OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Wednesday 4 June 2025 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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8	
9	
TOTAL	

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Answer **all** questions in the spaces provided.

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1 The delivery times in days for a company are known to have a normal distribution.

The company takes a random sample of 37 deliveries and records the delivery time for each.

The unbiased estimate of the standard deviation calculated using the sample is 0.35 days.

The company claims that the standard deviation of the delivery times is less than 0.5 days.

Test the company's claim using the 1% level of significance.

[6 marks]

6

Turn over ►

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2 The continuous random variable X has probability density function

$$f(x) = \begin{cases} 0.1x & 0 \leq x \leq 4 \\ 2 - 0.4x & 4 < x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

2 (a) Show that $M_X(t)$, the moment generating function of X, is given by

$$M_X(t) = \frac{1 - 5e^{4t} + 4e^{5t}}{10t^2}$$

[6 marks]

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2 (b) The random variable Y has moment generating function

$$\mathrm{M}_Y(t) = \frac{\mathrm{e}^{5t} - 1}{5t}$$

The random variables X and Y are independent.

Find $\mathrm{M}_{X+Y}(t)$, the moment generating function of $X + Y$

Give your answer in the form

$$\mathrm{M}_{X+Y}(t) = \frac{a\,\mathrm{e}^{10t} + b\,\mathrm{e}^{9t} + c\,\mathrm{e}^{5t} + d\,\mathrm{e}^{4t} - 1}{50t^3}$$

where a, b, c and d are integers.

[3 marks]

Answer _____

9

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3 The total distance in miles that a particular type of electric car can travel on one complete charge is known to have a normal distribution.

A random sample of 5 electric cars of this type is taken.

The total distance travelled on one complete charge by each car is measured on two days: one hot day and one cold day.

The cars are driven by the same drivers and follow the same route on each day.

The results are given in the table below.

	Total distance travelled (miles)	
Car	**Hot**	**Cold**
A	218	210
B	291	298
C	193	187
D	232	230
E	208	197

The producer of the car claims that the car can travel a greater distance on one complete charge on a hot day than on a cold day.

Test the producer's claim using the 5% level of significance.

[10 marks]

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10

Turn over for the next question

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4 The price of a precious metal changes each day.

The random variable X is the logarithm of the price of the precious metal at the end of each day. X is known to have a normal distribution.

A random sample of 4 observations of X is taken and $\sum(x-\bar{x})^2 = 0.0075$

The random sample is used to construct a 90% confidence interval for the population mean of X

4 (a) Find the width of the confidence interval.

Give your answer to three significant figures.

[4 marks]

Answer _____

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4 (b) The confidence interval supports the claim that the population mean of X is 2.7

Find the smallest possible value for the sample mean of the random sample.

Give your answer to three significant figures.

[2 marks]

Answer _____

6

Turn over for the next question

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5 A television show is available to all users of an online service.

The makers of the show claim that 20% of users watch the show.

A random sample of 20 users is taken and each is asked if they watch the show.

A hypothesis test is carried out with the hypotheses

$$H_0 : p = 0.2$$
$$H_1 : p \neq 0.2$$

The test is carried out at the 5% level of significance.

5 (a) It is given that $p = 0.15$

Find the probability that a Type II error is made.

Give your answer to three decimal places.

[5 marks]

Answer _____

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5 (b) Find the power of the test.

Give your answer to three decimal places.

[1 mark]

Answer _____

6

Turn over for the next question

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6 A bag contains white balls and black balls.

There are a total of 120 balls in the bag.

In a game, a ball is taken out of the bag at random and its colour is recorded.

The ball is then returned to the bag.

If the colour of the ball is black then the player scores 10 points.

If the colour of the ball is white then the player scores 0 points.

The game is played three times.

The sampling distribution M of the highest score achieved in any one of the three games is given in the table below.

m	0	10
$\mathrm{P}(M = m)$	0.042875	0.957125

Find the number of black balls in the bag.

[5 marks]

Answer _____

5

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7 The length of films released in cinemas in country *A* and the length of films released in cinemas in country *B* are both known to have normal distributions.

7 (a) Gan takes a random sample of 160 films released in cinemas in country *A* and their length X minutes is recorded.

For this sample

$$\sum x = 18\,000 \quad \text{and} \quad \sum x^2 = 2\,502\,000$$

He also takes a random sample of 150 films released in cinemas in country *B* and their length Y minutes is recorded.

For this sample

$$\sum y = 15\,480 \quad \text{and} \quad \sum y^2 = 2\,033\,212$$

Gan assumes that the population variance of X is **not** equal to the population variance of Y

He claims that films released in cinemas in country *A* are on average longer than films released in cinemas in country *B*

Test Gan's claim using the 5% level of significance.

[10 marks]

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7 (b) Jenna takes a random sample of 6 films released in cinemas in country *A* and their length U minutes is recorded.

She also takes a random sample of 7 films released in cinemas in country *B* and their length V minutes is recorded.

The difference between Jenna's sample mean for country *A* and her sample mean for country *B* is 10.2 minutes.

She uses her sample to calculate unbiased estimates for the population variances.

$$s_U^2 = 3005 \quad \text{and} \quad s_V^2 = 2939$$

She assumes that the population variances for the lengths of films released in cinemas are the same for both countries.

Jenna uses her sample to test Gan's claim using the 5% level of significance.

Determine whether Jenna reaches the same conclusion as Gan.

[5 marks]

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15

Turn over for the next question

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8 The random variable X has population mean μ and population variance σ^2

8 (a) The random variables A and B are unbiased estimators of μ

The variance of A is $6\sigma^2$

The variance of B is $c\sigma^2$ where c is a constant.

The efficiency of B relative to A is equal to 10

Find the value of c

[3 marks]

Answer _____

8 (b) A random sample X_1, X_2, X_3, …, X_n is taken from X

The random variable

$$Y = 1^2 \times X_1 + 2^2 \times X_2 + 3^2 \times X_3 + ... + n^2 \times X_n$$

is a pooled estimator of μ

Show that Y is a **biased** estimator of μ

[3 marks]

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8 (c) The random variable kY is an **unbiased** estimator of μ where k is a constant.

Find k in terms of n

[4 marks]

Answer _____

10

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9 A company records how many online chat questions per hour they receive from customers over a randomly selected sample period of 38 hours.

The results are shown in the following table.

Questions per hour	0	1	2	3 or more
Frequency	11	7	8	12

The sample mean number of questions per hour is 1.5

The company's data manager claims that the number of online chat questions per hour follows a Poisson distribution.

9 (a) Explain why the test statistic for a hypothesis test investigating the data manager's claim has 2 degrees of freedom.

[2 marks]

9 (b) Test the data manager's claim using the 5% level of significance.

[8 marks]

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9 (c) A member of the data manager's team claims that the number of on-line chat questions per hour follows a Poisson distribution with mean 1.5

The team member carries out a hypothesis test with the hypotheses

H_0 : Number of online chat questions per hour follows a Poisson distribution with mean 1.5

H_1 : Number of online chat questions per hour does not follow a Poisson distribution with mean 1.5

The team member uses the 5% level of significance and the same data as the manager.

Determine whether the team member rejects their null hypothesis.

[3 marks]

13

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