt{\frac{25}{28}}$
	Area $=5\sqrt{3}$	**A1**	**AG** Must be convincingly shown.
		9	

	Question 11 Total	**9**	

Q	Answer	Marks	Comments
12(a)	$\cos 4\theta + \text{i}\sin 4\theta = (\cos\theta + \text{i}\sin\theta)^4$ $= \cos^4\theta + \text{i}4\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta$ $- \text{i}4\cos\theta\sin^3\theta + \sin^4\theta$	M1	Explicitly uses de Moivre's theorem and correctly expands $(\cos\theta + \text{i}\sin\theta)^4$ **PI** correct real part of the expansion
	$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$		
	$\cos 4\theta = \cos^4\theta - 6\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$	M1	Equates real parts and uses $\sin^2\theta = 1-\cos^2\theta$
	$\cos 4\theta = \cos^4\theta - 6\cos^2\theta + 6\cos^4\theta + 1 - 2\cos^2\theta + \cos^4\theta$		Must see this line **oe**
	$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	A1	**AG** Must be convincingly shown using de Moivre's theorem, including starting with $\cos 4\theta + \text{i}\sin 4\theta = (\cos\theta + \text{i}\sin\theta)^4$
		3	

Q	Answer	Marks	Comments
12(b)	Let $\cos 4\theta = 0$ $\therefore \quad 4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$	B1	Finding the values of θ
	Let $x = \cos\theta$ $\therefore$ $8x^4 - 8x^2 + 1 = 0$	M1	Showing the link to the quadratic in x^2 **PI** correct roots
	Where $x = \cos\left(\frac{\pi}{8}\right), \cos\left(\frac{3\pi}{8}\right)$ $, \cos\left(\frac{5\pi}{8}\right), \cos\left(\frac{7\pi}{8}\right)$	A1	
			If **M0M0A0** **SC1** for verifying $\theta = \frac{\pi}{8}$ gives $\cos 4\theta = 0$
		3	

Q	Answer	Marks	Comments
12(c)(i)	$\alpha\beta\gamma\delta = \frac{e}{a}$ $\cos\left(\frac{\pi}{8}\right) \times \cos\left(\frac{3\pi}{8}\right) \times \cos\left(\frac{5\pi}{8}\right) \times \cos\left(\frac{7\pi}{8}\right) = \frac{1}{8}$	M1	Uses $\alpha\beta\gamma\delta = \frac{e}{a}$
	$\cos\left(\frac{7\pi}{8}\right) = -\cos\left(\frac{\pi}{8}\right)$ and $\cos\left(\frac{5\pi}{8}\right) = -\cos\left(\frac{3\pi}{8}\right)$	M1	
	$\cos\left(\frac{\pi}{8}\right) \times \cos\left(\frac{3\pi}{8}\right) \times \left(-\cos\left(\frac{3\pi}{8}\right)\right) \times \left(-\cos\left(\frac{\pi}{8}\right)\right) = \frac{1}{8}$		Must see this line **oe**
	$\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{3\pi}{8}\right) = \frac{1}{8}$	A1	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
12(c)(ii)	$x^2 = \frac{2+\sqrt{2}}{4}$ or $x^2 = \frac{2-\sqrt{2}}{4}$	M1	Solves the quadratic in x^2 stating **BOTH** solutions of x^2
	Now $\cos\frac{\pi}{8} > \cos\frac{3\pi}{8} > 0$ $\therefore$ $\cos^2\frac{\pi}{8} = \frac{2+\sqrt{2}}{4}$	m1	Justifies the rejection of $x^2 = \frac{2-\sqrt{2}}{4}$ by considering relative values of $\cos\frac{\pi}{8}$, $\cos\frac{3\pi}{8}$ and 0
	Now $\cos\frac{\pi}{8} > 0 > \cos\frac{7\pi}{8}$ $\therefore$ $\cos\frac{\pi}{8} = \sqrt{\frac{2+\sqrt{2}}{4}}$	A1	Justifies the rejection of $x = -\sqrt{\frac{2+\sqrt{2}}{4}}$ by considering relative values of $\cos\frac{\pi}{8}$, 0 and $\cos\frac{7\pi}{8}$ **AG** Must be convincingly shown
		3	

	Question 12 Total	**12**	

Q	Answer	Marks	Comments
13(a)	$y = \text{sech}\, x = \dfrac{1}{\cosh x}$		
	$\left(\dfrac{dy}{dx} = \right)\dfrac{(0 \times \cosh x) - 1 \times \sinh x}{(\cosh x)^2}$	M1	oe
	$\dfrac{dy}{dx} = \dfrac{-\sinh x}{\cosh^2 x}$ $= -\text{sech}\, x \tanh x$	A1	**AG** Must be convincingly shown, including $\dfrac{dy}{dx}$ and $-\text{sech}\, x \tanh x$
		2	

Q	Answer	Marks	Comments
13(b)	$\dfrac{4\sinh\theta - 2}{\cosh\theta} = \cosh\theta$		
	$4\sinh\theta - 2 = \cosh^2\theta = 1 + \sinh^2\theta$	M1	Equating C_1 & C_2 and use of $\cosh^2\theta = 1 + \sinh^2\theta$ to obtain an equation in only $\sinh\theta$
	$0 = \sinh^2\theta - 4\sinh\theta + 3$		
	$\sinh\theta = 1, 3$	A1	Forming a quadratic in $\sinh\theta$ and solving to get $\sinh\theta = 1, 3$
	$A_1 = \dfrac{1}{2}\displaystyle\int \left(\dfrac{4\sinh\theta - 2}{\cosh\theta}\right)^2 d\theta$ or $A_2 = \dfrac{1}{2}\displaystyle\int \cosh^2\theta\, d\theta$	B1	Use of $\dfrac{1}{2}\displaystyle\int r^2 [d\theta]$ for either part
	$A_1 = k\displaystyle\int \left(8\tanh^2\theta - 8\tanh\theta\,\text{sech}\,\theta + 2\text{sech}^2\theta\right) d\theta$	M1	
	$A_1 = k\displaystyle\int \left(8 - 8\text{sech}^2\theta - 8\tanh\theta\,\text{sech}\,\theta + 2\text{sech}^2\theta\right) d\theta$ $= k\displaystyle\int \left(8 - 6\text{sech}^2\theta - 8\tanh\theta\,\text{sech}\,\theta\right) d\theta$	M1	Use of $\tanh^2\theta = 1 - \text{sech}^2\theta$

	$A_1 = k\left(8\theta - 6\tanh\theta + 8\operatorname{sech}\theta\right)$	A1	
	$A_1 = k\left(8\ln\left(3+\sqrt{10}\right) - 6\times\frac{3}{\sqrt{10}} + \frac{8}{\sqrt{10}} - 8\ln\left(1+\sqrt{2}\right) + 6\times\frac{1}{\sqrt{2}} - \frac{8}{\sqrt{2}}\right)$		
	$A_1 = 8\ln\left(\frac{3+\sqrt{10}}{1+\sqrt{2}}\right) - \sqrt{10} - \sqrt{2}$	A1	
	$A_2 = \frac{k}{4}\int\left(1+\cosh 2\theta\right)\mathrm{d}\theta$	M1	Use of $\cosh^2\theta = \frac{1}{2}\left(1+\cosh 2\theta\right)$ or $\cosh^2\theta = \frac{1}{4}\left(e^{2\theta} + e^{-2\theta} + 2\right)$
	$A_2 = \frac{k}{4}\left(\theta + \frac{1}{2}\sinh 2\theta\right)$	A1	$\frac{k}{16}\left(e^{2\theta} - e^{-2\theta} + 4\theta\right)$
	$A_2 = \frac{k}{4}\left(\theta + \sinh\theta\cosh\theta\right)$ $= \frac{k}{4}\left(\ln\left(3+\sqrt{10}\right) + 3\sqrt{10} - \ln\left(1+\sqrt{2}\right) - \sqrt{2}\right)$		
	$A_2 = \frac{1}{4}\ln\left(\frac{3+\sqrt{10}}{1+\sqrt{2}}\right) + \frac{3\sqrt{10}}{4} - \frac{\sqrt{2}}{4}$	A1	
	$R = A_1 - A_2$ $= 8\ln\left(\frac{3+\sqrt{10}}{1+\sqrt{2}}\right) - \sqrt{10} - \sqrt{2} - \frac{1}{4}\ln\left(\frac{3+\sqrt{10}}{1+\sqrt{2}}\right) - \frac{3\sqrt{10}}{4} + \frac{\sqrt{2}}{4}$		
	$R = \frac{31}{4}\ln\left(\frac{3+\sqrt{10}}{1+\sqrt{2}}\right) - \frac{7\sqrt{10}}{4} - \frac{3\sqrt{2}}{4}$	A1	**AG** Must be convincingly shown
		11	

	Question 13 Total	**13**	