OXFORD AQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Monday 13 January 2025   07:00 GMT   Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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ANSWER IN THE SPACES PROVIDED**

Answer **all** questions in the spaces provided.

1 The polynomial $f(x)$ is defined by

$$f(x) = 12x^3 + bx^2 + cx + 6$$

where b and c are constants.

When $f(x)$ is divided by $(2x-1)$ the remainder is 10

When $f(x)$ is divided by $(2x+3)$ the remainder is -24

Find the value of b and the value of c

[4 marks]

$b = \underline{\hspace{3cm}}$ $c = \underline{\hspace{3cm}}$

4

Turn over ▶

2 (a) Use the mid-ordinate rule with 5 strips to find an estimate for

$$\int_0^3 5^{(1-x)}\,\mathrm{d}x$$

Give your answer to three decimal places.

[4 marks]

Answer _____

2 (b) A curve has equation $y = 5^{(1-x)}$

2 (b) (i) The curve intersects the line $y = 2x - 3$ at a single point where $x = \alpha$

Show that α lies between 1.6 and 1.7

[2 marks]

2 (b) (ii) The equation $5^{(1-x)} = 2x - 3$ can be rearranged into the form $x = 0.5\left(3 + 5^{(1-x)}\right)$

Use the iterative formula $x_{n+1} = 0.5\left(3 + 5^{(1-x_n)}\right)$ with $x_1 = 1.6$ to find the value of x_2 and the value of x_3

Give your values to three decimal places.

[2 marks]

$x_2 =$ _____ $x_3 =$ _____

Turn over for the next question

Turn over ▶

3 (a) It is given that

$$y = \left(3 - 4x^2\right)^{10}$$

Find $\dfrac{dy}{dx}$

[2 marks]

Answer _____

3 (b) It is given that

$$y = x\ln\left(4x\right) + 2\tan\left(3x\right)$$

Find $\dfrac{dy}{dx}$

[4 marks]

Answer _____

3 (c) It is given that

$$y = \frac{(3x-2)^2}{(1-4x)^3}$$

Show that

$$\frac{dy}{dx} = \frac{6(px-q)(qx-p)}{(1-4x)^k}$$

where p, q and k are positive integers.

[4 marks]

Turn over ▶

4 The coordinates of the point A are $(2, -3, -1)$

The coordinates of the point B are $(-2, -1, 4)$

4 (a) Find the vector $\overrightarrow{AB}$

[1 mark]

Answer

4 (b) Find $\left|\overrightarrow{AB}\right|$

[2 marks]

Answer

4 (c) The line l has equation $\mathbf{r} = \begin{bmatrix} 3 \\ b \\ -15 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ where b is a constant.

4 (c) (i) Calculate the acute angle between $\overrightarrow{AB}$ and the line l

Give your answer to the nearest $0.1°$

[3 marks]

Answer

4 (c) (ii) The line through A and B intersects the line l

Find the value of b

[3 marks]

Answer

Turn over ▶

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5 The region bounded by the curve $y = 1 + \sin x$, the lines $x = 0$, $x = \dfrac{\pi}{3}$ and the x-axis

is rotated through 2π radians about the x-axis to form a solid.

Find the exact value of the volume of this solid.

[5 marks]

Answer _____

5

Turn over ▶

6   The functions f and g are defined by

$$f(x) = 4\sin\left(\frac{x}{3}\right) \qquad \text{for} \quad -\frac{3\pi}{2} \leq x \leq \frac{3\pi}{2}$$

$$g(x) = |x| \qquad \text{for all real values of } x$$

6 (a)   The inverse of f is f^{-1}

Find $f^{-1}(x)$

[3 marks]

Answer _____

6 (b) (i) Write down an expression for $gf(x)$

[1 mark]

Answer _____

6 (b) (ii) Find the range of $gf(x)$

[2 marks]

Answer _____

6 (c) Describe a sequence of **two** geometrical transformations that maps the graph

of $y = \sin x$ onto the graph of $y = 4\sin\left(\dfrac{x}{3}\right)$

[4 marks]

10

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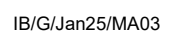
7 (a) (i) Express $16\cos\theta - 30\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the value of α in radians to three significant figures.

[3 marks]

Answer _____

7 (a) (ii) Hence, or otherwise, solve the equation

$$16\cos(y+2) - 30\sin(y+2) = 17$$

Give all values of y to two decimal places in the interval $-\pi < y < \pi$

[3 marks]

Answer _____

7 (b) Solve the equation

$$2\tan^2\left(x-15°\right) = 4 + \sec\left(x-15°\right)$$

Give all solutions to the nearest degree in the interval $-90° < x < 270°$

[5 marks]

Answer _____

Turn over ▶

8 Find $\int_0^1 \ln(2x+1)\,dx$

Give your answer in an exact form.

[7 marks]

Answer _____

9 It is given that

$$\tan \alpha + \tan \beta = 2$$

$$\cot \alpha + \cot \beta = -2$$

Find the possible exact values of $\tan \alpha$

[4 marks]

Answer _____

4

Turn over ▶

10 (a) Find $\int\left(\dfrac{1}{4}e^{2x} - 3e^{-2x}\right)dx$

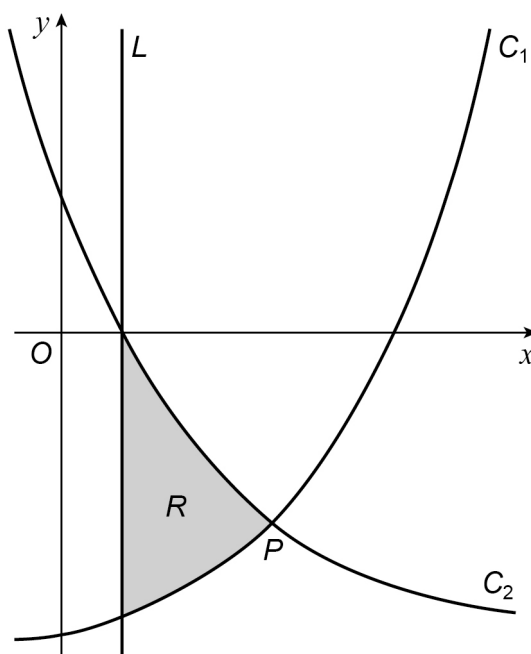**[2 marks]**

Answer _____

10 (b) The diagram shows:

the curve C_1 with equation $y = \dfrac{1}{4}\left(e^{2x} - 9\right)$

the curve C_2 with equation $y = 3e^{-2x} - 2$

the line L with equation $x = \dfrac{1}{2}\ln\left(\dfrac{3}{2}\right)$

the shaded region R

the point P, where the curves C_1 and C_2 intersect.

10 (b) (i) Show that the x-coordinate of the point P is $\ln 2$

[4 marks]

10 (b) (ii) Find the exact value of the area of the shaded region R which is bounded by

the curves $y = \dfrac{1}{4}\left(e^{2x} - 9\right)$, $y = 3e^{-2x} - 2$ and the line $x = \dfrac{1}{2}\ln\left(\dfrac{3}{2}\right)$

Give your answer in the form $a + b\ln c$ where a, b and c are rational numbers.

[5 marks]

Answer _____

Turn over ▶

11 The equation of a curve is

$$2e^{4x} + 6y^2 = e^x y^3$$

The curve has exactly one stationary point.

The coordinates of the stationary point of the curve are (a, b) where a and b are constants.

11 (a) Find $\dfrac{dy}{dx}$

[3 marks]

Answer

11 (b) Show that $b = 2e^a$

[2 marks]

11 (c)   Find the exact value of a and the exact value of b

[4 marks]

$a =$ _____ $b =$ _____

9

Turn over for the next question

Turn over ▶

12 (a) Use the substitution $u^2 = x - 2$ to find $\int x\sqrt{(x-2)}\, dx$

[5 marks]

Answer _____

12 (b) It is given that

$$\frac{1}{x}\frac{dy}{dx} = \sqrt{2y(x-2)}$$

Solve the differential equation given that $y = 2$ when $x = 3$

Give your answer in the form $y = \frac{2}{225}(f(x))^2$

[7 marks]

Answer _____

Turn over ►

13   A curve is defined by the parametric equations

$$x = \frac{2t}{t-1} \quad \text{and} \quad y = t - \frac{1}{t}$$

13 (a)   Find an equation of the tangent to the curve at the point where $t = 2$

Give your answer in the form $px + qy = r$ where p, q and r are integers.

[7 marks]

Answer _____

13 (b) Find a Cartesian equation of this curve.

Give your answer in the form $y = \dfrac{a(x-1)}{x(x-b)}$ where a and b are integers.

[3 marks]

Answer _____

10

Turn over ▶

14 It is given that

$$\frac{4x+4}{(a-bx)(b-ax)} = \frac{P}{a-bx} + \frac{Q}{b-ax}$$

where:

a and b are constants such that $a \neq b$

P is a rational function of a and b

Q is a rational function of a and b

14 (a) Find P and Q in terms of a and b

[4 marks]

$P =$ _____ $Q =$ _____

14 (b) Find the binomial expansion of $(a-bx)^{-1}$ up to and including the term in x^2

[2 marks]

Answer _____

14 (c) (i) Use your answers to **part (a)** and **part (b)** to show that

$$\frac{4x+4}{(a-bx)(b-ax)} = \frac{4}{b-a}\left(D+Ex+Fx^2\right)$$

for small values of x

Give D, E and F in terms of a and b

[2 marks]

14 (c) (ii) In the case where $b = 2a$ show that

$$\frac{4x+4}{(a-bx)(b-ax)} = \frac{1}{a^2}\left(K+Lx+Mx^2\right)$$

where K, L and M are rational numbers.

[2 marks]

10

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