OXFORD AQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS
MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

January 2025

Version: 1.0 Final

2 5 1 X M A 0 5 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from www.oxfordaqa.com

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	Maximum resultant force that can act on truck		
	$P = Fv$	M1	Use of or stating $P = Fv$ **PI** by 28 750 [N]
	$\frac{P}{v} - kv^{\frac{9}{4}} = ma$	A1	Forms a correct equation of motion
	$k = \frac{\frac{P}{v} - ma}{v^{\frac{9}{4}}}$		
	$k = \frac{\frac{345 \times 10^3}{12} - 18000 \times 1.5}{12^{\frac{9}{4}}}$ $\left[= \frac{28750 - 27000}{12^{\frac{9}{4}}} \right]$		
	$k = 6.529...$ or $k = \frac{1750}{12^{\frac{9}{4}}}$		
	$k = 6.53$	A1	**AG** Must see value to at least 4 sf or k equal to a correct fraction before the printed result
		3	

Q	Answer	Marks	Comments
1(b)	Work done against the resistance force $W = kv^{\frac{9}{4}} \times (vt)$ $W = 6.53 \times 18^{\frac{9}{4}} \times (18 \times 30)$ $W = 4357.89... \times 540$ $W = 2.4 \times 10^6 \quad [\text{J}]$	 M1 A1	 Note: unrounded answer is $W = 2.353082432 \times 10^6\,[\text{J}]$ if unrounded value for k is used **AWRT** $2.4 \times 10^6 \quad [\text{J}]$
		2	

Q	Answer	Marks	Comments
1(c)	At maximum speed, resultant force is zero $\frac{P}{v} = kv^{\frac{9}{4}}$ $\frac{P}{k} = v^{\frac{13}{4}}$ $v = \left(\frac{P}{k}\right)^{\frac{4}{13}}$ $v = \left(\frac{345 \times 10^3}{6.53}\right)^{\frac{4}{13}} \quad \left[= (52833.07...)^{\frac{4}{13}} \right]$ $v = 28 \quad [\text{m s}^{-1}]$	 M1 A1	 Note: unrounded answer is $28.39304389 \text{ m s}^{-1}$ if $k = 6.529\ldots$ used **AWRT** 28
		2	

	Question 1 Total	**7**	

Q	Answer	Marks	Comments
2(a)	$82 = 100\cos\alpha$	**M1**	Use of equilibrium perpendicular to slope, with at least one side correct
	$\cos\alpha = \dfrac{82}{100}$		
	$\alpha = 35^\circ$	**A1**	**CAO** to nearest degree
		2	

Q	Answer	Marks	Comments
2(b)	$F = 100\sin\alpha$	**M1**	Use of equilibrium parallel to slope, with at least one side correct **ft** their α
	$F = 57\ [\text{N}]$	**A1ft**	**ft** their α **AWRT** 57
		2	

Q	Answer	Marks	Comments
2(c)	$F = \mu R$		
	$\mu = \dfrac{F}{R} = \dfrac{57}{82}$		
	$\mu = 0.70$	**B1**	**oe** **AWRT** 0.7 Condone one significant figure answer
		1	

	Question 2 Total	**5**	

Q	Answer	Marks	Comments
3(a)	Equilibrium of forces in vertical direction $R = mg$ $F = \mu mg$ Resultant force towards *O* $m\omega^2 r = \mu mg$ $\omega = \sqrt{\frac{\mu g}{r}}$ or $\omega^2 = \frac{\mu g}{r}$ $\omega = \sqrt{\frac{0.45 \times 9.8}{0.3}}$ or $\omega^2 = \frac{0.45 \times 9.8}{0.3} [= 14.7]$ $\omega = 3.834...$ or $\omega = \frac{7\sqrt{30}}{10}$ $\omega = 3.83$ [3 sf]	 B1 M1 m1 A1	 R = normal reaction F = friction Allow $F \leq \mu mg$ **PI** by 88.2 [N] Sets their friction equal to the resultant force for circular motion Rearranges to make ω or ω^2 the subject and substitutes in **AG** Must see value to at least 4 sf or the exact value of $\frac{7\sqrt{30}}{10}$ before the printed result
		4	

Q	Answer	Marks	Comments
3(b)(i)	$\mathbf{r} = 0.3\cos(3t)\mathbf{i} + 0.3\sin(3t)\mathbf{j}$ $\mathbf{v} = -0.9\sin(3t)\mathbf{i} + 0.9\cos(3t)\mathbf{j}$	**M1 A1**	**M1**: At least one component correct **A1**: Both components correct
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$\mathbf{v} = -0.9\sin(3t)\mathbf{i} + 0.9\cos(3t)\mathbf{j}$ $\mathbf{a} = -2.7\cos(3t)\mathbf{i} - 2.7\sin(3t)\mathbf{j}$ $\mathbf{F} = -54\cos(3t)\mathbf{i} - 54\sin(3t)\mathbf{j}$	**M1 A1** **A1ft**	**M1**: At least one component correct **ft** their velocity vector **A1**: Both components correct **ft** their acceleration vector
		3	

	Question 3 Total	**9**	

Q	Answer	Marks	Comments
4(a)	$x=(u\cos\alpha)t$	B1	**oe**
	$y=(u\sin\alpha)t-\frac{1}{2}gt^2$	B1	
	$t=\frac{x}{u\cos\alpha}$		
	$y=(u\sin\alpha)\times\frac{x}{u\cos\alpha}-\frac{1}{2}g\times\left(\frac{x}{u\cos\alpha}\right)^2$	M1 A1	**M1**: Eliminates t in both terms with at least one term correct **A1**: All correct
	$(u\sin\alpha)\times\frac{x}{u\cos\alpha}=x\tan\alpha$		
	$-\frac{1}{2}g\times\left(\frac{x}{u\cos\alpha}\right)^2=-\frac{gx^2}{2u^2}\times\frac{1}{\cos^2\alpha}$		
	$-\frac{gx^2}{2u^2}\times\frac{1}{\cos^2\alpha}=-\frac{gx^2}{2u^2}\sec^2\alpha$		
	$y=x\tan\alpha-\frac{gx^2}{2u^2}\sec^2\alpha$		
	$\frac{g\sec^2\alpha}{2u^2}x^2-x\tan\alpha+y=0$	A1	**AG** Must be convincingly shown
		5	

Q	Answer	Marks	Comments
4(b)(i)	$\frac{9.8\sec^2(60°)}{2\times15^2}x^2 - x\tan(60°) + 7 = 0$	M1	Forms a three-term quadratic equation by substituting in the values of g, u and α **PI** by two correct times, **AWRT** 1.9 and **AWRT** 0.75
	$\frac{98}{1125}x^2 - \sqrt{3}x + 7 = 0$		$0.087111...x^2 - 1.732...x + 7 = 0$
	$x = 5.64$ [m]	A1	**AWRT** 5.64
	$x = 14.2$ [m]	A1	**AWRT** 14.2
		3	

Q	Answer	Marks	Comments
4(b)(ii)	$T = \frac{x_1}{u\cos\alpha} - \frac{x_2}{u\cos\alpha}$		
	$T = \frac{14.2 - 5.64}{15\cos(60°)}$	M1	$T = 1.89... - 0.752$ Possible **ft** from **4(b)(i)** **PI** by correct final answer
	$T = 1.1$ [seconds]	A1	Unrounded answer is 1.1463… seconds **AWRT** 1.1 Condone 1.15 but not 1.2
		2	

	Question 4 Total	**10**	

Q	Answer	Marks	Comments
5(a)	40 [newtons]	**B1**	Condone omission of units
	There is zero resultant force on the rod in the horizontal direction	**E1**	Any correct reason
		2	

Q	Answer	Marks	Comments
5(b)	Anticlockwise moments about *X*		
	$25gd\cos(65°)+40\times3$	**B1**	Correct two-term anticlockwise moment expression
	Clockwise moments about *X*		
	$40\times11\sin(65°)+0.4\times40\times11\cos(65°)$	**B1**	Correct two-term clockwise moment expression **PI** by 473.156… [N m]
	Principle of Moments		
	$25gd\cos(65°)+40\times3$ $=40\times11\sin(65°)+0.4\times40\times11\cos(65°)$	**M1**	Sets their anticlockwise moments equal to their clockwise moments **PI** by correct final answer
	$d=\dfrac{11\times(40\sin(65°)+16\cos(65°))-120}{25\times9.8\times\cos(65°)}$		
	$d=3.4$ [m]	**A1**	Note: **u**nrounded answer is 3.410770837 [m]
		4	

	Question 5 Total	**6**	

Q	Answer	Marks	Comments
6(a)	The mass per unit area of the lamina is constant throughout the entire lamina	E1	Any valid explanation
		1	

Q	Answer	Marks	Comments
6(b)(i)	$\bar{X} = \dfrac{2\times -5 + 4\times 5\ [+7\times 0 + 6\times 0]}{2+4+7+6}$	M1	Allow one error eg omission of lamina
	$\bar{X} = \dfrac{10}{19}$	A1	**CAO** in exact form
		2	

Q	Answer	Marks	Comments
6(b)(ii)	$\bar{Y} = \dfrac{7\times d + 6\times \frac{d}{3}\ [+2\times 0 + 4\times 0]}{2+4+7+6}$	M1	Allow one error eg omission of lamina
	$\bar{Y} = \dfrac{9d}{19}$	A1	**CAO** in exact form
		2	

Q	Answer	Marks	Comments
6(c)	$\tan(5°) = \dfrac{\left(\frac{10}{19}\right)}{\left(\frac{9d}{19}\right)}$	M1 A1ft	**M1**: Correct LHS or their $\bar{X}$ divided by their $\bar{Y}$ **A1ft**: Fully correct, **ft** their $\bar{X}$ and their $\bar{Y}$
	$\tan(5°) = \dfrac{10}{9d} \Rightarrow d = \dfrac{10}{9\tan(5°)}$		
	$d = 13$	A1	Note: unrounded answer is 12.70005811 **AWRT** 13 **CSO**
		3	

	Question 6 Total	**8**	

Q	Answer	Marks	Comments
7(a)	Loss in GPE $mg\Delta h = 3.2 \times 9.8 \times (2.5\sin 40^\circ)$ $mg\Delta h = 50.3945486\ [\text{J}]$ Conservation of Energy $50.3945486 = \frac{1}{2}mu^2$ $u = \sqrt{\frac{2 \times 50.3945486}{3.2}}$ $u = 5.6$	 **B1** **M1** **A1**	 Correct loss in GPE **PI** By correct working later on Sets their loss in GPE equal to gain in KE Note: unrounded answer is 5.612182541 **AWRT** 5.6
		3	

Q	Answer	Marks	Comments
7(b)	Time to fall 6 metres vertically $s = ut + \frac{1}{2}at^2$ $-6 = (-5.6...\sin 40^\circ)t - \frac{1}{2} \times 9.8 \times t^2$ $t = 0.798[0806916\ \text{s}]$ $x = ut$ $x = (5.6...\cos 40^\circ) \times (0.7980806916)$ $x = 3.4$	 **M1 A1ft** **A1** **M1** **A1**	 **M1**: Use of $s = ut + \frac{1}{2}at^2$ with at least two of s, u and a correct and consistent **A1ft**: Fully correct use of $s = ut + \frac{1}{2}at^2$ with their u Ignore inclusion of –1.534[293222] **ft** their u and their t **CAO, AWRT** 3.4
		5	

Q	Answer	Marks	Comments
7(c)	Vertical component of velocity upon colliding with the ground $= \sqrt{u^2 + 2as}$ $= \sqrt{\left(-5.6...\sin 40^\circ\right)^2 + 2 \times -9.8 \times -6}$ $= 11.4\left[2863218 \text{ m s}^{-1}\right]$	B1	**PI** by **AWRT** 20.6 or **AWRT** 69.4
	Magnitude of velocity upon colliding with the ground $v = \sqrt{11.42863218^2 + \left(5.6...\cos 40^\circ\right)^2}$	M1	**ft** their vertical component of velocity and their value of u
	$v = 12.2$	A1	Note: unrounded value is 12.21051157
	$\tan\alpha = \dfrac{5.6...\cos 40^\circ}{11.42863218}$ $[= 0.3761763597]$ $\alpha = \tan^{-1}\left(\dfrac{5.6...\cos 40^\circ}{11.42863218}\right)$	M1	**ft** their vertical component of velocity and their value of u Allow **M1** for final angle of **AWRT** 69.4
	$\alpha = 20.6$	A1	Note: unrounded value is 20.61511315
		5	

	Question 7 Total	**13**	

Q	Answer	Marks	Comments
8(a)	Magnitude of *Y*'s momentum if *X* **does not** change direction during the collision		
	Conservation of Momentum		
	$19.2 + 7.5 = 2.4 + P_Y$	M1	Either conservation of momentum equation correct
	$P_Y = 24.3 \ [\text{kg m s}^{-1}]$	A1	**AWRT** 24
	Magnitude of *Y*'s momentum if *X* **does** change direction during the collision		
	Conservation of Momentum		
	$19.2 + 7.5 = -2.4 + P_Y$		
	$P_Y = 29.1 \ [\text{kg m s}^{-1}]$	A1	**AWRT** 29
		3	

Q	Answer	Marks	Comments
8(b)	KE of Sphere *X* before collision		
	$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times \left(\frac{19.2}{2}\right)^2$	M1	$\text{KE} = \frac{1}{2} \times 2 \times 9.6^2$
	$\text{KE} = 92.16 \ [\text{J}]$		
	KE of Sphere *Y* before collision		
	$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times \left(\frac{7.5}{5}\right)^2$	M1	$\text{KE} = \frac{1}{2} \times 2 \times 1.5^2$
	$\text{KE} = 5.625 \ [\text{J}]$		
	Sum of KE before collision		
	$E = 92.16 + 5.625$		
	$E = 97.785$	A1	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
8(c)	KE of Sphere X after collision $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\times 2\times\left(\frac{2.4}{2}\right)^2$ $\text{KE} = 1.44\ [\text{J}]$	B1	**PI**
	Possible speeds of Sphere Y after collision $v_Y = 4.86\ [\text{m s}^{-1}]$ or $v_Y = 5.82\ [\text{m s}^{-1}]$	B1	At least one correct, **PI**
	KE of Sphere Y after collision (no change in direction) $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\times 5\times(4.86)^2$ $\text{KE} = 59.049\ [\text{J}]$	B1ft	**ft** their speed for Y **PI** by 60.489
	Sum of KE after collision $F = [1.44 + 59.049 =]\ 60.489$ $E - F = [97.785 - 60.489 =]\ 37.3$	B1	Note: unrounded value is 37.296
	KE of Sphere Y after collision (a change in direction) $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\times 5\times(5.82)^2$ $\text{KE} = 84.681$	B1ft	**ft** their speed for Y **PI** by 86.121
	Sum of KE after collision $F = [1.44 + 84.681 =]\ 86.121\ [\text{J}]$ $E - F = [97.785 - 86.121 =]\ 11.7$	B1	Note: unrounded value is 11.664 [J]
		6	

	Question 8 Total	**12**	

Q	Answer	Marks	Comments
9	$\mathbf{F} = \left(36t^2 - 18t\right)\mathbf{i} + \left(6\cos t - 24\sin 2t\right)\mathbf{j} + 3\mathrm{e}^{-\frac{t}{3}}\mathbf{k}$		
	$\mathbf{a} = \left(12t^2 - 6t\right)\mathbf{i} + \left(2\cos t - 8\sin 2t\right)\mathbf{j} + \mathrm{e}^{-\frac{t}{3}}\mathbf{k}$	B1	Correct acceleration vector
	$\mathbf{v} = \left(4t^3 - 3t^2\right)\mathbf{i} + \left(2\sin t + 4\cos 2t\right)\mathbf{j} - 3\mathrm{e}^{-\frac{t}{3}}\mathbf{k} + \mathbf{c}$	M1 A1ft	**M1**: At least one component correct from their acceleration vector **A1ft**: All components correct from their acceleration vector Condone omission of constant(s) of integration
	$\mathbf{v}(0) = 0\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mathbf{c} = \mathbf{0}$	M1	Substitutes $t = 0$ into their velocity vector and sets equal to zero vector, **oe**
	$\mathbf{v} = \left(4t^3 - 3t^2\right)\mathbf{i} + \left(2\sin t + 4\cos 2t - 4\right)\mathbf{j} + \left(3 - 3\mathrm{e}^{-\frac{t}{3}}\right)\mathbf{k}$	A1	Correct velocity vector
	$\mathbf{r} = \left(t^4 - t^3\right)\mathbf{i} + \left(-2\cos t + 2\sin 2t - 4t\right)\mathbf{j} + \left(3t + 9\mathrm{e}^{-\frac{t}{3}}\right)\mathbf{k} + \mathbf{c}$	M1 A1ft	**M1**: At least one component correct from their velocity vector **A1ft**: All components correct from their velocity vector Condone omission of constant(s) of integration
	$\mathbf{r}(0) = 0\mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mathbf{c} = \mathbf{0}$		
	$\mathbf{r} = \left(t^4 - t^3\right)\mathbf{i} + \left(-2\cos t + 2\sin 2t - 4t + 2\right)\mathbf{j} + \left(3t + 9\mathrm{e}^{-\frac{t}{3}} - 9\right)\mathbf{k}$	A1	Correct position vector
	$\mathbf{r}(2) = 8\mathbf{i} - 6.681...\mathbf{j} + 1.620...\mathbf{k}$	M1	Substitutes $t = 2$ into their 3-dimensional position vector
	$\lvert\mathbf{r}(2)\rvert = \sqrt{8^2 + (-6.681...)^2 + (1.620...)^2}$		
	$\lvert\mathbf{r}(2)\rvert = 10.5$ [m]	A1	Note: unrounded answer is 10.54830625 [m]

	Question 9 Total	**10**	