OXFORD AQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS
FM03

(9665/FM03) Unit FP2 Pure Mathematics

Mark scheme

January 2025

Version: Final 1.0

251XFM03/MS

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$\overrightarrow{AB} = \begin{bmatrix} k+1 \\ 2 \\ 1 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$, $\overrightarrow{AD} = \begin{bmatrix} k+2 \\ -1 \\ 3 \end{bmatrix}$	**B1**	Obtains three non-coplanar direction vectors, eg $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$
	$\overrightarrow{AB} \times \overrightarrow{AC} \bullet \overrightarrow{AD} = \begin{bmatrix} k+1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} k+2 \\ -1 \\ 3 \end{bmatrix}$	**M1**	Calculates scalar triple product of their three direction vectors. Do not accept position vectors
	$\overrightarrow{AB} \times \overrightarrow{AC} \bullet \overrightarrow{AD} = \begin{bmatrix} -1 \\ -k+2 \\ 3k-3 \end{bmatrix} \bullet \begin{bmatrix} k+2 \\ -1 \\ 3 \end{bmatrix}$	**A1**	Correct vector product
	$\overrightarrow{AB} \times \overrightarrow{AC} \bullet \overrightarrow{AD} = 9k - 13$	**A1**	Correct scalar triple product. Do not accept in vector form
	$9k - 13 = 4$ $9k - 13 = -4$	**M1**	Their scalar triple product $= (\pm)4$
	$k = \frac{17}{9}$ $k = 1$	**A1**	Correct values of k

	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2(a)	$\cosh^2 x - \sinh^2 x = \left\{\frac{1}{2}\left(e^x + e^{-x}\right)\right\}^2 - \left\{\frac{1}{2}\left(e^x - e^{-x}\right)\right\}^2$	M1	Uses correct expressions for $\cosh x$ and $\sinh x$ to simplify $\cosh^2 x - \sinh^2 x$
	$= \frac{1}{4}\left(e^{2x} + 2 + e^{-2x}\right) - \frac{1}{4}\left(e^{2x} - 2 + e^{-2x}\right)$ $= 1$	A1	Correct working to required answer **AG** Must include the LHS, at least two intermediate steps, and the RHS
		2	

Q	Answer	Marks	Comments
2(b)	$\cosh^2 x - 1 - \cosh x - 5 = 0$	M1	Correct use of identity from part **(a)**
	$\cosh^2 x - \cosh x - 6 = 0$	M1	Forms three term quadratic equation in $\cosh x$ **PI** Correct solution(s)
	$\cosh x = 3$ or $\cosh x = -2$	A1	Correct answers from correct quadratic. Must see both.
	$\cosh x \neq -2$ $[\text{as } \cosh x \geq 1]$	E1	Rejection of $\cosh x = -2$
	$x = \pm\cosh^{-1}(3)$		
	$x = \ln\left(3 \pm \sqrt{8}\right)$	A1	Both correct solutions and no others, **ACF** Eg $x = \pm\ln\left(3 + \sqrt{8}\right)$ Must score **M1M1A1** to get final **A1** Can score **B1M1A1E0A1**
		5	

	Question 2 Total	**7**	

Q	Answer	Marks	Comments
3(a)	$\mathbf{R}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$	B1	Obtains correct matrix for $\mathbf{R}^{-1}$
	$\mathbf{A} = \mathbf{B}\mathbf{R}^{-1}$ $\mathbf{A} = \begin{bmatrix} 0 & \cos\theta & -\sin\theta \\ 1 & 0 & 0 \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$	M1	Correct method to find **A**, by right multiplication by $\mathbf{R}^{-1}$, can be algebraic
	$\mathbf{A} = \begin{bmatrix} 0 & \cos^2\theta + \sin^2\theta & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \cos^2\theta + \sin^2\theta \end{bmatrix}$		
	$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **A** is independent of θ	A1	Correct result including conclusion that **A** is independent of θ
		3	

Q	Answer	Marks	Comments
3(b)	Reflection	M1	Reflection
	in $y = x$ plane	A1	Correct plane **SC1** for ‘reflect’ or ‘reflected’ or ‘reflects’ in $y = x$ plane
		2	

	Question 3 Total	**5**	

Q	Answer	Marks	Comments
4(a)	The integrand $\frac{1}{\sqrt{4-x^2}}$ is not defined at $x=2$	E1	**oe**
		1	

Q	Answer	Marks	Comments
4(b)	$\int \frac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\left(\frac{x}{2}\right)$	B1	Correct integration, with or without limits
	$\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx = \lim_{a\to 2}\int_0^a \frac{1}{\sqrt{4-x^2}}\,dx$ $= \lim_{a\to 2}\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^a$	M1	Evidence of limit 2 replaced by a and limit taken somewhere **oe** $\lim_{a\to 2}$ seen at any stage Do not accept the limit as x
	$\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx = \lim_{a\to 2}\left[\sin^{-1}\left(\frac{a}{2}\right) - \sin^{-1}(0)\right]$ $= \sin^{-1}(1)$ $= \frac{\pi}{2}$	A1	Correct with limiting process clearly shown. Do not accept 90°
		3	

	Question 4 Total	**4**	

Q	Answer	Marks	Comments
5	$\text{f}(1) = 7^2 + 11^1 = 60 [= 4 \times 15]$ [True for $n = 1$]	B1	
	$n = k + 1$ Assume true $n = k$ (*) Makes 11^k or 7^{k+1} the subject of an equation eg $11^k = 4M - 7^{k+1}$ (I) or Considers $\text{f}(k+1) + p\,\text{f}(k) = \ldots$	M1	$\text{f}(k) = 7^{k+1} + 11^k = 4M$ $7^{k+1} = 4M - 11^k$ (I) Do not condone $q\,\text{f}(k+1) + p\,\text{f}(k) \quad q \neq 1$
	$\text{f}(k+1) = 7^{k+2} + 11^{k+1}$ $= 7^{k+2} + 11(4M - 7^{k+1})$ or Rearranges to get $\text{f}(k+1) = \ldots - p\,\text{f}(k)$	M1	
	$\text{f}(k+1) =$ a correct function that is a multiple of 4	A1	Shows that $\text{f}(k+1)$ is a multiple of 4 Eg $\text{f}(k+1) = 4 \times 11M + 7^{k+1}(7-11)$ $= 4(11M - 7^{k+1})$ Must include $\text{f}(k+1) =$
	Hence true for $n = k + 1$ (**) and since true for $n = 1$ (***) By **induction** $\text{f}(n)$ is a multiple of 4 for all integers $n \geq 1$ (****)	E1	Must have (*), (**), (***) present, previous 4 marks scored and a final statement (****) clearly indicating that it relates to all integers $n \geq 1$ and induction

	Question 5 Total	5	

Q	Answer	Marks	Comments
6(a)	$(r+1)^5-(r-1)^5$ $=r^5+5r^4+10r^3+10r^2+5r+1$ $-\left(r^5-5r^4+10r^3-10r^2+5r-1\right)$ $=10r^4+20r^2+2$	B1	Correct result from correct working, **AG**. Must have LHS, RHS and at least one line of intermediate working
		1	

Q	Answer	Marks	Comments
6(b)	$S_n = \sum_{r=1}^{n}\left\{(r+1)^5 - (r-1)^5\right\}$ $= 2^5 - 0^5$ $+ 3^5 - 1^5$ $+ 4^5 - 2^5$ $\vdots$ $+(n-1)^5 - (n-3)^5$ $+n^5 - (n-2)^5$ $+(n+1)^5 - (n-1)^5$	M1	Uses method of differences showing the first and last lines and at least two other lines so that a pair of values which cancel are seen
	$= n^5 + (n+1)^5 - 1$	A1	Correct answer
	States or uses $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ and $\sum_{r=1}^{n} 1 = n$	B1	
	$10\sum_{r=1}^{n} r^4 = n^5 + (n+1)^5 - 1$ $-20\times\frac{1}{6}n(n+1)(2n+1) - 2n$	m1	Use of $\sum_{r=1}^{n} r^2$, $\sum_{r=1}^{n} 1$ and their S_n to obtain an equation for $\sum_{r=1}^{n} r^4$
	$10\sum_{r=1}^{n} r^4 = 2n^5 + 5n^4 + 10n^3 + 10n^2 + 3n$ $-20\times\frac{1}{6}n(n+1)(2n+1)$	m1	$\sum_{r=1}^{n} 1$ and their S_n simplified
	$10\sum_{r=1}^{n} r^4 = n(n+1)(2n^3 + 3n^2 + 7n + 3)$ $-\frac{10}{3}n(n+1)(2n+1)$	m1	Attempts to factorise out $n(n+1)$ or $n(2n+1)$ or $(n+1)(2n+1)$
	$\sum_{r=1}^{n} r^4 = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$	A1	Correct result from correct working
		7	

	Question 6 Total	**8**	

Q	Answer	Marks	Comments
7	$\text{Area} = \frac{1}{2}\int_0^{\frac{\pi}{6}} 2(1+\tan\theta)\sec^4\theta\,d\theta$ $= \int_0^{\frac{\pi}{6}} \sec^4\theta\,d\theta + \int_0^{\frac{\pi}{6}} \tan\theta\sec^4\theta\,d\theta$	B1	$\frac{1}{2}\int_0^{\frac{\pi}{6}} r^2\,d\theta$ used with correct integrand, correct limits and $d\theta$. All seen here
	$\int_0^{\frac{\pi}{6}} \sec^4\theta\,d\theta = \int_0^{\frac{\pi}{6}} (1+\tan^2\theta)\sec^2\theta\,d\theta$ $= \int_0^{\frac{\pi}{6}} (\sec^2\theta + \tan^2\theta\sec^2\theta)\,d\theta$	M1	Valid method to integrate $\sec^4\theta$
	$\int_0^{\frac{\pi}{6}} \tan\theta\sec^4\theta\,d\theta = \int_0^{\frac{\pi}{6}} (\tan\theta\sec\theta)\sec^3\theta\,d\theta$	M1	Valid method to integrate $\tan\theta\sec^4\theta$
	$\text{Area} = \left[\tan\theta + \frac{1}{3}\tan^3\theta + \frac{1}{4}\sec^4\theta\right]_0^{\frac{\pi}{6}}$	A1	Integrates to obtain $p\tan\theta + q\tan^3\theta + r\sec^4\theta$
	$\text{Area} = \left[\frac{\sqrt{3}}{3} + \frac{1}{3}\left(\frac{\sqrt{3}}{3}\right)^3 + \frac{1}{4}\left(\frac{2\sqrt{3}}{3}\right)^4 - \frac{1}{4}\right]$ $= \frac{21+40\sqrt{3}}{108}$	A1	Can score **B0 M1M1A1A1**

Q	Answer	Marks	Comments
7 ALT	Area $= \frac{1}{2}\int_0^{\frac{\pi}{6}} 2(1+\tan\theta)\sec^4\theta \, d\theta$	B1	$\frac{1}{2}\int_0^{\frac{\pi}{6}} r^2 \, d\theta$ used with correct integrand, correct limits and $d\theta$. All seen here
	$t = \tan\theta$ $dt = \sec^2\theta \, d\theta$ Area $= \int_0^{\frac{1}{\sqrt{3}}} (1+t)(1+t^2) \, dt$	M1	Valid method using $t = \tan\theta$ to integrate $k\int_0^{\frac{\pi}{6}} (1+\tan\theta)\sec^4\theta \, d\theta$. Condone lack of or wrong limits
	Area $= \int_0^{\frac{1}{\sqrt{3}}} (1+t+t^2+t^3) \, dt$ $= \left[t + \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{4}t^4 \right]_0^{\frac{1}{\sqrt{3}}}$	A1	Correct Integration. Condone lack of or wrong limits
		A1	Correct limits, seen at any point PI correct substitution
	Area $= \left[\frac{1}{\sqrt{3}} + \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^3 + \frac{1}{4}\left(\frac{1}{\sqrt{3}}\right)^4 \right]$ $= \frac{21+40\sqrt{3}}{108}$	A1	Can score **B0 M1A1A1A1**

	Question 7 Total	**5**	

Q	Answer	Marks	Comments
8(a)	$0=\begin{vmatrix} 2 & -1 & 3 \\ 1 & k-1 & -1 \\ k-3 & -1 & 1 \end{vmatrix}$ $=2(k-1-1)+(1+k-3)+3\{-1-(k-1)(k-3)\}$	M1	Sets determinant $=0$ **and** attempts to expand
	$0=3k-6-12+12k-3k^2$ $=-3k^2+15k-18$ $0=k^2-5k+6$	B1	Obtains a correct quadratic expression
	$k=2$ or $k=3$	A1	Obtains correct values of k
		3	

Q	Answer	Marks	Comments
8(b)(i)	$2x - y + 3z = 1 \quad (\mathrm{I})$ $x + 2y - z = 3 \quad (\mathrm{II})$ $-y + z = 1 \quad (\mathrm{III})$	B1	Correct system of equations in the case $k=3$
	$(\mathrm{I})-2(\mathrm{II}) \Rightarrow -5y + 5z = -5$ $5(\mathrm{III}) \Rightarrow -5y + 5z = 5$	M1	Eliminating one variable in order to compare two simultaneous equations
	(Inconsistent) no solutions	A1	From comparing **correct** equations
		3	

Q	Answer	Marks	Comments
8(b)(ii)	Three planes form a (triangular) prism	E1	Must come from correct working in part **(b)(i)**
		1	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9	$\frac{dx}{d\theta} = \cos^2\theta - \sin^2\theta$		
	$\frac{dy}{d\theta} = 2\sin\theta\cos\theta$	B1	Correct expression for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$
	$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta$ $+4\sin^2\theta\cos^2\theta$	M1	Correctly expands their $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$
	$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta$ $= \left(\cos^2\theta + \sin^2\theta\right)^2$ $= 1$	A1	$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 1$ from correct working
	$S = 2\pi\int_0^{\frac{\pi}{6}} \sin^2\theta \times 1\,d\theta$	B1ft	Their correct integrand, with 2π, $d\theta$ and correct limits. All seen here
	$= \pi\int_0^{\frac{\pi}{6}} (1 - \cos 2\theta)\,d\theta$	M1	Use of $2\sin^2\theta = 1 - \cos 2\theta$
	$= \pi\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{6}}$	M1	Correctly integrating $k(1 \pm \cos 2\theta)$
	$= \pi\left[\frac{\pi}{6} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right]$	M1	Correct use of limits in an expression of the form $k\left(\theta \pm \frac{1}{2}\sin 2\theta\right)$
	$= \frac{\pi}{12}\left[2\pi - 3\sqrt{3}\right]$	A1	**CSO** – form of answer given

	Question 9 Total	8	

Q	Answer	Marks	Comments
10(a)	$(\beta-d)+\beta+(\beta+d)+(\beta+2d)=\frac{2}{1}$ $4\beta+2d=2$	M1	Use of $\sum\alpha=\frac{-b}{a}$ with arithmetic sequence structure. If $\alpha,\alpha+d,\alpha+2d,\alpha+3d$ $4\alpha+6d=2$ Condone –2 instead of 2
	$\frac{-21}{1}=(\beta-d)\beta+(\beta-d)(\beta+d)$ $+(\beta-d)(\beta+2d)+\beta(\beta+d)$ $+\beta(\beta+2d)+(\beta+d)(\beta+2d)$ $-21=6\beta^2+6\beta d-d^2$	M1	Use of $\sum\alpha\beta=\frac{c}{a}$ with arithmetic sequence structure. If $\alpha,\alpha+d,\alpha+2d,\alpha+3d$ $-21=6\alpha^2+18\alpha d+11d^2$ Condone +21 instead of –21
		A1	Obtains two correct, simultaneous equations
	$10\beta^2-10\beta-20=0$	M1	Eliminates one variable to get a three term quadratic equation If $\alpha,\alpha+d,\alpha+2d,\alpha+3d$ $10\alpha^2-10\alpha-200=0$
	$\beta=2, d=-3$ **or** $\beta=-1, d=3$	A1	Finds correct β and d If $\alpha,\alpha+d,\alpha+2d,\alpha+3d$ $\alpha=5, d=-3$ or $\alpha=-4, d=3$
	$\alpha=5$, $\beta=2$, $\gamma=-1$, $\delta=-4$ **or** $\alpha=-4$, $\beta=-1$, $\gamma=2$, $\delta=5$	A1	Correct values of α, β, γ and δ
		6	

Q	Answer	Marks	Comments
10(b)	$\frac{-p}{1} = 5\times2\times-1+5\times2\times-4$ $+5\times-1\times-4+2\times-1\times-4$ $p=22$	M1	Use of $\sum\alpha\beta\gamma = \frac{-d}{a}$ and their roots to get a value of p Condone p instead of – p
	$\frac{q}{1} = 5\times2\times-1\times-4$ $q=40$	M1	Use of $\alpha\beta\gamma\delta = \frac{e}{a}$ and their roots to get a value of q Condone $-q$ instead of q
	$p=22$ $q=40$	A1	Obtains the correct values of p and q If 0/3 scored **SC1** for writing $p=-\sum\alpha\beta\gamma$ and $q=\alpha\beta\gamma\delta$
		3	

	Question 10 Total	**9**	

Q	Answer	Marks	Comments
11(a)	$\mathbf{r}_1 = \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 0.5 \\ 2 \\ -1 \end{bmatrix}$	B1	Correct general point for L_1
	$\mathbf{r}_2 = \begin{bmatrix} 8 \\ w \\ 6 \end{bmatrix} + \mu \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$	B1	Correct general point for L_2 Condone same parametric variable as used for L_1
	$\begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 0.5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ w \\ 6 \end{bmatrix} + \mu \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$		
	x $0.5\lambda + 3\mu = 2$ z $\lambda + \mu = -1$	M1	Uses x and z general points to form a pair of simultaneous equations in two different parametric variables
	$\lambda = -2, \mu = 1$	A1	Correct values for the parametric variables
	y $7 - 2\times 2 = w + 1\times 2$ $w = 1$	A1	Correct w
		5	

Q	Answer	Marks	Comments
11(b)	$\mathbf{n} = \begin{bmatrix} 0.5 \\ 2 \\ -1 \end{bmatrix} \times \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$	M1	Vector product of their direction vectors from part **(a)**
	$d = \begin{bmatrix} 8 \\ 5 \\ 14 \end{bmatrix} \bullet \begin{bmatrix} 6 \\ 7 \\ 5 \end{bmatrix} = 153$	M1	Forms the scalar product of multiple of their normal vector and a position vector of a point in Π Condone use of a point on either of their lines
	$\mathbf{r} \bullet \begin{bmatrix} 8 \\ 5 \\ 14 \end{bmatrix} = 153$	A1	A correct equation of Π, in the specified form.
		3	

Q	Answer	Marks	Comments
11(c)(i)	$153=\begin{bmatrix}5+8\sigma\\-3+5\sigma\\1+14\sigma\end{bmatrix}\cdot\begin{bmatrix}8\\5\\14\end{bmatrix}$ $=40+64\sigma-15+25\sigma+14+196\sigma$	**M1**	Substitutes equation of perpendicular line from P to Π into equation of Π
	$\sigma=\frac{2}{5}$	**A1**	Correct parametric variable PI $\left(\frac{41}{5},-1,\frac{33}{5}\right)$
	$\sigma'=\frac{4}{5}$	**M1**	Doubles their parametric variable PI $\overrightarrow{PQ}=2\,\overrightarrow{PM}$
	$Q=\begin{bmatrix}\frac{57}{5}\\1\\\frac{61}{5}\end{bmatrix}$	**A1**	Correct position vector of Q Condone coordinates
		4	

Q	Answer	Marks	Comments
11(c)(ii)	$\frac{2}{5}\begin{bmatrix}8\\5\\14\end{bmatrix}$	**M1**	Uses a correct method to find the required distance. Eg their $\sigma\times\mathbf{n}$
	$\frac{2}{5}\sqrt{285}$	**A1**	**CAO**
		2	

	Question 11 Total	**14**	

Q	Answer	Marks	Comments
12	Auxiliary equation $m^2 - m - 2 = 0 \quad m = -1, 2$	M1	Finds solutions to the correct auxiliary equation
	$[y_{CF} =] \quad Ae^{-x} + Be^{2x}$	A1	Correct **CF**
	Particular Integral		
	$[y_{PI} =] \quad C\cos x + D\sin x + Exe^{-x}$	B1	Correct particular integral
	$[y'_{PI} =] \quad -C\sin x + D\cos x + Ee^{-x} - Exe^{-x}$ $[y''_{PI} =] \quad -C\cos x - D\sin x - 2Ee^{-x} + Exe^{-x}$	M1	Obtains first and second derivatives of their y_{PI}
	$\left(-C\cos x - D\sin x - 2Ee^{-x} + Exe^{-x}\right)$ $-\left(-C\sin x + D\cos x + Ee^{-x} - Exe^{-x}\right)$ $-2\left(C\cos x + D\sin x + Exe^{-x}\right)$ $= 6e^{-x} - 10\cos x$	M1	Substitutes their first and second derivatives of y_{PI} into DE **and** attempts to compare their coefficients
	$\cos x$ $\sin x$ $3C + D = 10$ $C - 3D = 0$	M1	Forms a pair of simultaneous equations from their coefficients of $\cos x$ and $\sin x$
	$C = 3, D = 1$	A1	Correct coefficients of $\cos x$ and $\sin x$ from correct equations
	e^{-x} $E = -2$	B1	Correct coefficient of xe^{-x}
	General Solution		
	$[f(x) =] Ae^{-x} + Be^{2x} + 3\cos x + \sin x$ $-2xe^{-x}$		
	$[f'(x) =] -Ae^{-x} + 2Be^{2x} - 3\sin x + \cos x$ $-2e^{-x} + 2xe^{-x}$	M1	Differentiates their $f(x)$, but must have exactly two arbitrary constants
	$A + B = 7$ $-A + 2B = 2$	M1	Uses given conditions to form a pair of simultaneous equations for their two arbitrary constants
	$A = 4, B = 3$	A1	Correct values for A and B, from correct equations
	$f(x) = 4e^{-x} + 3e^{2x} + 3\cos x + \sin x - 2xe^{-x}$	A1	Correct **ISW**

	Question 12 Total	**12**	

Q	Answer	Marks	Comments
13(a)(i)	$\lvert\mathbf{M}\rvert = 4(3-k-2)-(1-2k-4)-(k+1)(1-6)$ $=12+3k$	M1	Obtains a linear expression for $\lvert\mathbf{M}\rvert$
	$k \neq -4$	A1	Correct restriction on k
		2	

Q	Answer	Marks	Comments
13(a)(ii)	Cofactor matrix $\begin{bmatrix} 1-k & 3+2k & -5 \\ -2-k & 6+2k & -2 \\ 5+4k & -9-5k & 11 \end{bmatrix}$	M1 A1 A1	At least three entries correct At least six entries correct All nine entries correct
	Inverse matrix $\mathbf{M}^{-1}$ $\mathbf{M}^{-1} = \dfrac{1}{3(4+k)}\begin{bmatrix} 1-k & -2-k & 5+4k \\ 3+2k & 6+2k & -9-5k \\ -5 & -2 & 11 \end{bmatrix}$	M1 A1	Transpose of their cofactors with no more than one further slip **and** division by their $\lvert\mathbf{M}\rvert$ in terms of k Correct $\mathbf{M}^{-1}$
		5	

Q	Answer	Marks	Comments
13(b)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3(4+k)} \begin{bmatrix} 1-k & -2-k & 5+4k \\ 3+2k & 6+2k & -9-5k \\ -5 & -2 & 11 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$		
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3(4+k)} \begin{bmatrix} 4-4k-6-3k+5+4k \\ 12+8k+18+6k-9-5k \\ -20-6+11 \end{bmatrix}$	**M1**	Multiplies their $\mathbf{M}^{-1}$ onto $\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$ to form a single unsimplified vector
		A1ft	At least 2 entries correct for their $\mathbf{M}^{-1}$
	$x = \frac{1-k}{4+k}, \quad y = \frac{7+3k}{4+k}, \quad z = -\frac{5}{4+k}$	**A1**	Correct and fully simplified
		3	

	Question 13 Total	**10**	

Q	Answer	Marks	Comments
14(a)	$z - z^{-1} = 2\text{i}\sin\theta$	B1	Stated or used
	$\left(z - z^{-1}\right)^6 = z^6 - 6z^4 + 15z^2 - 20 + 15z^{-2} - 6z^{-4} + z^{-6}$	M1	Expands $\left(z - z^{-1}\right)^6$
	$(2\text{i}\sin\theta)^6 = z^6 + z^{-6} - 6\left(z^4 + z^{-4}\right) + 15\left(z^2 + z^{-2}\right) - 20$	A1	Forms a correct equation for $\sin^6\theta$ and expanded $\left(z - z^{-1}\right)^6$
	$z^n + z^{-n} = 2\cos n\theta$	B1	Stated or used for $n = 2$, 4 **or** 6
	$-64\sin^6\theta = 2\cos 6\theta - 6 \times 2\cos 4\theta + 15 \times 2\cos 2\theta - 20$	M1	RHS of previous line collected and $z^n + z^{-n} = 2\cos n\theta$ used for $n = 2$, 4 **and** 6
	$\sin^6\theta = \frac{1}{32}(-\cos 6\theta + 6\cos 4\theta - 15\cos 2\theta + 10)$	A1	Correct answer from correct working
		6	

Q	Answer	Marks	Comments
14(b)	$\int_0^{\frac{\pi}{6}} \sin^6\theta \, d\theta = \frac{1}{32}\left[-\frac{1}{6}\sin 6\theta + \frac{6}{4}\sin 4\theta - \frac{15}{2}\sin 2\theta + 10\theta\right]_0^{\frac{\pi}{6}}$	M1	Integrates their answer to part **(a)** correctly, provided all terms in integrand are of the form $\cos(n\theta)$
		A1	Correct integration
	$= \frac{1}{32}\left(-\frac{1}{6} \times 0 + \frac{3}{2} \times \frac{\sqrt{3}}{2} - \frac{15}{2} \times \frac{\sqrt{3}}{2} + \frac{10\pi}{6}\right)$ $= \frac{1}{32}\left(-3\sqrt{3} + \frac{5\pi}{3}\right)$ $= \frac{1}{96}\left(5\pi - 9\sqrt{3}\right)$	A1	Correct answer in the requested form, convincingly done **AG**
		3	

	Question 14 Total	**9**	

Q	Answer	Marks	Comments
15(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}+y\tan x=\frac{2x+5}{x^2+4x+5}\cos x$	M1	Dividing by $\cos x$ to obtain Standard Form
	I.F. is $\mathrm{e}^{\int \tan x\,\mathrm{d}x}=\mathrm{e}^{\ln(\sec x)}$	M1	Their I.F identified and integration attempted
	$=\sec x$	A1	Correct integrating factor
	$y\sec x=\int\left(\frac{2x+5}{x^2+4x+5}\right)\mathrm{d}x$	m1	Multiplying both sides of the **Standard Form** by their I.F and integrating LHS to get $y\times$I.F
	$\frac{2x+5}{x^2+4x+5}=\frac{2x+4}{x^2+4x+5}+\frac{1}{x^2+4x+5}$	M1	Splits RHS integrand into form $\frac{\mathrm{f}'(x)}{\mathrm{f}(x)}+\frac{A}{\mathrm{f}(x)}$
	$y\sec x=\int\left(\frac{2x+4}{x^2+4x+5}\right)\mathrm{d}x+\int\left(\frac{1}{(x+2)^2+1}\right)\mathrm{d}x$		
	$\int\left(\frac{2x+4}{x^2+4x+5}\right)\mathrm{d}x=\ln\left(x^2+4x+5\right)$	A1	Integrates $\frac{2x+4}{x^2+4x+5}$ to get $\ln\left(x^2+4x+5\right)$
	$\frac{1}{x^2+4x+5}=\frac{1}{(x+2)^2+1}$	M1	Completes the square in the denominator of $\frac{A}{\mathrm{f}(x)}$
	$\int\left(\frac{1}{(x+2)^2+1}\right)\mathrm{d}x=\tan^{-1}(x+2)$	A1	Integrates $\frac{A}{(x+2)^2+1}$ to get $A\tan^{-1}(x+2)$ Condone lack of $+c$
	$[\mathrm{f}(x)=]\cos x\left\{\ln\left(x^2+4x+5\right)+\tan^{-1}(x+2)+c\right\}$	A1	Correct GS
		9	

Q	Answer	Marks	Comments
15(b)	$\tan^{-1}(2) = \ln(5) + \tan^{-1}(2) + c$ $c = -\ln(5)$	M1	Substituting $y = \tan^{-1}(2)$ and $x = 0$ into their GS (with a constant of integration) to find a value of c
	$y = \cos x\left\{\ln\left(x^2+4x+5\right) + \tan^{-1}(x+2) - \ln(5)\right\}$ $= \cos x\left\{\ln\left(\dfrac{x^2+4x+5}{5}\right) + \tan^{-1}(x+2)\right\}$	A1	Correct PS **ACF**, must be $y = \cdots$
		2	

	Question 15 Total	**11**	