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FURTHER MATHEMATICS

FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$H_0 : \sigma = 0.5$ $H_1 : \sigma < 0.5$	**B1**	Both hypotheses, **oe**
	$\frac{(37-1)\times 0.35^2}{0.5^2}$	**M1**	**PI** Condone not squaring 0.35 **and** 0.5
	$= 17.64$	**A1**	**oe**
	$\chi^2_{36}(0.01) = 19.223$	**B1**	Finds correct critical value or correct probability, **AWRT** 0.0044 Allow 19.233
	$17.64 < 19.223$ Reject H_0	**A1ft**	Correctly compares their chi squared test statistic and their critical value or their probability and 0.01 and makes the correct conclusion as to whether to reject the null hypothesis
	Sufficient evidence to suggest that the standard deviation of delivery times is less than 0.5 days	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and the correct critical value Condone definite conclusions

	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2(a)	$\mathrm{M}_X(t) = \int_0^4 0.1x\,\mathrm{e}^{tx}\,\mathrm{d}x + \int_4^5 (-0.4x+2)\,\mathrm{e}^{tx}\,\mathrm{d}x$	M1	Applies mgf formula
	$= \left[\frac{0.1x}{t}\mathrm{e}^{tx}\right]_0^4 - \int_0^4 \frac{0.1}{t}\mathrm{e}^{tx}\,\mathrm{d}x + \left[\frac{-0.4x+2}{t}\mathrm{e}^{tx}\right]_4^5 + \int_4^5 \frac{0.4}{t}\mathrm{e}^{tx}\,\mathrm{d}x$	M1	Applies integration by parts formula correctly to one of the two integrals **oe**
	$= \left[\frac{0.1x}{t}\mathrm{e}^{tx} - \frac{0.1}{t^2}\mathrm{e}^{tx}\right]_0^4$	A1	Correctly integrates first integral Correct integrals of components may be seen on separate lines
	$+ \left[\frac{-0.4x+2}{t}\mathrm{e}^{tx} + \frac{0.4}{t^2}\mathrm{e}^{tx}\right]_4^5$	A1	Correctly integrates second integral Correct integrals of components may be seen on separate lines
	$= \frac{0.1}{t^2} - \frac{0.1}{t^2}\mathrm{e}^{4t} + \frac{0.4}{t^2}\mathrm{e}^{5t} - \frac{0.4}{t^2}\mathrm{e}^{4t}$	M1	Applies limits May be applied to the different components on separate lines
	$= \frac{0.1}{t^2} - \frac{0.5}{t^2}\mathrm{e}^{4t} + \frac{0.4}{t^2}\mathrm{e}^{5t}$ $= \frac{1-5\mathrm{e}^{4t}+4\mathrm{e}^{5t}}{10t^2}$	A1	**AG** Must be convincingly shown
		6	

Q	Answer	Marks	Comments
2(b)	$\mathrm{M}_{X+Y}(t) = \frac{1-5\mathrm{e}^{4t}+4\mathrm{e}^{5t}}{10t^2} \times \frac{\mathrm{e}^{5t}-1}{5t}$	M1	Identifies $\mathrm{M}_{X+Y}(t)$ as equal to the mgfs multiplied together
	$= \frac{\mathrm{e}^{5t}-1-5\mathrm{e}^{9t}+5\mathrm{e}^{4t}+4\mathrm{e}^{10t}-4\mathrm{e}^{5t}}{50t^3}$	M1	Multiplies mgfs together to achieve an unsimplified form
	$= \frac{4\mathrm{e}^{10t}-5\mathrm{e}^{9t}-3\mathrm{e}^{5t}+5\mathrm{e}^{4t}-1}{50t^3}$	A1	**CAO**
		3	

	Question 2 Total	**9**	

Q	Answer	Marks	Comments
3	$H_0 : \mu_C = \mu_H$ $H_1 : \mu_C < \mu_H$	**B1**	Both hypotheses
	Differences: 8, –7, 6, 2, 11	**M1**	Calculates correct differences, either way round
	$\bar{d} = \frac{20}{5} = 4$	**A1**	**oe**
	$s_d^2 = \frac{1}{4}\left(274 - \frac{20^2}{5}\right)$	**m1**	**oe** **PI** **ft** Their differences
	$= 48.5$	**A1**	**oe** **AWRT** $s_d = 6.96$
	$t = \frac{4 - 0}{\sqrt{\frac{48.5}{5}}}$	**M1**	Applies formula with their values
	$= 1.28$	**A1ft**	**AWRT** 1.3 **ft** Their values
	$t_4(0.95) = 2.132$	**B1**	**AWRT** 2.1
	$1.28 < 2.132$ Do not reject H_0	**A1ft**	Correctly compares their t test statistic and their critical value and makes the correct conclusion as to whether to reject the null hypothesis
	Insufficient evidence to suggest that on average, the car travels further on a complete charge on a hot day than a cold day	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion

	Question 3 Total	**10**	

Q	Answer	Marks	Comments
4(a)	$s^2 = \frac{0.0075}{3} = 0.0025$	**B1**	**oe** Eg $s = 0.05$
	$t_3(0.95) = 2.353$	**B1**	**AWRT** 2.35
	$\text{Width} = 2 \times 2.353\sqrt{\frac{0.0025}{4}}$	**M1**	Applies correct formula with their values Condone missing 2 ×
	$= 0.118$	**A1**	**AWRT** 0.118
		4	

Q	Answer	Marks	Comments
4(b)	$2.7 - 0.5 \times 0.118$	**M1**	Correct calculation with their width
	$= 2.64$	**A1ft**	**AWRT** 2.64 **ft** Their width
		2	

	Question 4 Total	**6**	

Q	Answer	Marks	Comments
5(a)	X ~ B(20, 0.2) P($X \leq 0$) = 0.0115 P($X \leq 1$) = 0.0692	**M1**	Finds P($X \leq 0$) or P($X \leq 1$)
	P($X \geq 8$) = 1 – 0.9679 = 0.0321 P($X \geq 9$) = 1 – 0.99 = 0.01	**M1**	Finds P($X \geq 8$) or P($X \geq 9$) **oe**
	Acceptance region $1 \leq X \leq 8$	**A1**	Identifies acceptance region
	Y ~ B(20, 0.15) P($1 \leq Y \leq 8$) = P($Y \leq 8$) – P($Y \leq 0$) = 0.9987 – 0.0388	**M1**	Attempts to find probability of their acceptance region for Y ~ B(20, 0.15)
	= 0.960	**A1**	**AWRT** 0.960 Condone 0.96
		5	

Q	Answer	Marks	Comments
5(b)	1 – 0.960 = 0.040	**B1ft**	**ft** Their Type II probability **AWRT** 0.040 Condone 0.04
		1	

	Question 5 Total	**6**	

Q	Answer	Marks	Comments
6	$\mathrm{P}(M=0)=\left(\mathrm{P}(\text{White})\right)^3$	M1	Recognises $\mathrm{P}(M=0)=\left(\mathrm{P}(\text{White})\right)^3$ **PI**
	$\mathrm{P(White)}=\sqrt[3]{0.042875}=0.35$	A1	**oe**
	Number of white balls $=120\times0.35=42$	M1	Multiplies their P(White) by 120 or subtracts their P(White) from 1
	Number of black balls $=120-42$	M1	Subtract their number of white balls from 120 or multiplies their P(Black) by 120
	Number of black balls $=78$	A1	

	Question 6 Total	**5**	

Q	Answer	Marks	Comments
7(a)	$H_0 : \mu_X = \mu_Y$ $H_1 : \mu_X > \mu_Y$	**B1**	Both hypotheses
	$\bar{x} = \frac{18000}{160} = 112.5$ and $\bar{y} = \frac{15480}{150} = 103.2$	**B1**	Finds both sample means
	$s_X^2 = \frac{1}{159}\left(2502000 - \frac{18000^2}{160}\right)$ $s_Y^2 = \frac{1}{149}\left(2033212 - \frac{15480^2}{150}\right)$	**M1**	Applies formula to find one of s_X^2 or s_Y^2 **oe**
	$s_X^2 = 3000$	**A1**	Correct s_X^2 or **AWRT** $s_X = 54.8$
	$s_Y^2 = 2924$	**A1**	Correct s_Y^2 or **AWRT** $s_Y = 54.1$
	$z = \frac{112.5 - 103.2}{\sqrt{\frac{3000}{160} + \frac{2924}{150}}}$	**M1**	Applies formula
	$= 1.50$	**A1**	**AWRT** 1.50
	z critical value = 1.6449	**B1**	**AWRT** 1.64
	$1.50 < 1.6449$ Do not reject H_0	**A1ft**	Correctly compares their z or t test statistic and their critical value and makes the correct conclusion as to whether to reject the null hypothesis
	Insufficient evidence to suggest that the length of films released in cinemas in country *A* are longer than those released in country *B*	**E1ft**	Gives a conclusion in context based on a comparison of their test statistic and their critical value Conclusion must not be definite
		10	

Q	Answer	Marks	Comments
7(b)	$s_P^2 = \frac{(6-1)\times 3005 + (7-1)\times 2939}{6+7-2} = 2969$	B1	
	$t = \frac{10.2}{\sqrt{2969\left(\frac{1}{6}+\frac{1}{7}\right)}}$	M1	Applies formula
	$= 0.336$	A1	**AWRT** 0.34
	$t_{11}(0.95) = 1.796$	B1	**AWRT** 1.80
	$0.336 < 1.796$ Jenna reaches the same conclusion as Gan	A1ft	Correctly compares their t test statistic and the correct critical value and makes the correct conclusion as to whether Jenna reaches the same conclusion as Gan
		5	

	Question 7 Total	**15**	

Q	Answer	Marks	Comments
8(a)	Relative Efficiency $= \dfrac{\frac{1}{\text{Var}(B)}}{\frac{1}{\text{Var}(A)}} = \dfrac{6\sigma^2}{c\sigma^2}$	**M1**	Applies relative efficiency formula either way round **oe**
	$\dfrac{6\sigma^2}{c\sigma^2} = 10 \Rightarrow \dfrac{6}{c} = 10$	**M1**	Sets their relative efficiency equal to 10 and eliminates σ^2 **oe**
	$c = 0.6$	**A1**	**oe**
		3	

Q	Answer	Marks	Comments
8(b)	$\text{E}(Y) = 1^2 \times \text{E}(X_1) + 2^2 \times \text{E}(X_2) + ... + n^2 \times \text{E}(X_n)$	**M1**	Finds $\text{E}(Y)$ in terms of $\text{E}(X_i)$ **oe**
	$\text{E}(Y) = \left(1^2 + 2^2 + ... + n^2\right)\mu$	**A1**	Finds $\text{E}(Y)$ in terms of μ **oe**
	$\neq \mu$ so biased estimator	**A1**	**AG** Must be convincingly shown and must see conclusion
		3	

Q	Answer	Marks	Comments
8(c)	$\text{E}(kY) = k\text{E}(Y)$	**M1**	**PI**
	$= k\left(1^2 + 2^2 + ... + n^2\right)\mu$	**A1**	Finds $\text{E}(kY)$ in terms of μ, k and n **oe**
	$= k\left(\dfrac{n(n+1)(2n+1)}{6}\right)\mu$	**M1**	Uses $\sum_{r=1}^{n} r^2 = \dfrac{n(n+1)(2n+1)}{6}$
	$k\left(\dfrac{n(n+1)(2n+1)}{6}\right)\mu = \mu$		
	$k = \dfrac{6}{n(n+1)(2n+1)}$	**A1**	**oe**
		4	

	Question 8 Total	**10**	

Q	Answer	Marks	Comments
9(a)	Start with the number of categories and minus 1 [for limitation on frequency]	E1	
	Subtract 1 for estimating the population mean from the data so $4 - 1 - 1 = 2$	E1	
		2	

Q	Answer	Marks	Comments
9(b)	H_0 : Number of online chat questions per hour follows a Poisson distribution	B1	Both hypotheses **oe**
	H_1 : Number of online chat questions per hour does not follow a Poisson distribution		
	Questions per hour / **Probability** / **Expected frequency** 0 / 0.2231 / 8.48 1 / 0.3347 / 12.7 2 / 0.2510 / 9.54 3 or more / 0.1912 / 7.26	M1 A1	**M1**: One correct probability or expected frequency found to 3 sf **A1**: All correct expected frequencies found to 3 sf
	$\sum \frac{(O-E)^2}{E} = \frac{(11-8.48)^2}{8.48} + \frac{(7-12.7)^2}{12.7} + \frac{(8-9.54)^2}{9.54} + \frac{(12-7.26)^2}{7.26}$	M1	Attempts to calculate test statistic
	$= 6.7$	A1ft	**AWRT** 6.7 **ft** their expected frequencies calculated from a Poisson distribution
	$\chi^2_2(0.95) = 5.991$	B1	**AWRT** 6.0
	6.7 > 5.991 Reject H_0	A1ft	Correctly compares their χ^2 test statistic and their critical value and rejects null hypothesis
	Sufficient evidence to suggest that the number of on-line chat queries per hour does not follow a Poisson distribution	E1	Gives a conclusion in context based on a comparison of the correct test statistic and the correct critical value Conclusion must not be definite
		8	

Q	Answer	Marks	Comments
9(c)	$\chi^2_3(0.95) = 7.815$	B1	**AWRT** 7.8
	$6.7 < 7.815$	M1	Compares their test statistic with their critical value
	The team member does not reject the null hypothesis and so reaches different conclusion	A1	Makes the correct conclusion based a comparison of their test statistic and the correct critical value
		3	

	Question 9 Total	**13**	