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(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$h\mathrm{f}(2,1) = 0.1\times\left(2-\sqrt{\sin 1}\right)$	M1	Correct substitution into RHS of this expression **PI**
	$= 0.108268...$	A1	**PI**
	$y_2 = 1 + 0.108268... = 1.108268...$	A1ft	**AWRT** 1.11 **ft** sin 1 evaluated in degrees to achieve **AWRT** 1.19
	$y_3 = 1.108268 + 0.1\times\left(2.1-\sqrt{\sin(1.108268)}\right)$	M1	Correct substitution using their x_2 and their y_2 into second term here
	$[= 1.223667675]$		
	1.224	A1	**CAO** to 3 dp
		5	

Q	Answer	Marks	Comments
1(b)	By using a smaller step length	B1	Reference to smaller step length, or suggests a smaller step Allow increase the number of steps
		1	

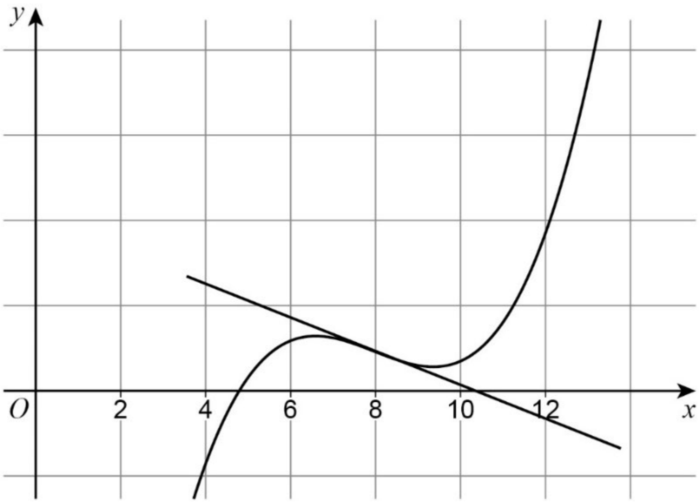
	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2(a)	$2\times(-2)-3p=0$	M1	Obtains $\pm2\times2\pm3p=0$ **PI**
	$p=-\dfrac{4}{3}$	A1	**CAO**
		2	

Q	Answer	Marks	Comments
2(b)	$\mathbf{A}^2=\begin{bmatrix}2 & 3\\ p & -2\end{bmatrix}\times\begin{bmatrix}2 & 3\\ p & -2\end{bmatrix}$		
	$=\begin{bmatrix}4+3p & 2\times3-2\times3\\ 2p-2p & 4+3p\end{bmatrix}$		
	$=\begin{bmatrix}4+3p & 0\\ 0 & 4+3p\end{bmatrix}$	M1	For a 2 × 2 matrix with at least 3 correct entries
	$=(4+3p)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $=(4+3p)\mathbf{I}$	A1	Obtains correct result, including factoring of $4+3p$ Allow $(4+3p)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ or $(4+3p)\mathbf{I}$ **CSO**
		2	

Q	Answer	Marks	Comments
2(c)	$[x' =]\ 2x+3(mx+c)$ $[y' =]\ -x-2(mx+c)$	**M1**	Valid attempt to find x', y' Condone sign errors Condone using x for x' and y for y' Condone only considering mx Or correctly finds eigenvalues $\lambda = \pm 1$
	$-x-2(mx+c)$ $= m(2x+3(mx+c))+c$	**m1**	**ft** their $y' = m$ (their x') $+ c$
	$3m^2+4m+1=0$	**m1**	Attempt to find m by comparing coefficients or setting coefficients = 0 or finding gradient associated with eigenvectors Dependent on first **M** mark
	$m=-1$, $m=-\frac{1}{3}$	**A1**	Finds correct values of m
	$3c(m+1)=0$ $m=-\frac{1}{3} \Rightarrow c=0$ $m=-1 \Rightarrow c \in \mathbb{R}$	**m1**	Attempts to find c for one value of m by comparing coefficients Dependent on all previous **M** marks
	$y=-\frac{1}{3}x$, $y=-x+c$	**A1**	Correct equations, with no restrictions on c for the second
		6	

	Question 2 Total	**10**	

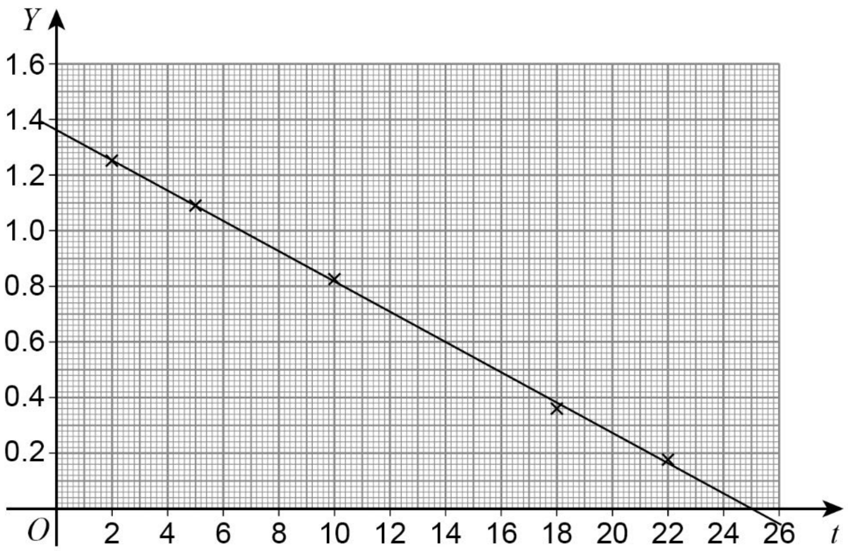
Q	Answer	Marks	Comments
3(a)		M1	Draws tangent at $x = 8$
	Tangent intersects the x-axis further from the root	A1	Correct conclusion Accept x_2 is further away from α than x_1
			
		2	

Q	Answer	Marks	Comments
3(b)	$\mathrm{f}'(x) = x^2 - 16x + 62$	M1	Differentiates expression for y **PI**
	$\mathrm{f}'(5) = 7$	A1	**PI**
	$x_2 = 5 - \dfrac{\frac{1}{3}\times 5^3 - 8\times 5^2 + 62\times 5 - 150}{7}$	M1	Correct substitution into NR formula
	$= 5 - \dfrac{\left(\frac{5}{3}\right)}{7}$ $\quad [= 4.761904762]$		
	$x_2 = 4.762$	A1	**CAO** to 3 decimal places
		4	

	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4(a)	$\log_{10} y = \log_{10}\left(a \times b^t\right)$ $= \log_{10} a + \log_{10} b^t$	M1	Takes logs of both sides and applies a correct law of logarithms to achieve $\log_{10} a + \log_{10} b^t$
	$= \log_{10} a + t\log_{10} b$ [which is a linear relationship between x and $\log_{10} y$]	A1	Obtains correct relationship **CSO**
		2	

Q	Answer	Marks	Comments
4(b)(i)	… | 1.09 | 0.83 | 0.36 | 0.18	B1	At least two correct, condone more than 2 dp
		B1	All correct, condone more than 2 dp
		2	

Q	Answer	Marks	Comments
4(b)(ii)		B1ft	Points plotted **ft** provided all of their points can be plotted on the grid Half a square tolerance
		B1ft	Line of best fit seen **ft** at least 3 points plotted
			
		2	

Q	Answer	Marks	Comments
4(c)	$y-$intercept $=1.36$ gradient $= -0.054$	B1	Sight of the y–intercept or the gradient for their straight line of best fit
	$a=10^{1.36}$ or $b=10^{-0.054}$	M1	At least one correct equation using their y-intercept or gradient for a or b A straight line must have been plotted in part **(b)(ii)**
	$a=23$ $b=0.88$	A1	**AWFW** [19.9, 25] for a **AWFW** [0.87, 0.89] for b
		3	

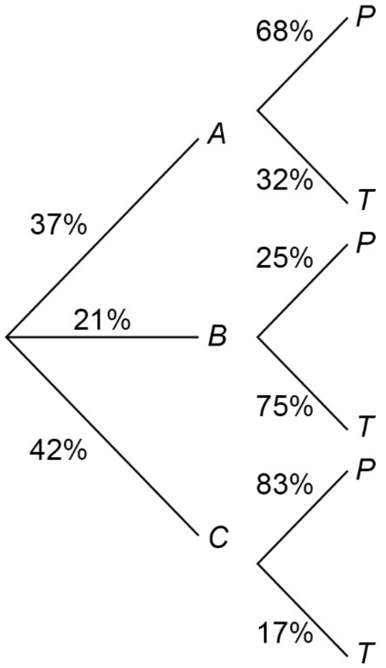
Q	Answer	Marks	Comments
4(d)	$=23\times0.88^{13}$	M1	Substitutes values into $y=a\times b^t$ **ft** their exact or rounded a, b
	$=4.4$	A1ft	Answer must be given to at least 2 sf **ft** their exact or rounded a, b
		2	

	Question 4 Total	**11**	

Q	Answer	Marks	Comments
5(a)	$\tan\theta = \frac{1}{2} \Rightarrow \cos 2\theta = 0.6,\ \sin 2\theta = 0.8$	M1	Uses $\tan\theta = \frac{1}{2}$ to find θ or $\cos 2\theta$ or $\sin 2\theta$ **PI** By correct answer
	$\mathbf{M} = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$	A1	**CAO**, **oe**
		2	

Q	Answer	Marks	Comments
5(b)	$\mathbf{NM} = \begin{bmatrix} -\frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$	M1	Multiplication of **NM** in correct order, **ft** their **M**
	$= \begin{bmatrix} \frac{33}{65} & -\frac{56}{65} \\ \frac{56}{65} & \frac{33}{65} \end{bmatrix} \quad \left[= \begin{bmatrix} 0.507... & -0.861... \\ 0.861... & 0.507... \end{bmatrix} \right]$	A1ft	**ft** Their **M** Elements must be given to at least 3 sf
	Rotation	B1	Correct transformation named
	$[\phi =]\ \tan^{-1}\left(\frac{56}{33}\right)$	M1	Attempts to calculate angle for a rotation
	$= 59.5°$, anticlockwise about the origin	A1	**oe** eg “1.04 anticlockwise”; “5.24 (or $300.5°$) clockwise”; “-2.10”; “$-300.5°$” condone “$+1.04$” or “$+59.5°$” **CSO** **AWRT** 59° or 59.5° or 301° or 1.0 or 5.2 for the angle
		5	

	Question 5 Total	**7**	

Q	Answer	Marks	Comments
6(a)		**M1** **A1**	Correct structure Fully correct
		2	

Q	Answer	Marks	Comments
6(b)	$\dfrac{0.21\times 0.75}{0.37\times 0.32+0.21\times 0.75+0.42\times 0.17}$ $=\dfrac{1575}{3473}$	**M1** **M1** **A1**	Correct calculation for the numerator **PI** by sight of 0.1575 **oe** **ft** their **(a)** Correct calculation for the denominator **PI** by sight of 0.3473 **oe** **ft** their **(a)** **AWRT** 0.453 or 0.4535 Allow 0.45 with a correct calculation seen
		3	

	Question 6 Total	**5**	

Q	Answer	Marks	Comments
7(a)	$\text{Var}(X)=\dfrac{1-0.3}{0.3^2}=\dfrac{70}{9}$	B1	**oe** May be unsimplified
	$\text{Var}(Y)=20\times0.3\times(1-0.3)=\dfrac{21}{5}$	B1	**oe** May be unsimplified
	$\text{Var}(T)=\dfrac{70}{9}a^2+\dfrac{21}{5}(1-a)^2$	M1	Finds $\text{Var}(T)=a^2\text{Var}(X)+(1-a)^2\text{Var}(Y)$ for their $\text{Var}(X)$ and $\text{Var}(Y)$
	$\text{Var}(T)=\dfrac{539}{45}a^2-\dfrac{42}{5}a+\dfrac{21}{5}$	M1	Expands their expression for Var(T) to achieve a three-term quadratic in a
	$\dfrac{1078}{45}a-\dfrac{42}{5}=0\Rightarrow a=\dfrac{27}{77}$	A1	**AG** Correctly differentiates correct expression for Var(T) and solves to find a with no errors seen or Completes the square to achieve $\text{Var}(T)=\dfrac{539}{45}\left(a-\dfrac{27}{77}\right)^2+\dfrac{30}{11}$ and identifies that $a=\dfrac{27}{77}$ with no errors seen
		5	

Q	Answer	Marks	Comments
7(b)	$\text{E}(X)=\dfrac{1}{0.3}=\dfrac{10}{3}$	M1	Finds one of E(X) or E(Y) May be unsimplified
	$\text{E}(Y)=20\times0.3=6$	A1	Finds both E(X) and E(Y) May be unsimplified
	$\text{E}(T)=\dfrac{27}{77}\times\dfrac{10}{3}+\left(1-\dfrac{27}{77}\right)\times6=\dfrac{390}{77}$	A1	**AWRT** 5.06
		3	

	Question 7 Total	**8**	

Q	Answer	Marks	Comments
8(a)	$\mathrm{G}_{X_i}(t)=\frac{2}{5}t^{-1}+\frac{3}{5}t$	B1	**oe** or calculates the probability of one of the possible outcomes
	$\mathrm{G}_Y(t)=\left(\frac{2}{5}t^{-1}+\frac{3}{5}t\right)^3$	M1	Cubes their $\mathrm{G}_{X_i}(t)$ or calculates the probabilities of all four possible outcomes
	$\mathrm{G}_Y(t)=\left(\frac{2}{5}t^{-1}+\frac{3}{5}t\right)\left(\frac{4}{25}t^{-2}+\frac{12}{25}+\frac{9}{25}t^2\right)$	M1	Multiplies out to reach form $\left(at^{-1}+bt\right)\left(ct^{-2}+d+et^2\right)$ or expands using binomial expansion formulae or uses their probabilities to find $\mathrm{G}_Y(t)$ in the required form
	$\mathrm{G}_Y(t)=\frac{8}{125}t^{-3}+\frac{36}{125}t^{-1}+\frac{54}{125}t+\frac{27}{125}t^3$	A1	**CAO oe**
		4	

Q	Answer	Marks	Comments
8(b)	$\frac{8}{125}+\frac{27}{125}=\frac{7}{25}$	B1ft	**oe** **ft** Their $A+D$
		1	

Q	Answer	Marks	Comments
8(c)	$\mathrm{G}'_Y(t)=-\frac{24}{125}t^{-4}-\frac{36}{125}t^{-2}+\frac{54}{125}+\frac{81}{125}t^2$	M1	Correct differentiation of their $\mathrm{G}_Y(t)$ Condone a different letter being used for t
	Mean $=\mathrm{G}'_Y(1)=-\frac{24}{125}-\frac{36}{125}+\frac{54}{125}+\frac{81}{125}=\frac{3}{5}$	A1	**oe** Answer must come from $\mathrm{G}'_Y(1)$
		2	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)	$L=[x][\tan\theta]$ $L=L[\tan\theta]$	M1	Recognises $[x]=L$ and $[y]=L$ and sets up equation to find $[\tan\theta]$ Condone use of units Condone missing square brackets
	$[\tan\theta]=1$	A1	**AG** Must be convincingly shown Units must not be used Condone missing square brackets in working but must be seen in final line
	$L=\dfrac{[g][x^2]}{[2V^2][\cos^2\theta]}$ $L=\dfrac{LT^{-2}L^2}{L^2T^{-2}[\cos^2\theta]}$	M1	Recognises $[g]=LT^{-2}$, $[x^2]=L^2$ and $[2V^2]=L^2T^{-2}$ and sets up equation to find $[\cos^2\theta]$ Condone use of units Condone missing square brackets
	$1=\dfrac{1}{[\cos^2\theta]}$ $[\cos^2\theta]=1$	A1	**AG** Must be convincingly shown An intermediate line after $L=\dfrac{LT^{-2}L^2}{L^2T^{-2}[\cos^2\theta]}$ must be seen Units must not be used Condone missing square brackets in working but must be seen in final line Condone for all four marks eg $L=L[\tan\theta]-\dfrac{LT^{-2}L^2}{L^2T^{-2}[\cos^2\theta]}$ $1=[\tan\theta]-\dfrac{1}{[\cos^2\theta]}$ $1=[\tan\theta]=[\cos^2\theta]$
		4	

Q	Answer	Marks	Comments
9(b)	$\tan\theta$ is dimensionless	B1	Correct statement Allow "no units" Condone dimensionless constant
		1	

	Question 9 Total	5	

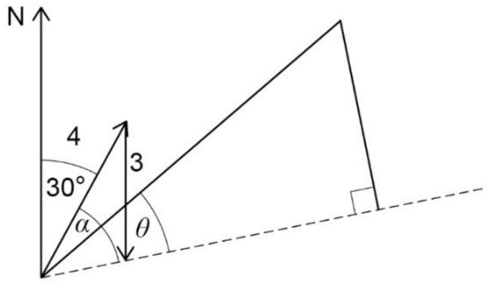
Q	Answer	Marks	Comments
10(a)	$[e =]\ \dfrac{2}{5}$	**B1**	Correct value
		1	

Q	Answer	Marks	Comments
10(b)	$I = 0.04\times 2 - 0.04\times(-5)$	**M1**	Uses $\pm 0.04\times 2 \pm 0.04\times 5$
	$= 0.28\ [\text{N s}]$	**A1**	Correct magnitude If unit seen it must be correct
		2	

Q	Answer	Marks	Comments
10(c)(i)	$0.28 = F\times 0.2$	**M1**	Uses their $I = 0.2Ft$
	$F = 1.4$ N	**A1ft**	**ft** their answer to part **(b)** If a negative value is given in part **(b)**, the final answer must be positive If unit seen it must be correct
		2	

Q	Answer	Marks	Comments
10(c)(ii)	$[0.28 =]\ \int_0^{0.2} kt(1-5t)\,\mathrm{d}t$	**M1**	Identifies correct integral to find impulse Condone missing limits
	$[0.28 =]k\left[\dfrac{t^2}{2}-\dfrac{5t^3}{3}\right]_0^{0.2}$	**A1**	Correct integration **PI** By a correct equation for k Condone missing limits
	$0.28 = \dfrac{k}{150}$	**m1**	Forms equation to find k using their answer to part **(b)**
	$k = 42$	**A1ft**	**ft** their answer to part **(b)** Ignore any units given
		4	

	Question 10 Total	**9**	

Q	Answer	Marks	Comments
11			
	$v^2 = 4^2 + 3^2 - 2\times4\times3\cos30°$ or ${}_A\mathbf{v}_B = \begin{bmatrix} 4\sin30° \\ 4\cos30° - 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2\sqrt{3} - 3 \end{bmatrix}$	M1	Uses cosine rule to find v or finds correct relative velocity
	$v = 2.053...$ or $\tan\beta = \dfrac{2\sqrt{3}-3}{2}$	A1	Correct v given to 2 sf or obtains $\tan\beta = \dfrac{2\sqrt{3}-3}{2}$
	$\dfrac{\sin\alpha}{3} = \dfrac{\sin30°}{2.053...}$ or $\tan^{-1}\left(\dfrac{2\sqrt{3}-3}{2}\right)$	M1	Uses sine rule to find α or attempts $\tan^{-1}\left(\dfrac{2\sqrt{3}-3}{2}\right)$
	$\alpha = 46.935...$ or $\tan^{-1}\left(\dfrac{2\sqrt{3}-3}{2}\right) = 13.064\ldots$	A1	Correct α given to 2 sf or $\tan^{-1}\left(\dfrac{2\sqrt{3}-3}{2}\right) =$ **AWRT** 13
	$\theta = 30 + 46.935... - 45 = 31.935...$	A1	Correct θ given to 2 sf
	Min Distance $= 1000\times\sin31.935... = 529$ [m] or	A1	Correct distance given to nearest metre

	${}_A\mathbf{v}_B = \begin{bmatrix} 4\sin 30° \\ 4\cos 30° - 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2\sqrt{3} - 3 \end{bmatrix}$	**(M1)**	Finds correct relative velocity **PI**
	${}_A\mathbf{r}_B = \begin{bmatrix} 2 \\ 2\sqrt{3} - 3 \end{bmatrix} t - \begin{bmatrix} 500\sqrt{2} \\ 500\sqrt{2} \end{bmatrix}$ $s^2 = \left(2t - 500\sqrt{2}\right)^2 + \left(\left(2\sqrt{3} - 3\right)t - 500\sqrt{2}\right)^2$ $\left[s^2 = (2t - 707.10...)^2 + (0.464...t - 707.10...)^2\right]$	**(A1)**	Correct relative position or distance²
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(s^2\right) = 4\left(2t - 500\sqrt{2}\right) + 2\left(2\sqrt{3} - 3\right)\left(\left(2\sqrt{3} - 3\right)t - 500\sqrt{2}\right)$	**(M1)**	Differentiates Condone errors if intent clear **PI**
	$0 = 8t + 2\left(2\sqrt{3} - 3\right)^2 t - 2000\sqrt{2} - 1000\sqrt{2}\left(2\sqrt{3} - 3\right)$	**(A1)**	Correct equation **PI**
	$t = \frac{2000\sqrt{2} + 1000\sqrt{2}\left(2\sqrt{3} - 3\right)}{8 + 2\left(2\sqrt{3} - 3\right)^2} = \frac{2000\sqrt{6} - 1000\sqrt{2}}{50 - 24\sqrt{3}} = \frac{3484.765...}{8.430...} = 413.338...$	**(A1)**	Correct time given to 2 sf **PI**
	$s = \sqrt{\left(2 \times 413.338 - 500\sqrt{2}\right)^2 + \left(\left(2\sqrt{3} - 3\right) \times 413.338 - 500\sqrt{2}\right)^2} = 529\ [\mathrm{m}]$	**(A1)**	Correct distance given to nearest metre

	Question 11 Total	**6**	