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FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$8+5\text{i}$	B1	oe
		1	

Q	Answer	Marks	Comments
1(b)	$-b = 8-5\text{i} + 8+5\text{i}$ $c = (8-5\text{i})(8+5\text{i})$	M1	Forms an equation in b (or c) only eg $-\frac{b}{2} = 8$ or Writes the sum of the roots or the product of the roots May be seen in an expansion of $(z-(8-5\text{i}))(z-(8+5\text{i}))$
	$b = -16$	A1	
	$c = 89$	A1	
		3	

	Question 1 Total	4	

Q	Answer	Marks	Comments
2(a)	When $x = a,\ y = 5a^2 - 4a$		
	When $x = a + h,\ y = 5(a+h)^2 - 4(a+h)$	**M1**	Writes a correct expression for the y-coordinate when $x = a + h$
	$y = 5a^2 + 10ah + 5h^2 - 4a - 4h$		
	Gradient $= \dfrac{5a^2 + 10ah + 5h^2 - 4a - 4h - (5a^2 - 4a)}{a + h - a}$	**M1**	Writes a correct expression for the gradient of the line
	$= \dfrac{10ah + 5h^2 - 4h}{h}$		
	$= 10a + 5h - 4$	**A1**	**oe**
		3	

Q	Answer	Marks	Comments
2(b)	Let $a = 3$ and $h \to 0$	**M1**	Considers $h \to 0$ Condone $h = 0$ seen **PI** by a correctly evaluated part **(a)** using $h = 0$
	Gradient $= 10 \times 3 + 5 \times 0 - 4$ $= 26$	**A1**	Must see **M1** Condone poor notation and poor bracket use **A0** if $h = 0$ seen
		2	

	Question 2 Total	**5**	

Q	Answer	Marks	Comments
3	$\dfrac{5+\mathrm{i}}{7-3\mathrm{i}}=\dfrac{(5+\mathrm{i})(7+3\mathrm{i})}{(7-3\mathrm{i})(7+3\mathrm{i})}$	M1	Multiplies numerator and denominator by the conjugate of $7-3\mathrm{i}$
	$=\dfrac{35+15\mathrm{i}+7\mathrm{i}-3}{49+21\mathrm{i}-21\mathrm{i}+9}$	M1	Correct numerator **or** denominator with an i^2 replaced with –1 May be simplified or unsimplified
	$=\dfrac{32+22\mathrm{i}}{58}$		
	$=\dfrac{16+11\mathrm{i}}{29}$	A1	Must include at least one intermediate fraction after the first **M1**

	Question 3 Total	**3**	

Q	Answer	Marks	Comments
4(a)(i)	$\frac{7}{3}$	B1	oe
		1	

Q	Answer	Marks	Comments
4(a)(ii)	$\frac{c}{3}$	B1	oe
		1	

Q	Answer	Marks	Comments
4(b)(i)	$\left(\alpha+\frac{5}{\beta}\right)\left(\beta+\frac{5}{\alpha}\right)=\alpha\beta+5+5+\frac{25}{\alpha\beta}$	M1	Correctly writes $\left(\alpha+\frac{5}{\beta}\right)\left(\beta+\frac{5}{\alpha}\right)$ in terms of $\alpha\beta$ **PI** by a correct substitution of their $\alpha\beta$
	$\frac{c}{3}+10+\frac{75}{c}$	A1	**oe** expression with no fractions within the numerator or denominator
		2	

Q	Answer	Marks	Comments
4(b)(ii)	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$ $=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$ $=\frac{\left(\frac{7}{3}\right)^2-2\left(\frac{c}{3}\right)}{\frac{c}{3}}$	M1	Correctly writes $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ in terms of $\alpha+\beta$ and $\alpha\beta$ **PI** by a correct substitution of their $\alpha+\beta$ and $\alpha\beta$
	$=\frac{49-6c}{3c}$	A1	**oe** expression with no fractions within the numerator or denominator
		2	

Q	Answer	Marks	Comments
4(b)(iii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1	Correctly writes $\alpha^3+\beta^3$ in terms of $\alpha+\beta$ and $\alpha\beta$ **PI** by a correct substitution of their $\alpha+\beta$ and $\alpha\beta$
	$=\left(\frac{7}{3}\right)^3-3\left(\frac{c}{3}\right)\left(\frac{7}{3}\right)$		
	$=\frac{343}{27}-\frac{7c}{3}$	A1	**oe** expression
		2	

	Question 4 Total	8	

Q	Answer	Marks	Comments
5(a)	$\tan^{-1}(1) = \frac{\pi}{4}$	M1	Correctly calculates $\tan^{-1}(1)$ **PI** by a correct solution eg $-\frac{\pi}{12}$
	$x + \frac{\pi}{3} = n\pi + \frac{\pi}{4}$ where $n \in \mathbb{Z}$	M1	Correct non-trigonometric general equation in x Condone n (or eg k) not defined
	$[x =]\ n\pi - \frac{\pi}{12}$	A1	**oe** eg $x = n\pi + \frac{11\pi}{12}$ Accept $\pm n\pi - \frac{\pi}{12}$
		3	

Q	Answer	Marks	Comments
5(b)	$\left(-\frac{13\pi}{12}\right) + \left(-\frac{\pi}{12}\right) + \left(\frac{11\pi}{12}\right) + \left(\frac{23\pi}{12}\right)$	M1	Identifies three correct roots May be unsimplified
	$= \frac{10\pi}{12} \times 2$		
	$= \frac{5\pi}{3}$	A1	
		2	

Q	Answer	Marks	Comments
5(c)	$\frac{10\pi}{12} \times m = 5\pi$	M1	Forms correct equation in m **ft** their part **(b)**
	$m = 6$	A1ft	**ft** their part **(b)** if $m > 0$
		2	

	Question 5 Total	7	

Q	Answer	Marks	Comments
6(a)	$x^2+2xy\mathrm{i}-y^2=45-28\mathrm{i}$ Comparing imaginary parts: $2xy=-28$	**M1**	Expands LHS and compares imaginary parts. Ignore sign errors. Accept $xy=-14$ (or $xy+14=0$) for this mark.
	$y=-\dfrac{14}{x}$	**A1**	**AG** Must be convincingly shown. If $xy=-14$ (or $xy+14=0$) is seen instead of $2xy=-28$ (or $2xy+28=0$), then either $x^2-y^2=45$ must also be seen or a statement that imaginary parts have been compared.
		2	

Q	Answer	Marks	Comments
6(b)	Comparing real parts: $x^2-y^2=45$ $k=45$	**B1**	
		1	

Q	Answer	Marks	Comments
6(c)	$x^2-\left(-\dfrac{14}{x}\right)^2=45$	**M1**	Forms a correct equation in x (or y) only **ft** their part **(b)**
	$x^4-45x^2-196=0$		
	$x^2=49$	**A1**	Correct value for x or y or x^2 or y^2 **PI** by a correct root
	$y^2=4$	**M1**	Substitutes their value of x or x^2 to find a value of y or y^2 (or vice versa)
	$z=7-2\mathrm{i}$, $z=-7+2\mathrm{i}$	**A1**	**oe** Accept $\pm 7\mp 2\mathrm{i}$ but not $\pm 7\pm 2\mathrm{i}$
		4	

	Question 6 Total	**7**	

Q	Answer	Marks	Comments
7(a)	$\log\left(1+\dfrac{2}{r}\right) = \log\left(\dfrac{r+2}{r}\right)$ $= \log(r+2) - \log(r)$	**B1**	Must include at least one intermediate stage eg $\log\left(\dfrac{r+2}{r}\right)$ or $\log\left(\dfrac{Ar+B}{r}\right)$ Accept RHS shown equal to $\log\left(A+\dfrac{B}{r}\right)$ with $A=1$ and $B=2$ given
		1	

Q	Answer	Marks	Comments
7(b)	$\sum_{r=1}^{2n}\log\left(1+\frac{2}{r}\right)=\sum_{r=1}^{2n}\left(\log(r+2)-\log(r)\right)$	M1	Writes at least 3 pairs of logs of the form $\log(r+2)-\log(r)$
	$=\log(3)-\log(1)$ $+\log(4)-\log(2)$ $+\log(5)-\log(3)$ $+\ldots$ $+\log(2n)-\log(2n-2)$ $+\log(2n+1)-\log(2n-1)$ $+\log(2n+2)-\log(2n)$	M1	Includes at least one pair of cancelling logs, eg $\log(3)$ and $-\log(3)$ **PI** by all four terms of $\log(2n+2)+\log(2n+1)-\log2-\log1$
	$=\log(2n+2)+\log(2n+1)-\log(2)-\log(1)$ $=\log\left(\frac{2n+2}{2}\right)+\log(2n+1)$	A1	Correctly expresses the required sum in terms of no more than 4 logs
	$=\log(n+1)+\log(2n+1)$	A1	**CAO** with M1M1 awarded **SC1** $\log\left(\frac{3}{1}\times\frac{4}{2}\times\frac{5}{3}\times\ldots\times\frac{2n}{2n-2}\times\frac{2n+1}{2n-1}\times\frac{2n+2}{2n}\right)$ **SC2** $\log\left(\frac{3}{1}\times\frac{4}{2}\times\frac{5}{3}\times\ldots\times\frac{2n}{2n-2}\times\frac{2n+1}{2n-1}\times\frac{2n+2}{2n}\right)$ $=\log\left(\frac{2n+1}{1}\times\frac{2n+2}{2}\right)$ **SC3** $\log\left(\frac{3}{1}\times\frac{4}{2}\times\frac{5}{3}\times\ldots\times\frac{2n}{2n-2}\times\frac{2n+1}{2n-1}\times\frac{2n+2}{2n}\right)$ $=\log\left(\frac{2n+1}{1}\times\frac{2n+2}{2}\right)$ $=\log(2n+1)+\log(n+1)$
		4	

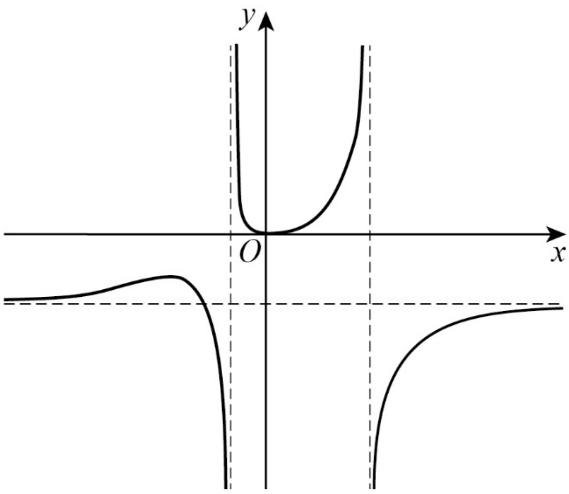
Q	Answer	Marks	Comments
7(c)	$\sum_{r=1}^{1200} \log\left(1+\frac{2}{r}\right) = \log(600+1)+\log(2\times 600+1)$	M1	Substitutes $n = 600$ into their part **(b)** of the correct form Allow $\log(Cn+D)+\log(En+F)$ for non-integer C, D, E, F
	$= \log(601\times 1201)$	A1	**ACF** **PI** by correct answer
	$= \log(721801)$	A1	May follow any correct expression in part **(b)** eg $\log(2n+2)+\log\left(n+\frac{1}{2}\right)$
		3	

	Question 7 Total	8	

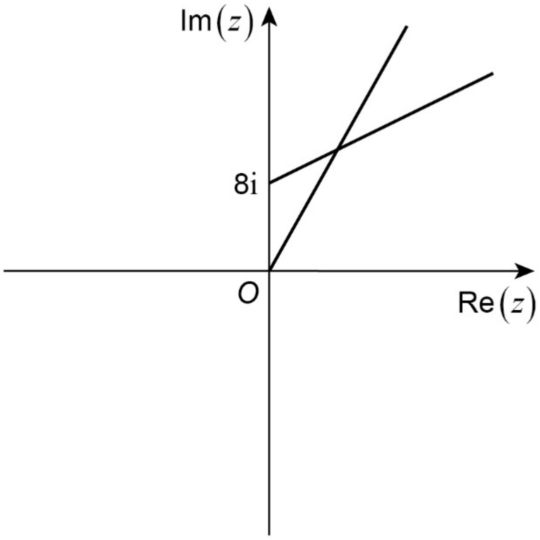
Q	Answer	Marks	Comments
8(a)	$y = -1$	B1	Correct horizontal asymptote
	$4 + 3x - x^2 = 0$	M1	Attempts to solve denominator = 0 **PI** by one correct vertical asymptote
	$\Rightarrow \quad x = -1$, $x = 4$	A1	Correct vertical asymptotes Maximum 2 marks if any incorrect asymptotes
		3	

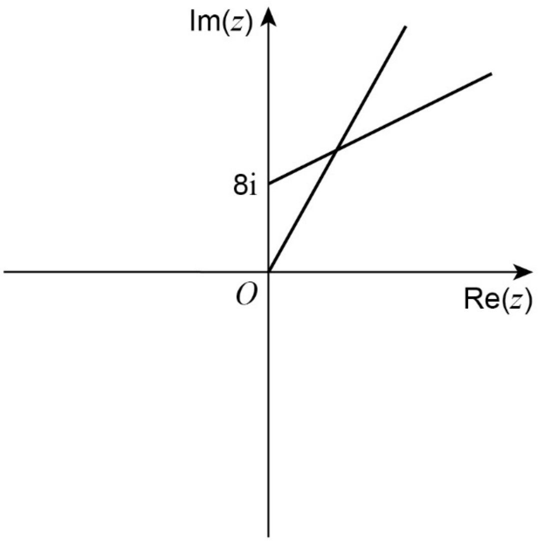
Q	Answer	Marks	Comments
8(b)	$k = \dfrac{x^2}{4 + 3x - x^2}$ $k\left(4 + 3x - x^2\right) = x^2$	M1	Forms a quadratic equation of the form $ax^2 + bx + c = 0$ with coefficients in terms of k
	$(k+1)x^2 - 3kx - 4k = 0$	A1	Correct quadratic equation in x with coefficients in terms of k Condone missing = 0 **PI** by correct further work
	At least one real root, so $(-3k)^2 - 4(k+1)(-4k) \geq 0$	M1	Correct discriminant in terms of k for their quadratic equation **PI** by $(-3k)^2 \geq 4(k+1)(-4k)$
	$25k^2 + 16k \geq 0$	M1	Sets their discriminant ≥ 0 Accept > 0 **PI** by a correct range for their quadratic equation
	$k \leq -\dfrac{16}{25}$ or $k \geq 0$	A1	Accept $<$ for $\leq$ and $>$ for $\geq$ **ACF** eg $k \in (-\infty, -0.64] \cup [0, \infty)$
		5	

Q	Answer	Marks	Comments
8(c)	Let $k=-1$, so $3x+4=0$	M1	Equates $\dfrac{x^2}{4+3x-x^2}$ with their y-value from part **(a)** **PI** by a correct **ft** x-value
	$\Rightarrow \quad x=-\dfrac{4}{3}$ Required point is $\left(-\dfrac{4}{3},-1\right)$	A1	Accept $x=-\dfrac{4}{3}$ and $y=-1$ if unambiguous
		2	

Q	Answer	Marks	Comments
8(d)		B1	Correct RHS and correctly approaching the asymptotes
		M1	Maximum point to the left of the left-hand vertical asymptote, above the horizontal asymptote and below the x-axis
		A1	Correct LHS and correctly approaching the asymptotes
		3	

	Question 8 Total	**13**	

Q	Answer	Marks	Comments
9(a)(i)		M1 A1	Line from (or through) *O* into the 1st quadrant Half-line from *O* into 1st quadrant at a steeper gradient than their other line If no other line is drawn then it must be at an angle of more than 45° from the real axis Condone an unruled line. Mark intention.
		2	

Q	Answer	Marks	Comments
9(a)(ii)		M1 A1	Line from (or through) the positive imaginary axis into the 1st quadrant at a positive angle from the real axis Half-line from 8i into 1st quadrant at a shallower gradient than their other line If no other line is drawn then it must be at an angle of less than 45° from the real axis Accept the imaginary intercept labelled as 8. Condone $(0, 8)$ Condone an unruled line. Mark intention.
		2	

Q	Answer	Marks	Comments
9(b)	$\dfrac{r}{\sin\left(\dfrac{2\pi}{3}\right)} = \dfrac{8}{\sin\left(\dfrac{\pi}{6}\right)}$	**M1**	Forms a correct equation in the modulus (or the real or imaginary part) only
	$r = 8\sqrt{3}$	**A1**	Calculates the correct value of the modulus (or the real or imaginary part)
	Required complex number is $8\sqrt{3}\left(\cos\left(\dfrac{\pi}{3}\right) + \mathrm{i}\sin\left(\dfrac{\pi}{3}\right)\right)$		
	$= 4\sqrt{3} + 12\mathrm{i}$	**A1**	**oe** eg $\sqrt{48} + \mathrm{i}12$
		3	

Q	Answer	Marks	Comments
9(c)(i)	$8\mathrm{i} - \left(4\sqrt{3} + 12\mathrm{i}\right)$	**M1**	Full method for *R* or at least one correct part
	$= -4\sqrt{3} - 4\mathrm{i}$	**A1**	**oe** eg $-\sqrt{48} - \mathrm{i}4$
		2	

Q	Answer	Marks	Comments
9(c)(ii)	Area of triangle *OPQ* $= \dfrac{1}{2} \times 8 \times 8\sqrt{3} \times \sin\left(\dfrac{\pi}{6}\right)$	**M1**	Full method for area of parallelogram *OPQR* or triangle *OPQ* (or *ORQ*)
	$= 16\sqrt{3}$		
	Area of parallelogram *OPQR* $= 32\sqrt{3}$	**A1**	**oe** eg $\sqrt{3072}$
		2	

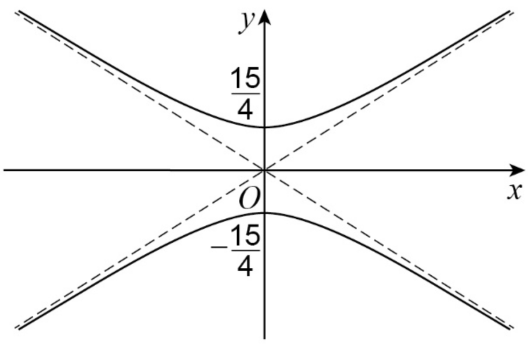
	Question 9 Total	**11**	

Q	Answer	Marks	Comments
10(a)	$\sqrt{(x-0)^2+(y-5)^2} = \pm 3(y+5)$	M1	Correct distance from P to $(0,5)$ or correct distance from P to $y=-5$ **PI** by $x^2+(y-5)^2=3^2(y+5)^2$
		M1	Forms a correct equation in terms of x and y
	$x^2+y^2-10y+25=9(y^2+10y+25)$	M1	Correctly removes square root **PI** by $x^2+(y-5)^2=3^2(y+5)^2$
	$x^2 = 8y^2+100y+200$	A1	**CAO** with M1M1M1 awarded Condone $y--5$ in the place of $\pm(y--5)$
		4	

10(b)(i)	$x^2 = 8\left(y^2+\frac{25}{2}y\right)+200$ $x^2 = 8\left(\left(y+\frac{25}{4}\right)^2-\frac{625}{16}\right)+200$	M1	Calculates $\frac{100}{2\times 8}$ Accept as part of an expression Accept $-\frac{100}{2\times 8}$
	Translation vector is $\begin{bmatrix} 0 \\ \frac{25}{4} \end{bmatrix}$	M1	Indicates the translation is parallel to the y-axis eg writes a vector of the form $\begin{bmatrix} 0 \\ y \end{bmatrix}$ for any non-zero y
		A1	**CAO**
		3	

Q	Answer	Marks	Comments
10(b)(ii)	$x^2 = 8\left(y+\frac{25}{4}\right)^2-\frac{625}{2}+200$		
	$b=-\frac{225}{2}$	B1ft	**ft** Their a value
		1	

Q	Answer	Marks	Comments
10(b)(iii)	$\dfrac{225}{2} = 8y^2 - x^2$		
	$1 = \dfrac{16y^2}{225} - \dfrac{2x^2}{225}$	**M1**	Writes the equation of H_2 in the form $py^2 - qx^2 = 1$ (or $px^2 - qy^2 = 1$ if their b is positive) where p and q are positive **PI** by a correct asymptote **ft** their part **(b)(ii)**
	Asymptotes of H_2 are $\dfrac{16y^2}{225} = \dfrac{2x^2}{225}$	**M1**	Writes at least one asymptote equation for their $py^2 - qx^2 = 1$ (or $px^2 - qy^2 = 1$ if their b is positive) Award M1M1 for $x^2 = 8y^2$ oe
	$y = \pm \dfrac{x}{4}\sqrt{2}$	**A1**	
		3	

Q	Answer	Marks	Comments
10(b)(iv)	 	**B1ft**	Correct shape for their b
		B1ft	Two asymptotes drawn through the origin with correct curve behaviour relative to the asymptotes **ft** Their b
		B1	Correct y-intercepts
		3	

	Question 10 Total	**14**	