OXFORD AQA

INTERNATIONAL QUALIFICATIONS

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I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Thursday 9 January 2025 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 A curve passes through the point $(2, 1)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x - \sqrt{\sin y}$$

where y is measured in radians.

1 (a) Use Euler's step-by-step method with a step length of 0.1 to estimate the value of y when $x = 2.2$

Give your answer to three decimal places.

[5 marks]

Answer _____

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1 (b) Explain how you could use Euler's step-by-step method to find an improved estimate for the value of y when $x = 2.2$

[1 mark]

6

Turn over for the next question

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2 The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ p & -2 \end{bmatrix}$ where p is a constant.

2 (a) In the case when $\mathbf{A}$ is singular, find the value of p

[2 marks]

Answer _____

2 (b) Show that $\mathbf{A}^2 = (k + np)\mathbf{I}$ for all values of p, where $\mathbf{I}$ is the 2×2 identity matrix and k and n are constants.

[2 marks]

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2 (c) In the case when $p = -1$ find the equations of the invariant lines of the transformation represented by the matrix **A**

[6 marks]

Answer _____

10

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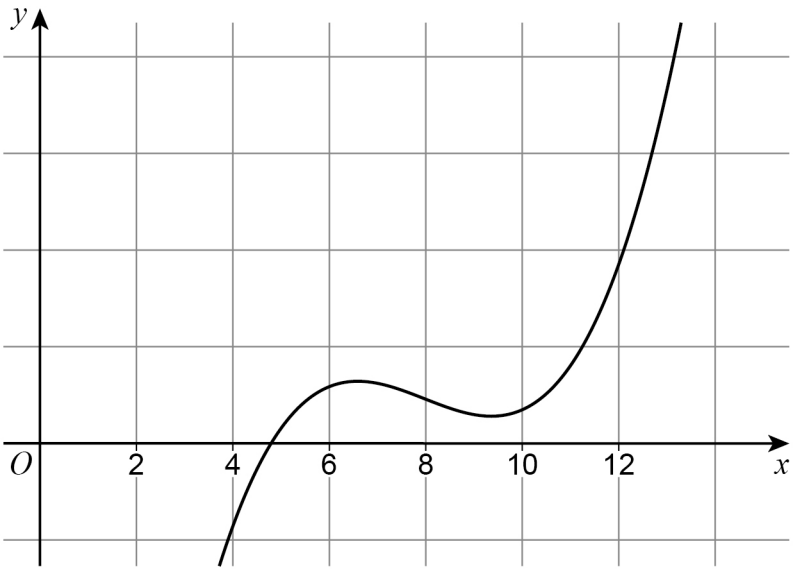
3 The equation $\frac{1}{3}x^3 - 8x^2 + 62x - 150 = 0$ has one real root α

The Newton–Raphson method is to be used once to find an improved approximation to α

3 (a) Explain why an initial value of $x_1 = 8$ would **not** give an improved approximation to α after one iteration.

Draw an appropriate straight line on the graph of $y = \frac{1}{3}x^3 - 8x^2 + 62x - 150$ below as part of your explanation.

[2 marks]



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3 (b) Use the Newton–Raphson method once with the initial value $x_1 = 5$ to find an improved approximation, x_2, to α

Give your answer to three decimal places.

[4 marks]

Answer _____

6

Turn over for the next question

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4 The variables t and y are believed to be related by the equation

$$y = a \times b^t$$

where a and b are constants.

4 (a) Show that there is a linear relationship between t and Y where $Y = \log_{10} y$

[2 marks]

4 (b) The table below shows some values of t and y obtained in an experiment.

t	2	5	10	18	22
y	17.9	12.4	6.7	2.3	1.5

4 (b) (i) Complete the table below.

Give your values of Y to two decimal places.

[2 marks]

t	2	5	10	18	22
Y	1.25				

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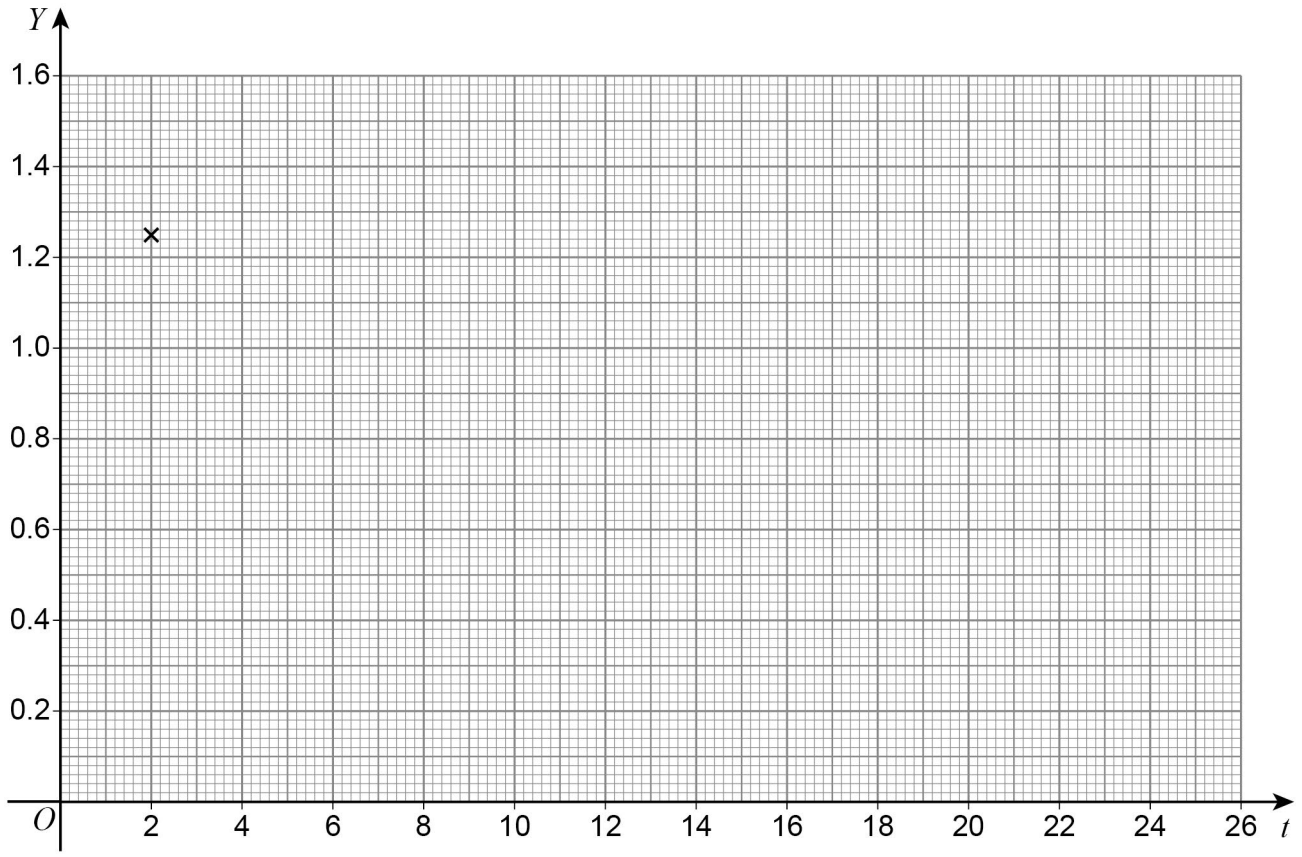
4 (b) (ii) Plot the remaining values of t and Y on the grid below.

Draw a line of best fit.

[2 marks]



4 (c) Use your line of best fit to estimate the value of a and the value of b

[3 marks]

$a =$ _____ $b =$ _____

4 (d) Use your value of a and value of b to estimate the value of y when $t = 13$

[2 marks]

Answer _____

11

Turn over ►

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5 The matrix **M** represents a reflection in the line $y = \frac{1}{2}x$

5 (a) Find the matrix **M**

[2 marks]

Answer _____

Do not write outside the box

5 (b) The matrix **N** is defined by $\mathbf{N} = \begin{bmatrix} -\frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{bmatrix}$

The single transformation T is the combination of a reflection in the line $y = \frac{1}{2}x$ followed by the transformation represented by **N**

Describe fully the transformation T

[5 marks]

7

Turn over for the next section

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 In a company there are three departments: *A*, *B* and *C*

Each employee works in exactly one department.

37% of employees work in department *A*

21% of employees work in department *B*

Employees of the company either have a permanent contract or a temporary contract.

68% of employees in department *A* have permanent contracts.

25% of employees in department *B* have permanent contracts.

83% of employees in department *C* have permanent contracts.

6 (a) Draw a tree diagram to represent this information.

[2 marks]

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6 (b) An employee with a temporary contract is chosen at random.

Find the probability that the employee works in department *B*

[3 marks]

Answer _____

5

Turn over for the next question

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7 The random variable X has a geometric distribution with parameter 0.3

The random variable Y has a binomial distribution with parameters $n = 20$ and $p = 0.3$

The random variables X and Y are independent.

The random variable T is given by

$$T = aX + (1 - a)Y$$

where a is a constant.

The value of a is chosen so that the variance of T is as small as possible.

7 (a) Show that the value of a is $\frac{27}{77}$

[5 marks]

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7 (b) Find the mean of T

[3 marks]

Answer _____

8

Turn over for the next question

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8 A player plays a game which consists of three rounds.

In each round, the player either moves

- forward one step with probability $\frac{3}{5}$

  or

- backward one step with probability $\frac{2}{5}$

The movement of the player in any round is independent of their movement in all previous rounds.

The random variable X_i is such that

$$X_i = \begin{cases} +1 & \text{if the player moves forward in round } i \\ -1 & \text{if the player moves backward in round } i \end{cases}$$

The random variable Y is given by $Y = X_1 + X_2 + X_3$

8 (a) Find $G_Y(t)$, the probability generating function of Y

Give your answer in the form $At^{-3} + Bt^{-1} + Ct + Dt^3$ where A, B, C and D are constants.

[4 marks]

Do not write outside the box

Answer _____

8 (b) The player wins the game if after three rounds they are three steps from the starting position.

Find the probability that the player wins the game.

[1 mark]

Answer _____

8 (c) Use differentiation to find the mean of Y

[2 marks]

Answer _____

7

Turn over ►

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 The equation

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

can be used to find the path of a projectile, where

$x =$ horizontal displacement

$y =$ vertical displacement

$V =$ speed of projection

$\theta =$ angle of projection

$g =$ acceleration due to gravity

9 (a) Use the given equation to show that $[\tan\theta] = 1$ and $[\cos^2\theta] = 1$

[4 marks]

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9 (b) Explain the meaning of $\left[\tan\theta\right] = 1$

[1 mark]

5

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10 A ball of mass 0.04 kg falls vertically from rest and bounces on a horizontal surface.

The ball collides with the surface with speed 5 m s^{-1} and leaves the surface with speed 2 m s^{-1}

10 (a) Write down the coefficient of restitution between the ball and the surface.

[1 mark]

Answer _____

10 (b) Find the magnitude of the impulse exerted on the ball by the surface.

[2 marks]

Answer _____

10 (c) The ball is in contact with the surface for 0.2 seconds.

10 (c) (i) A simple model assumes that a constant force of magnitude F newtons acts on the ball while it is in contact with the surface.

Find the value of F

[2 marks]

Answer _____

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10 (c) (ii) Another model assumes that a variable force acts on the ball while it is in contact with the surface.

The magnitude of this force is given by

$$kt(1-5t)$$

where k is a constant and t seconds is the time after the ball first makes contact with the surface.

Find the value of k

[4 marks]

Answer _____

9

Turn over ►

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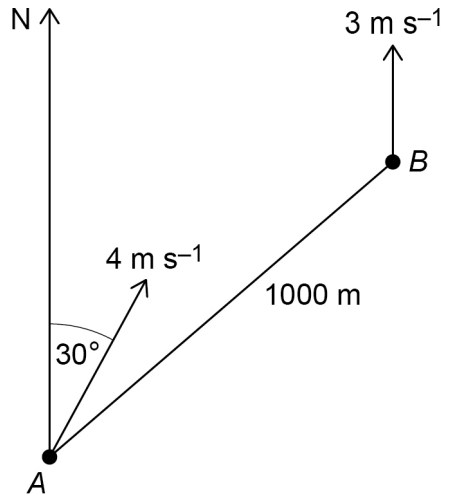
11 Two boats A and B are moving with constant velocity on still water.

Boat A is moving on a bearing of 030° with a velocity of $4\ \text{m s}^{-1}$

Boat B is moving due north with a velocity of $3\ \text{m s}^{-1}$

Boat B is initially 1000 metres north east of boat A

The diagram shows the initial positions and velocities of the two boats.



Find the minimum distance between the two boats.

Give your answer to the nearest metre.

[6 marks]

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Answer _____

6

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