

**INTERNATIONAL AS  
FURTHER MATHEMATICS**

**FM01**

(9665/FM01) Unit FP1 Pure Mathematics

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Mark scheme

June 2025

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Version: 0.1 Pre-Standardisation



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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**Key to mark scheme abbreviations**

<b>M</b>	Mark is for method
<b>m</b>	Mark is dependent on one or more M marks and is for method
<b>A</b>	Mark is dependent on M or m marks and is for accuracy
<b>B</b>	Mark is independent of M or m marks and is for method and accuracy
<b>E</b>	Mark is for explanation
<b>√ or ft</b>	Follow through from previous incorrect result
<b>CAO</b>	Correct answer only
<b>CSO</b>	Correct solution only
<b>AWFW</b>	Anything which falls within
<b>AWRT</b>	Anything which rounds to
<b>ACF</b>	Any correct form
<b>AG</b>	Answer given
<b>SC</b>	Special case
<b>oe</b>	Or equivalent
<b>A2, 1</b>	2 or 1 (or 0) accuracy marks
<b>-x EE</b>	Deduct x marks for each error
<b>NMS</b>	No method shown
<b>PI</b>	Possibly implied
<b>SCA</b>	Substantially correct approach
<b>sf</b>	Significant figure(s)
<b>dp</b>	Decimal place(s)
<b>ISW</b>	Ignore subsequent working

Q	Answer	Marks	Comments
1	$(5+ai)a = (7+bi)(1+i)$ $5a+a^2i = 7+7i+bi-b$ $5a = 7-b \text{ and } a^2 = 7+b$ $a^2 + 5a - 14 = 0$ $a = -7 \text{ or } a = 2$ $b = 42 \text{ or } b = -3$ $a = -7 \text{ and } b = 42$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Rewrites the equation to a form allowing parts comparison</p> <p>Replaces <math>i^2</math> with <math>-1</math></p> <p>Equates real <b>or</b> imaginary parts to form at least one equation in <math>a</math> and <math>b</math></p> <p>Equates real <b>and</b> imaginary parts and correctly solves a quadratic equation to find at least one value of <math>a</math> or <math>b</math></p> <p>Obtains the correct values of <math>a</math> and <math>b</math></p>
	<b>Question 1 Total</b>	<b>5</b>	

Q	Answer	Marks	Comments
2	$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$ $300 = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{225}{\pi}}$ $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $\frac{dr}{dt} = \frac{1}{4\pi r^2} \times 12$ $\frac{dr}{dt} = \frac{3}{\pi \left( \sqrt[3]{\frac{225}{\pi}} \right)^2}$ $\left[ \frac{dr}{dt} \right] = 0.0554 \text{ cm s}^{-1} \text{ (3 sf)}$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Differentiates to find an expression for <math>\frac{dV}{dr}</math> or <math>\frac{dr}{dV}</math></p> <p>Rearranges <math>V = \frac{4}{3}\pi r^3</math> to find the radius in terms of the volume, or the radius when <math>V = 300</math>, eg <math>r = 4.15</math></p> <p>Writes a correct chain rule linking volume, radius and time</p> <p>Correctly substitutes into a correct chain rule</p> <p>Obtains the correct value Condone missing units Accept <b>AWRT</b> 0.0554</p>
<b>Question 2 Total</b>		<b>5</b>	

Q	Answer	Marks	Comments
3(a)	$2x + \frac{\pi}{3} = n\pi + (-1)^n \frac{\pi}{4} \quad n \in \mathbb{Z}$ $2x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{3}$ $x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{8} - \frac{\pi}{6}$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Obtains a correct angle for <math>2x + \frac{\pi}{3}</math></p> <p>Writes a non-trigonometric equation in <math>x</math></p> <p>eg <math>\frac{\pi}{2} - \left(2x + \frac{\pi}{3}\right) = 2n\pi \pm \frac{\pi}{4}</math></p> <p>Correctly rearranges their non-trigonometric equation to make <math>x</math> the subject</p> <p>Obtains a correct general expression (or pair of expressions) for <math>x</math></p> <p>eg <math>x = n\pi + \frac{\pi}{12} \pm \frac{\pi}{8}</math></p> <p>Condone a missing definition of <math>n</math> Accept two separate expressions, eg <math>x = n\pi - \frac{\pi}{24}</math>, <math>x = n\pi + \frac{5\pi}{24}</math></p>
		<b>4</b>	

Q	Answer	Marks	Comments
3(b)	$x = \frac{5\pi}{24}, \frac{23\pi}{24}, \frac{29\pi}{24}, \frac{47\pi}{24}$ $\text{sum} = \frac{13\pi}{3}$	<p><b>M1</b></p> <p><b>A1</b></p>	<p>Identifies at least two correct positive solutions</p> <p><b>ACF</b> Obtains the correct sum</p>
		<b>2</b>	

	<b>Question 3 Total</b>	<b>6</b>	
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Q	Answer	Marks	Comments
<b>4(a)</b>	$\alpha + \beta = -4$ and $\alpha\beta = c$	<b>B1</b>	Identifies a correct value of $\alpha + \beta$ or $\alpha\beta$
		<b>B1</b>	Identifies the correct values of both $\alpha + \beta$ and $\alpha\beta$
	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	<b>M1</b>	Writes/uses a correct expression for $\alpha^3 + \beta^3$ in terms of $\alpha + \beta$ and $\alpha\beta$
	$\alpha^3 + \beta^3 = (-4)^3 - 3c(-4)$	<b>A1</b>	Obtains the correct simplified expression for $\alpha^3 + \beta^3$
		<b>4</b>	

Q	Answer	Marks	Comments
<b>4(b)(i)</b>	$\text{sum} = \alpha^2 - \beta + \beta^2 - \alpha$	<b>M1</b>	Writes any expression for the sum (or $-\text{sum}$ )
	$= \alpha^2 + \beta^2 - (\alpha + \beta)$		
	$= (\alpha + \beta)^2 - 2\alpha\beta - (\alpha + \beta)$	<b>M1</b>	Writes the sum (or $-\text{sum}$ ) in terms of $\alpha + \beta$ and $\alpha\beta$
	$= (-4)^2 - 2c - (-4)$		
	$= 20 - 2c$		
	$p = 2c - 20$	<b>A1</b>	Obtains the correct simplified expression for $p$
		<b>3</b>	

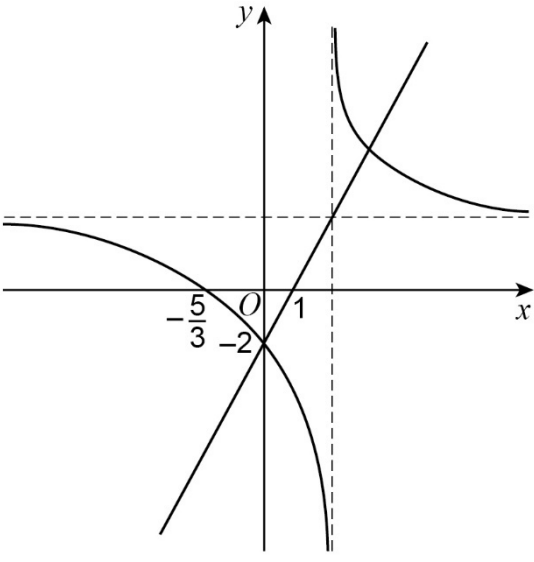
Q	Answer	Marks	Comments
4(b)(ii)	$\text{product} = (\alpha^2 - \beta)(\beta^2 - \alpha)$ $= (\alpha\beta)^2 - (\alpha^3 + \beta^3) + \alpha\beta$ $= c^2 - (12c - 64) + c$ $q = c^2 - 11c + 64$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Writes any correct expression for the product</p> <p>Writes the product in terms of <math>\alpha + \beta</math> and <math>\alpha\beta</math></p> <p><b>ft</b> Their <math>\alpha^3 + \beta^3</math> from <b>part (a)</b></p> <p>Obtains the correct simplified expression for <math>q</math></p>
		<b>3</b>	
	<b>Question 4 Total</b>	<b>10</b>	

Q	Answer	Marks	Comments
5(a)	$\sum_{r=1}^n (3r^2 - 3r + 1) = 3 \sum_{r=1}^n r^2 - 3 \sum_{r=1}^n r + \sum_{r=1}^n 1$ $= 3 \times \frac{1}{6} n(n+1)(2n+1) - 3 \times \frac{1}{2} n(n+1) + n$ $= \frac{1}{2} n(n+1)(2n+1-3) + n$ $= \frac{1}{2} n(2n^2 - 2) + n$ $= n^3 - n + n$ $= n^3, \text{ [which is a cube number for all } n\text{]}$	M1	Writes the expression in terms of $\sum r^2$ , $\sum r$ and $\sum 1$
		M1	Replaces $\sum r^2$ with $\frac{1}{6} n(n+1)(2n+1)$ and $\sum r$ with $\frac{1}{2} n(n+1)$
		M1	Replaces $\sum 1$ with $n$
		A1	Obtains a correct expression in terms of $n$ May be unsimplified
		A1	Correctly demonstrates that the expression is a cube
		5	

Q	Answer	Marks	Comments
5(b)	$\sum_{r=p+1}^{4p} (3r^2 - 3r + 1) = (4p)^3 - p^3$ $= 63p^3$ $= 3^2 \times 7 \times p^3$	M1	Writes $\sum_{r=p+1}^{4p}$ as the difference of two sums
		A1	Obtains a correct expression for $\sum_{r=p+1}^{4p}$ in terms of $p$ only May be unsimplified
		A1	Obtains the correct prime factor decomposition
		3	

	<b>Question 5 Total</b>	<b>8</b>	
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Q	Answer	Marks	Comments
<b>6(a)</b>	$x = \frac{5}{2}$ and $y = 3$	<b>M1</b>  <b>A1</b>	Identifies at least one correct asymptote Condone any incorrect asymptotes for this mark only  Identifies both correct asymptotes and no incorrect equations
		<b>2</b>	

Q	Answer	Marks	Comments
<b>6(b)</b>		<b>B1</b>  <b>B1</b>  <b>B1</b>  <b>B1</b>	Correct RHS with correct asymptotic behaviour  Correct LHS with correct asymptotic behaviour Must intersect the $x$ -axis to the left of the origin and the $y$ -axis below the origin  Includes the correct axis intercepts  Draws a line through the curve $y$ -intercept and the asymptote intersection
		<b>4</b>	

Q	Answer	Marks	Comments
6(c)	<p>C and L intersect when</p> $6x + 10 = (2x - 5)(2x - 2)$ $4x^2 - 20x = 0$ $x = 0, x = 5$ <p>The inequality solution is</p> $0 \leq x < \frac{5}{2}, x \geq 5$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Rearranges the equation/inequality into a 2 or 3-term quadratic equation/inequality <b>PI</b></p> <p>Obtains the correct <math>x</math>-coordinates of the intersections of C and L</p> <p>Identifies at least one region <b>ft</b> Their vertical asymptote or <b>ft</b> Their greatest <math>x</math>-intercept of C and L Condone <math>0 \leq x &lt; \frac{5}{2}</math> for this mark Condone any incorrect regions</p> <p>Identifies two regions <b>ft</b> Their vertical asymptote and their greatest <math>x</math>-intercept of C and L Condone <math>0 \leq x &lt; \frac{5}{2}</math> for this mark</p> <p>Obtains the correct regions</p>
		<b>5</b>	
	<b>Question 6 Total</b>	<b>11</b>	

Q	Answer	Marks	Comments
7(a)	The integrand is not defined for $x = 0$	E1	
		1	

Q	Answer	Marks	Comments
7(b)(i)	$a = 0$ and $b = 2$	B1	
		1	

Q	Answer	Marks	Comments
7(b)(ii)	$I = \lim_{p \rightarrow 0^-} \int_{-1}^p \left( \frac{2}{\sqrt[3]{x}} \right) dx + \lim_{q \rightarrow 0^+} \int_q^2 \left( \frac{2}{\sqrt[3]{x}} \right) dx$ $= \lim_{p \rightarrow 0^-} \left[ 3x^{\frac{2}{3}} \right]_{-1}^p + \lim_{q \rightarrow 0^+} \left[ 3x^{\frac{2}{3}} \right]_q$ $= \lim_{p \rightarrow 0^-} \left( 3p^{\frac{2}{3}} - 3 \right) + \lim_{q \rightarrow 0^+} \left( 3 \times 2^{\frac{2}{3}} - 3q^{\frac{2}{3}} \right)$ $= 0 - 3 + 3 \times 2^{\frac{2}{3}} - 0$ $= 3(\sqrt[3]{4} - 1)$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Replaces the zero limit of at least one of their integrals with a variable</p> <p>Substitutes a limit of <math>-1</math> or <math>2</math> into an expression of the form <math>kx^{\frac{2}{3}}</math> where <math>k</math> is a non-zero constant</p> <p>Correct use of the limiting notation at any stage</p> <p>Obtains the required result following the correct limiting process seen on both terms</p> <p>Accept <math>\rightarrow 0</math> instead of <math>\rightarrow 0^+</math> (or <math>\rightarrow 0^-</math>) throughout</p> <p>Condone the same variable used in both integrals</p>
		4	

	<b>Question 7 Total</b>	<b>6</b>	
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Q	Answer	Marks	Comments
8(a)	$4(x^2 - 6x) + y^2 - 4y = -p$ $4((x-3)^2 - 9) + (y-2)^2 - 4 = -p$ $a = 3 \quad \text{and} \quad b = 2$ $4(x-3)^2 + (y-2)^2 = 40 - p$ $40 - p = 8$ $p = 32$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Attempts to complete the square on either the <math>x</math> terms or the <math>y</math> terms.</p> <p><b>or</b></p> <p>Replaces <math>x</math> with <math>x-a</math> and <math>y</math> with <math>y-b</math> in <math>C_1</math> (or <math>x+a</math> and <math>y+b</math> in <math>C_2</math>)</p> <p>Obtains the correct value for one of <math>a</math> or <math>b</math></p> <p><b>or</b></p> <p>Compares coefficients to form two correct equations in <math>a</math> and <math>b</math></p> <p>Obtains the correct values for <math>a</math> and <math>b</math></p> <p>Forms a correct equation in <math>p</math></p> <p>PI by a correct value of <math>p</math> ft their <math>a</math> and <math>b</math></p> <p>Obtains the correct value of <math>p</math></p>
		<b>5</b>	



Q	Answer	Marks	Comments
8(c)	$\text{Area} = (\sqrt{2} + 3 + \sqrt{2}) \times (\sqrt{8} + 2 + \sqrt{8})$ $= 22 + 16\sqrt{2}$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Calculates the rectangle length <b>ft</b> Their <math>a</math> and <math>b</math> and their <math>C_1</math> <math>x</math>-intercepts</p> <p>Calculates the rectangle width <b>ft</b> Their <math>a</math> and <math>b</math> and their <math>C_1</math> <math>y</math>-intercepts</p> <p>Obtains the correct area in the required form</p>
		<b>3</b>	

	<b>Question 8 Total</b>	<b>14</b>	
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Q	Answer	Marks	Comments
9(a)		<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Draws a circle</p> <p>Draws a circle in the 1st quadrant which either touches the real axis or does not intercept the imaginary axis</p> <p>Draws a circle in the 1st quadrant touching the real axis but not the imaginary axis</p> <p>Accept any indication of the real axis intercept as 8</p>
		<b>3</b>	

Q	Answer	Marks	Comments
9(b)	least $\arg(z) = 0$	<b>B1</b>	States the correct argument
		<b>1</b>	

Q	Answer	Marks	Comments
9(c)	$\arg(8 + 6i) = \tan^{-1}\left(\frac{6}{8}\right)$ $\text{maximum } \arg(z) = 2 \times \tan^{-1}\left(\frac{6}{8}\right)$ $= 1.29 \text{ (2 dp)}$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Correct method for one of the acute angles in a 3,4,5 triangle</p> <p>Condone degrees</p> <p>Full method for required angle</p> <p>Condone degrees</p> <p>Obtains required angle</p> <p>Accept <b>AWRT</b> 1.29</p>
		<b>3</b>	

Q	Answer	Marks	Comments
9(d)(i)	$\cos \theta = \frac{10^2 + 12^2 - 6^2}{2 \times 10 \times 12}$	M1	Uses cosine rule on a 6,10,12 triangle to calculate an angle
	$\theta = \cos^{-1}\left(\frac{13}{15}\right)$	A1	Calculates the angle at the origin PI
	$\arg(z_2) = \tan^{-1}\left(\frac{6}{8}\right) \pm \cos^{-1}\left(\frac{13}{15}\right)$	M1	Full method for one of the required arguments
	= 0.12 or 1.17 (2 dp)	A1	Obtains both arguments Accept <b>AWRT</b> 0.12 and <b>AWRT</b> 1.17
		4	

Q	Answer	Marks	Comments
9(d)(ii)	$\cos \theta = \frac{10^2 + x^2 - 6^2}{2 \times 10 \times x}$	M1	Forms a correct equation in $ z_1 $
	$\frac{13}{15} = \frac{x^2 + 64}{20x}$	A1	Obtains a correct 3-term quadratic equation PI
	$3x^2 - 52x + 192 = 0$	M1	Solves their quadratic equation
	$x = \frac{16}{3}$ or 12	A1	Obtains the required modulus in exact form Must reject 12
		4	

	<b>Question 9 Total</b>	<b>15</b>	
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