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FURTHER MATHEMATICS

FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from www.oxfordaqa.com

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)(i)	$\|z\| = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}$	**M1**	Full method for the modulus of z
	$= 2$		
	$\arg(z) = \tan^{-1}\left(\frac{-1}{-\sqrt{3}}\right) - \pi$	**M1**	Sight of $\tan^{-1}\left(\frac{-1}{-\sqrt{3}}\right)$ **oe** **PI** by $\frac{\pi}{6}$ or $\frac{7\pi}{6}$ or $-\frac{5\pi}{6}$ Condone use of degrees
	$= -\frac{5\pi}{6}$		
	$z = 2\left(\cos\left(-\frac{5\pi}{6}\right) + \text{i}\sin\left(-\frac{5\pi}{6}\right)\right)$	**A1**	Accept unbracketed arguments Condone missing final bracket
		3	

Q	Answer	Marks	Comments
1(a)(ii)	$z^* = 2\left(\cos\left(\frac{5\pi}{6}\right) + \text{i}\sin\left(\frac{5\pi}{6}\right)\right)$	**B1ft**	**ft** their z in polar form condoning arguments outside the range $-\pi < \theta \le \pi$
		1	

Q	Answer	Marks	Comments
1(b)	$2 \times \frac{\pi}{6}$	**M1**	Full method for either angle *POQ* eg $2 \times \tan^{-1}\left(\frac{-1}{-\sqrt{3}}\right)$
	$= \frac{\pi}{3}$	**A1ft**	Accept 60° **ft** their θ and $-\theta$
		2	

	Question 1 total	**6**	

Q	Answer	Marks	Comments
2(a)	gradient $=\dfrac{8-3(7+h)-6(7+h)^2-(-307)}{(7+h)-7}$	M1 M1	Subtracts the y-coordinates May be unsimplified Writes an expression for the gradient of L May be unsimplified
	$=\dfrac{8-21-3h-6(49+14h+h^2)+307}{h}$		
	$=-87-6h$	A1	
		3	

Q	Answer	Marks	Comments
2(b)	gradient $=\lim\limits_{h\to 0}(-87-6h)$	M1	Considers their part **(a)** as $h\to 0$ Condone $h=0$ seen Do not accept $n\to 0$ or $x\to 0$
	$=-87$	A1ft	**ft** their $p+qh$ **A0** if $h=0$ seen and no limiting process used
		2	

	Question 2 total	**5**	

Q	Answer	Marks	Comments
3(a)	$\frac{1}{3r-1}-\frac{1}{3r+2}=\frac{3r+2-(3r-1)}{(3r-1)(3r+2)}$ $=\frac{3}{9r^2+3r-2}$	B1	Must include at least one intermediate stage
		1	

Q	Answer	Marks	Comments
3(b)	$\sum_{r=1}^{n}\frac{3}{9r^2+3r-2}=\sum_{r=1}^{n}\left(\frac{1}{3r-1}-\frac{1}{3r+2}\right)$		
	$=\frac{1}{2}-\frac{1}{5}$ $+\frac{1}{5}-\frac{1}{8}$	M1	Writes at least two pairs of fractions of the form $\frac{1}{3r-1}-\frac{1}{3r+2}$
	$+\frac{1}{8}-\frac{1}{11}$ $+$ .... $+\frac{1}{3n-1}-\frac{1}{3n+2}$	M1	Writes the first pair, the last pair, and at least one other pair of fractions of the form $\frac{1}{3r-1}-\frac{1}{3r+2}$ Must include at least one pair of cancelling fractions
	$=\frac{1}{2}-\frac{1}{3n+2}$	A1	Writes a correct expression in terms of n
	$=\frac{3n+2-2}{2(3n+2)}$		
	$=\frac{3n}{6n+4}$	A1	**CAO** from use of method of differences **SC1** for a correct expression with method of differences not explicitly shown **SC2** for a correct expression in the required form with method of differences not explicitly shown
		4	

Q	Answer	Marks	Comments
3(c)	$\sum_{r=1}^{\infty} \frac{3}{9r^2+3r-2} = \frac{3}{6} = \frac{1}{2}$	B1ft	Correctly calculates the sum to infinity **ft** their $\frac{3n}{cn+d}$ **PI** by a correct equation in terms of p only
	$\sum_{r=p}^{\infty} \frac{3}{9r^2+3r-2}$ $= \sum_{r=1}^{\infty} \frac{3}{9r^2+3r-2} - \sum_{r=1}^{p-1} \frac{3}{9r^2+3r-2}$ $= \frac{1}{2} - \frac{3(p-1)}{6(p-1)+4}$	M1	Substitutes $(p-1)$ into their $\frac{3n}{cn+d}$
	$\frac{1}{3p-1} = \frac{1}{1001}$ $p = 334$	A1	
		3	

	Question 3 total	**8**	

Q	Answer	Marks	Comments
4(a)	$k \le 0$	B1	
		1	

Q	Answer	Marks	Comments
4(b)	$\int\left(\frac{1}{x^2}\right)\mathrm{d}x = -\frac{1}{x}\ [+c]$ As $x \to 0$ then $[-]\frac{1}{x} \to [-]\infty$	E1	Identifies integrated result is undefined at $x = 0$
		1	

	Question 4 total	**2**	

Q	Answer	Marks	Comments
5	Let $z = x + \mathrm{i}y$ [where $x, y \in \mathbb{R}$]	M1	Chooses a suitable method **PI** by a rejection of imaginary roots for their x and y values
	$(x+\mathrm{i}y)^2 = 4 - 4\sqrt{15}\,\mathrm{i}$		
	$x^2 - y^2 + 2xy\mathrm{i} = 4 - 4\sqrt{15}\,\mathrm{i}$	A1	Obtains a correctly expanded expression for z^2 in terms of their x and y, with i^2 replaced with –1 **PI** by correctly compared real parts or correct values for their x and y
	Comparing real and imaginary parts:		
	$x^2 - y^2 = 4$ and $2xy = -4\sqrt{15}$	M1	Obtains at least one correct equation in x and y
	$x^2 - \left(-\dfrac{2\sqrt{15}}{x}\right)^2 = 4$ $x^4 - 4x^2 - 60 = 0$		
	$x^2 = 10$ or $x^2 = -6$		
	But x is real, so $x^2 \geq 0$		
	$x = \pm\sqrt{10}$	A1	Obtains at least one correct value for x **or** y
	$y = -\dfrac{2\sqrt{15}}{\pm\sqrt{10}} = \mp\sqrt{6}$		
	$z = \pm\left(\sqrt{10} - \sqrt{6}\,\mathrm{i}\right)$	A1	Writes both answers unambiguously with no incorrect answers
		5	

	Question 5 total	**5**	

Q	Answer	Marks	Comments
6(a)	$(2r)^3 - 3(2r)^2 + 3(2r) - 1$ $= 8r^3 - 12r^2 + 6r - 1$	B1	
		1	

Q	Answer	Marks	Comments
6(b)	$\sum_{r=1}^{n}(2r-1)^3$ $= 8\sum_{r=1}^{n} r^3 - 12\sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$		
	$= 8\times\frac{1}{4}n^2(n+1)^2 - 12\times\frac{1}{6}n(n+1)(2n+1)$ $+ 6\times\frac{1}{2}n(n+1) - n$	M1 M1	Substitutes the standard summation formulae for $\sum r^3$, $\sum r^2$ and $\sum r$ Replaces $\sum 1$ with n
	$= n\left(2n(n^2+2n+1) - 2(2n^2+3n+1) + 3(n+1) - 1\right)$	m1	Factorising their $\sum_{r=1}^{n}(2r-1)^3$ to give a factor of n or n^2 May be unsimplified Dependent on both previous **M1** marks
	$= n(2n^3 - n)$		
	$= n^2(2n^2 - 1)$	A1	**CAO**
		4	

Q	Answer	Marks	Comments
6(c)	$50^2\left(2\times 50^2-1\right)$ $=12\,497\,500$	**M1** **A1**	Substitutes $n=50$ into their **part (b)** **PI** by correct answer
		2	

	Question 6 total	**7**	

Q	Answer	Marks	Comments
7(a)	n can be any integer	**B1**	**oe**, such as $n \in$ ¢
		1	

Q	Answer	Marks	Comments
7(b)	$3x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{3}$	**M1**	Obtains a correct non-trigonometric general equation in x May be seen in part **(c)**
	$x = \frac{\pi}{36}\left(24n \pm 4 - 3\right)$	**A1**	Obtains a correct expression for x in the form $\frac{\pi}{36}\left(24n \pm p + q\right)$ where p and q are integers May be seen in part **(c)** **PI** by an integer answer of the form $24m - 3$ where m is an integer
	$x = \frac{\pi}{36}\left(24n \pm 4 - 3 + 24\right)$		
	$b = 21$	**A1**	**CAO**
		3	

Q	Answer	Marks	Comments
7(c) **ALT1**	$a = \frac{36}{3} + 4$ or $a = \frac{36}{3} - 4$	**M1**	Writes a value or expression equivalent to a multiple of 8 that is not a multiple of 3
	$a = \frac{36}{3} - 4$ or $a = \frac{36}{3} + 4 - 24$	**A1**	Writes an equation in a equivalent to $a = 8$ or $a = -8$ **PI** by an integer answer of the form $24m \pm 8$ or $24m \pm 16$ where m is an integer
	$a = 8$	**A1**	**CAO**
		3	

Q	Answer	Marks	Comments
7(c) ALT2		**M1**	Forms an equation of the form $+a+9=$ their $24n \pm p + q$ or $-a+9=$ their $24n \pm p + q$
	$+a+9=1$ or $+a+9=17$ or $-a+9=1$ or $-a+9=17$	**A1**	Writes an equation in a equivalent to $a=8$ or $a=-8$ **PI** by an integer answer of the form $24m \pm 8$ or $24m \pm 16$ where m is an integer
	$a=8$	**A1**	**CAO**
		3	

Q	Answer	Marks	Comments
7(c) ALT3	$3x + \frac{\pi}{4} = (2n+1)\pi \pm \frac{2\pi}{3}$	**M1**	Obtains a correct non-trigonometric general equation in x if not awarded in part **(b)**
	$x = \frac{\pi}{36}(24n \pm 8 + 9)$	**A1**	Obtains a correct expression for x in the form $\frac{\pi}{36}(24n \pm p + q)$ where p and q are integers if not awarded in part **(b)** **PI** by an integer answer of the form $24m \pm 8$ or $24m \pm 16$ where m is an integer
	$a=8$	**A1**	**CAO**
		3	

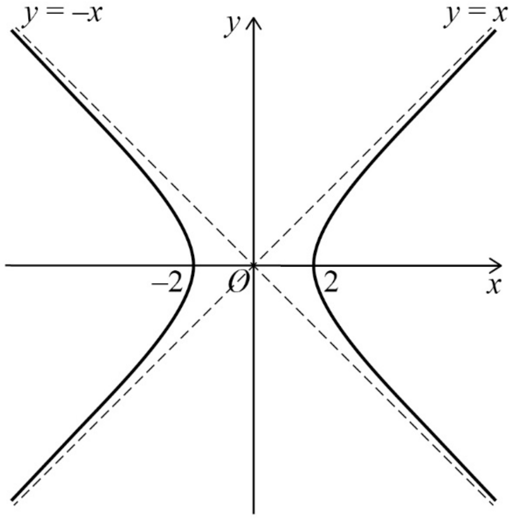
Q	Answer	Marks	Comments
7(d)	2×3	**M1**	Identifies at least three correct solutions in the interval $-\pi < x < \pi$ $-\frac{31\pi}{36}, -\frac{23\pi}{36}, -\frac{7\pi}{36}, \frac{\pi}{36}, \frac{17\pi}{36}, \frac{25\pi}{36}$
	$= 6$	**A1**	
		2	

Q	Answer	Marks	Comments
7(e)	$6 \times \frac{80}{2}$	**M1**	Multiplies their part **(d)** by 40
	$= 240$	**A1ft**	**ft** their part **(d)**
		2	

	Question 7 total	**11**	

Q	Answer	Marks	Comments
8(a)	$\sqrt{(x-2k)^2+y^2}=\pm\left(x-\frac{2}{k}\right)\times k$	M1	Writes a correct expression for RS or RS^2
		A1	Writes a correct equation for H Condone missing $\pm$
	$(x-2k)^2+y^2=k^2\left(x^2-\frac{4x}{k}+\frac{4}{k^2}\right)$	M1	Removes the square root correctly
	$x^2-4xk+4k^2+y^2=k^2x^2-4xk+4$		
	$(k^2-1)x^2-y^2=4k^2-4$	A1	Obtains the correct equation in the required form
		4	

Q	Answer	Marks	Comments
8(b)(i)	$\frac{x^2}{4}-\frac{y^2}{4(k^2-1)}=1$		
	Asymptotes are $\frac{x}{2}=\pm\frac{y}{2\sqrt{k^2-1}}$	M1	Identifies a correct asymptote equation or a gradient for their asymptotes **oe** eg $\pm x\sqrt{k^2-1}=y$
	$-\frac{2\sqrt{k^2-1}}{2}\times\frac{2\sqrt{k^2-1}}{2}=-1$	M1	Writes a correct equation in k for their $px^2-y^2=q$
	$k^2=2$		
	$k>1$ so $k=\sqrt{2}$	A1	
		3	

Q	Answer	Marks	Comments
8(b)(ii)		**B1** **B1** **B1**	Two asymptotes drawn with correct equations Correct x-axis intercepts Correct hyperbolic shape with correct curve behaviour relative to the asymptotes
		3	

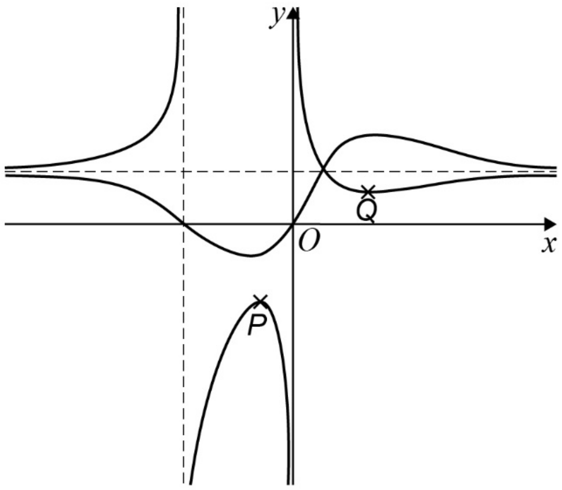
	Question 8 total	**10**	

Q	Answer	Marks	Comments
9(a)	$k=\dfrac{x^2+2}{x^2+3x}$		
	$kx^2+3kx-x^2-2=0$	M1	Forms a quadratic equation in x in terms of k
	$(k-1)x^2+3kx-2=0$		
	$(3k)^2-4(k-1)(-2)\geq 0$	M1	Obtains an unsimplified discriminant in terms of k for their quadratic equation
		m1	Sets their discriminant ≥ 0
	$9k^2+8k-8\geq 0$	A1	Must be convincingly shown
		4	

Q	Answer	Marks	Comments
9(b)	$k\leq\dfrac{-4-2\sqrt{22}}{9}$, $k\geq\dfrac{-4+2\sqrt{22}}{9}$	M1	Solves their $9k^2+8k+n=0$ **PI** by one correct **ft** value of k **PI AWRT** –1.5 or 0.6
	$y_{\text{max}}=\dfrac{-4-2\sqrt{22}}{9}$ $y_{\text{min}}=\dfrac{-4+2\sqrt{22}}{9}$	A1ft	Identifies the correct y-coordinates **ft** their **part (a)**
		2	

Q	Answer	Marks	Comments
9(c)(i)	1	B1	Condone $y=1$ only
		1	

Q	Answer	Marks	Comments
9(c)(ii)	1	B1	
		1	

Q	Answer	Marks	Comments
9(c)(iii)		B1 B1 B1 B1	Exactly two x-intercepts at the intersections of the vertical asymptotes with the x-axis A minimum point vertically above *P* and below the x-axis **and** A maximum point vertically above *Q* and above the horizontal asymptote **and** No other stationary points Only intersects C_1 at the horizontal asymptote intersection Graph tends to the horizontal asymptote from the correct side as $x \to \pm\infty$
		4	

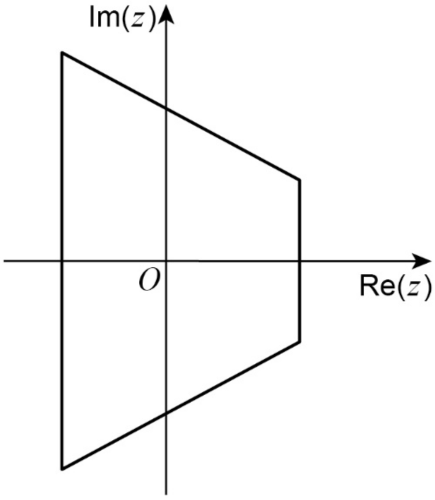
	Question 9 total	**12**	

Q	Answer	Marks	Comments
10(a) ALT1	The 2nd root is $4+5\text{i}$	**M1**	Correctly identifies the 2nd root
	$-m = 4+5\text{i}+4-5\text{i}$ and $n = (4+5\text{i})(4-5\text{i})$	**m1**	Correctly uses the sum or the product of roots
	$m=-8$	**A1**	
	$n = 41$	**A1**	
		4	

Q	Answer	Marks	Comments
10(a) ALT2	$(4-5\text{i})^2+m(4-5\text{i})+n=0$	**M1**	Substitutes $4-5\text{i}$ (or $4+5\text{i}$) into z^2+mz+n
	$16-40\text{i}-25+4m-5m\text{i}+n=0$ Comparing real and imaginary parts $-40-5m=0$ and $-9+4m+n=0$	**m1**	Compares real and imaginary parts to obtain at least one correct equation in m and/or n
	$m=-8$	**A1**	
	$n = 41$	**A1**	
		4	

Q	Answer	Marks	Comments
10(a) ALT3	$z = 4 \pm 5\mathrm{i}$	M1	Correctly identifies the 2nd root
	$z - 4 = \pm 5\mathrm{i}$		
	$(z-4)^2 = -25$	m1	Squares $z-4$ to form a quadratic equation with real coefficients
	$z^2 - 8z + 41 = 0$		
	$m = -8$	A1	
	$n = 41$	A1	
		4	

Q	Answer	Marks	Comments
10(b)	$-3+12\mathrm{i}$, $-3-12\mathrm{i}$	B1	Obtains both roots
		1	

Q	Answer	Marks	Comments
10(c)(i)		**M1** **A1**	Any pair of points vertically aligned and equidistant from the real axis Draws the correct trapezium symmetrical about the real axis
		2	

Q	Answer	Marks	Comments
10(c)(ii)	Area $= \frac{1}{2}\times(4--3)\times(10+24)$ $= 119$	**M1** **A1**	Full method for the required area for their conjugate roots
		2	

Q	Answer	Marks	Comments
10(d)(i) ALT1	$(\alpha--3)^2+12^2=(\alpha-4)^2+5^2$ $6\alpha+153=-8\alpha+41$ $\alpha=-8$	**B1** **M1** **A1**	Uses α as a real number **PI** by a correct answer or an equation in α with a real solution Full method for the centre of the circumcircle of their conjugate roots
		3	

Q	Answer	Marks	Comments
10(d)(i) ALT2	$y-\frac{12+5}{2}=-\frac{-3-4}{12-5}\left(x-\frac{-3+4}{2}\right)$	**B1**	Obtains an equation of at least one perpendicular bisector of a chord eg $y=0$ May be unsimplified
	$y=x+8$		
	$0=\alpha+8$	**M1**	Finds the intersection point of the perpendicular bisectors of two chords eg Substitutes $y=0$ into a Cartesian equation of a perpendicular bisector of a non-vertical chord
	$\alpha=-8$	**A1**	
		3	

Q	Answer	Marks	Comments
10(d)(ii)	Radius $=\sqrt{(-8+3)^2+12^2}$	**M1**	Full method for the radius (or radius^2) of the circumcircle of their conjugate roots
	$\beta=13$	**A1**	
		2	

	Question 10 total	**14**	