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INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

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251XFM05/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$0.5v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -5v^2$	**M1**	Forms differential equation with $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$
	$\dfrac{1}{v}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -10$	**M1**	Separates variables
	$\ln v = -10x + c$	**A1**	Correct integration
	$x = 0, v = 50 \Rightarrow c = \ln 50$	**A1**	Correct constant of integration
	$\ln\left(\dfrac{v}{50}\right) = -10x$ $v = 50\mathrm{e}^{-10x}$	**A1**	Correct expression Oe, for example $v = \mathrm{e}^{-10x+\ln 50}$
		5	

Q	Answer	Marks	Comments
1(b)	$25 = 50\mathrm{e}^{-10x}$ $\ln(0.5) = -10x$	**M1**	Equates 25 to their expression to form an equation
	$x = \dfrac{1}{10}\ln 2$ [m]	**A1**	Correct distance, **ACF** for example $x = -\dfrac{1}{10}\ln\left(\dfrac{1}{2}\right)$
		2	

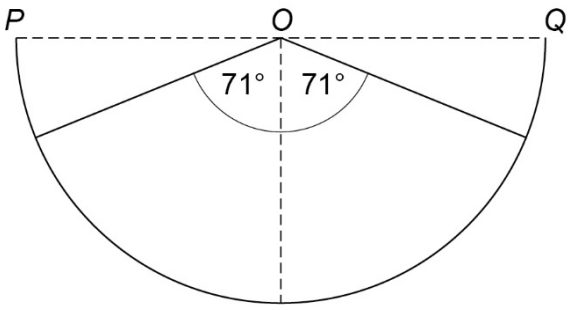
	Question 1 Total	**7**	

Q	Answer	Marks	Comments
2(a)	$3\begin{bmatrix}3\\2\end{bmatrix}+5\begin{bmatrix}1.6\\-1\end{bmatrix}=3\mathbf{v}+5\begin{bmatrix}1\\-0.4\end{bmatrix}$	M1 A1	**M1**: Equation for conservation of momentum **A1**: Correct equation
	$\mathbf{v}=\begin{bmatrix}4\\1\end{bmatrix}\ \left[\text{m s}^{-1}\right]$	A1	Correct velocity
		3	

Q	Answer	Marks	Comments
2(b)	$\mathbf{I}=5\begin{bmatrix}1\\-0.4\end{bmatrix}-5\begin{bmatrix}1.6\\-1\end{bmatrix}$	M1	Uses impulse formula
	$=\begin{bmatrix}-3\\3\end{bmatrix}$	A1	Correct impulse vector
	$\lvert\mathbf{I}\rvert=\sqrt{18}=3\sqrt{2}\ \left[\text{N s}\right]$	A1	Correct magnitude OE for example $\sqrt{18}$
		3	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$\frac{1}{2}mv^2 = mga\cos\theta$	**M1**	Uses conservation of energy
	$v^2 = 2ga\cos\theta$	**A1**	Correct energy equation
	$R - mg\cos\theta = \frac{mv^2}{a}$	**M1A1**	**M1** Resolves radially Must see $R - mg\cos\theta$ **A1** Correct equation
	$R - mg\cos\theta = 2mg\cos\theta$		
	$R = 3mg\cos\theta$	**A1**	Eliminates v^2 to obtain correct expression from correct working
		5	

Q	Answer	Marks	Comments
3(b)	$mg = 3mg\cos\theta$ $\cos\theta = \frac{1}{3}$ $\theta = 71°$	**B1**	Finds 71 or 70.5
		**B1**	Both correct positions with angles shown Do not award if more than two positions shown.
		2	

	Question 3 Total	**7**	

Q	Answer	Marks	Comments
4(a)	$3^2 = \omega^2\left(a^2 - 4^2\right)$	**M1**	Forms two equations
	$5^2 = \omega^2\left(a^2 - 2^2\right)$	**A1**	Two correct equations
	$\dfrac{3^2}{\left(a^2 - 4^2\right)} = \dfrac{5^2}{\left(a^2 - 2^2\right)}$	**M1**	Eliminates ω
	$9a^2 - 36 = 25a^2 - 400$ $16a^2 = 364$	**A1**	Correct equation in an exact form
	$a = \dfrac{\sqrt{91}}{2}$ [m]	**A1**	Correct amplitude
		5	

Q	Answer	Marks	Comments
4(b)	$9 = \omega^2\left(\dfrac{91}{4} - 16\right)$	**M1**	Forms equation for ω^2 using their amplitude
	$\omega = \dfrac{2}{\sqrt{3}}$	**A1**	Correct ω
	Period $= 2\pi \times \dfrac{\sqrt{3}}{2}$ $= \pi\sqrt{3}$ [s]	**A1**	Correct period
		3	

Q	Answer	Marks	Comments
4(c)	$-4 = \frac{\sqrt{91}}{2}\cos\left(\frac{2}{\sqrt{3}}t_1\right)$ or $-4 = \frac{\sqrt{91}}{2}\sin\left(\frac{2}{\sqrt{3}}t_1\right)$	**M1**	Forms equation for one displacement
	$t_1 = 2.2218...$ or $t_1 = -0.8615$	**A1**	Correct time
	$2 = \frac{\sqrt{91}}{2}\cos\left(\frac{2}{\sqrt{3}}t_2\right)$ or $2 = \frac{\sqrt{91}}{2}\sin\left(\frac{2}{\sqrt{3}}t_2\right)$		
	$t_2 = 0.9856...$ or $t_2 = 0.3747$	**A1**	Correct second time
	Time $= 2.2218... - 0.9856... = 1.24$ [s] or Time $= 0.3747 + 0.8615 = 1.24$ [s]	**A1**	Correct total time AWRT 1.24
		4	

	Question 4 Total	**12**	

Q	Answer	Marks	Comments
5(a)	Work Done $= \int_0^{\infty} 2\mathrm{e}^{-\frac{x}{10}}\mathrm{d}x$	**M1**	Sets up correct integral (Condone incorrect or missing limits)
	$= \lim_{a\to\infty} \int_0^{a} 2\mathrm{e}^{-\frac{x}{10}}\mathrm{d}x$	**B1**	Uses a limiting process
	$= \lim_{a\to\infty} \left[-20\mathrm{e}^{-\frac{x}{10}} \right]_0^{a}$	**A1**	Correct integration
	$= \lim_{a\to\infty} \left(20 - 20\mathrm{e}^{-\frac{a}{10}} \right)$ $= 20 \ [\mathrm{J}]$	**A1**	Correct value of the limit
		4	

Q	Answer	Marks	Comments
5(b)	$\frac{1}{2} \times 0.25v^2 = 20$	**M1**	Uses work done $=$ change in KE
	$v = 4\sqrt{10}$	**A1**	Correct max speed
	$0 \le v < 4\sqrt{10} \quad \left[\mathrm{m\ s^{-1}}\right]$	**A1**	Correct inequality Condone any inequalities with the correct orientation.
		3	

	Question 5 Total	**7**	

Q	Answer	Marks	Comments
6(a)	$\frac{14}{0.5}e = 3.2\times 9.8$	**M1**	Equation to find extension at maximum speed
	$e = 1.12$	**A1**	Correct extension
	$\frac{1}{2}\times 3.2v^2 + 3.2\times 9.8\times 1.38 + \frac{1}{2}\times\frac{14}{0.5}\times 1.12^2 = \frac{1}{2}\times\frac{14}{0.5}\times 2.5^2$	**M1**	Four term energy equation with at least two correct terms
	$1.6v^2 + 43.2768 + 17.5616 = 87.5$	**A1**	Correct energy equation
	$v = 4.082...$ $= 4.1$ $\left[\text{m s}^{-1}\text{ to 2sf}\right]$	**A1**	Correct maximum speed
		5	

Q	Answer	Marks	Comments
6(b)(i)	Initial EPE = 87.5 [J] GE gained $= 3.2\times 9.8\times 2.5 = 78.4$ $\left[\text{J}\right]$ KE = 9.1 [J] v = 2.38 [m s^{-1}]	**B1**	Compares EPE and GPE or obtains KE or speed.
	As the sphere still has kinetic energy when it has risen 2.5 metres, it will continue to rise and the string will become slack	**E1**	Correct explanation
		2	

Q	Answer	Marks	Comments
6(b)(ii)	$87.5 = 3.2\times 9.8h$	**M1**	Energy equation
	$h = 2.790...$ Minimum distance $= 0.2098\ldots$ $= 0.21$ [m to 2 sf]	**A1**	Correct minimum distance AWRT 0.21
		2	

Q	Answer	Marks	Comments
6(c)	Energy lost $= 3.2\times 9.8\times(0.41-0.21)$ $= 6.272$ [J]	M1	Finds energy lost using their height from **part (b)(i)**
	$6.272 = (3-0.41)\times A$	M1	Uses work done $= Fd$
	$A = 2.421...$ $= 2.4$ [N to 2sf]	A1	Correct air resistance **AWRT** 2.4 from correct working.
		3	

ALTERNATIVE

Q	Answer	Marks	Comments
6(c)	$87.5 = 3.2\times 9.8\times(3-0.41)+A(3-0.41)$	M1M1	**M1** Forms energy equation **M1** Uses work done $= Fd$
	$A = 2.4237...$ $= 2.4$ [N to 2sf]	A1	Correct air resistance **AWRT** 2.4
		3	

	Question 6 Total	**12**	

Q	Answer	Marks	Comments
7(a)	Let θ be the angle between the string and the vertical at time t		
	$m \times d\ \ddot{\theta} = -mg\sin\theta$	M1 A1	**M1**: Forms differential equation **A1**: Correct differential equation
	$\sin\theta \approx \theta$	B1	Uses small angle approximation
	$\ddot{\theta} \approx -\frac{g}{d}\theta$		
	As [angular] acceleration proportional to the [angular] displacement and in the opposite sense	E1	Concludes that motion is SHM from correct working
		4	

Q	Answer	Marks	Comments
7(b)	$\text{Period} = 2\pi\sqrt{\frac{0.8}{9.8}} = \frac{4\pi}{7}$	B1	Correct period **PI**
	$\alpha = \frac{\pi}{10}\sin\left(3.5 \times \frac{\pi}{14}\right) = 0.222...$	M1	Finds angle for half of motion PI by 0.444
	$\text{Distance} = 2 \times 0.8 \times 0.222... = 0.355\ [\text{m}]$	A1	Correct distance
		3	

	Question 7 Total	**7**	

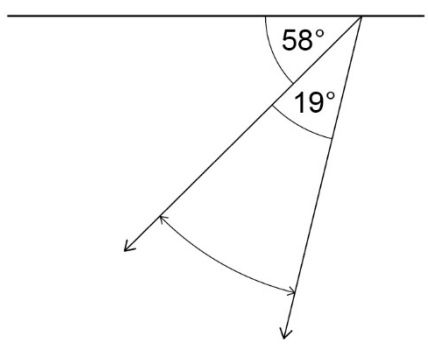
Q	Answer	Marks	Comments
8(a)	After impact $v_x = 10$ $v_y = e_1 10\sqrt{3}$	**B1**	Finds correct speeds after impact
	$\tan 30° = \dfrac{e_1 10\sqrt{3}}{10}$	**M1**	uses tan 30° to form an equation
	$\dfrac{1}{\sqrt{3}} = \dfrac{e_1 10\sqrt{3}}{10}$	**A1**	Correct equation
	$e_1 = \dfrac{1}{3}$	**A1**	Correct value for e_1
		4	

Q	Answer	Marks	Comments
8(b)	$2 = 10t - \frac{1}{2} \times 9.8\sin 60° t^2$	**M1A1**	**M1** Equation for motion parallel to the inclined plane **A1** Correct equation
	$t = 0.2206...$ or $2.1358...$	**A1**	Solves to get two correct times
	$10\sqrt{3}e_2 t - \frac{1}{2} \times 9.8\cos 60° t^2 = 0$	**M1**	Equation for motion perpendicular to the inclined plane
	$e_2 = \frac{4.9\cos 60° t}{10\sqrt{3}}$	**A1**	Expression for e_2
	$e_2 = 0.03$ or 0.30	**A1**	Two correct values for e_2
		6	

Q	Answer	Marks	Comments
8(b) ALT	$10\sqrt{3}e_2 t - \frac{1}{2} \times 9.8\cos 60° t^2 = 0$	**M1**	Equation for motion perpendicular to the inclined plane
	$t = \frac{20\sqrt{3}e_2}{9.8\cos 60°} = \frac{40\sqrt{3}e_2}{9.8}$	**A1** **A1**	Correct equation Expression for t
	$2 = 10\left(\frac{40\sqrt{3}e_2}{9.8}\right) - \frac{1}{2} \times 9.8\sin 60°\left(\frac{40\sqrt{3}e_2}{9.8}\right)^2$	**M1**	Equation for motion parallel to the inclined plane in terms of e_2
	$\left(\frac{1200\sqrt{3}}{9.8}\right)e_2{}^2 - \left(\frac{400\sqrt{3}}{9.8}\right)e_2 + 2 = 0$	**A1**	Correct equation
	$e_2 = 0.03$ or 0.30	**A1**	Two correct values for e_2
		6	

	Question 8 Total	**10**	

Q	Answer	Marks	Comments
9(a)	Along the line of centres		
	$3\times 7\cos 30^\circ - 2\times 6\cos 45^\circ = 3v_A + 2v_B$	**M1A1**	**M1** Equation for conservation of momentum **A1** Correct equation
	$\frac{21\sqrt{3}}{2} - 6\sqrt{2} = 3v_A + 2v_B$		
	$v_B - v_A = -e\left(-6\cos 45^\circ - 7\cos 30^\circ\right)$	**M1A1**	**M1** Equation based on coefficient of restitution **A1** Correct equation
	$v_B - v_A = e\left(3\sqrt{2} + \frac{7\sqrt{3}}{2}\right)$		
	$\frac{21\sqrt{3}}{2} - 6\sqrt{2} = 5v_A + 2e\left(3\sqrt{2} + \frac{7\sqrt{3}}{2}\right)$		
	$v_A = \frac{1}{5}\left(\frac{21\sqrt{3}}{2} - 6\sqrt{2} - 2e\left(3\sqrt{2} + \frac{7\sqrt{3}}{2}\right)\right)$	**M1**	Equation for v_A in terms of e
	$\frac{7\sqrt{3} - 24\sqrt{2}}{10} \le v_A \le \frac{7\sqrt{3} - 12\sqrt{2}}{6}$		
	$-2.1816... \le v_A \le -0.8077...$	**A1**	Correct inequality PI by ± 0.8077 and ± 2.1816
	Perpendicular to the line of centres		
	$v_A = 7\cos 60^\circ = 3.5$	**B1**	Perpendicular component
	Speed		
	$3.59 \le V_A \le 4.12$	**A1**	Correct inequality for the speed
		8	

Q	Answer	Marks	Comments
9(b)	$\tan\theta_1 = \dfrac{3.5}{0.8077}$	**M1**	Uses tan to find at least one angle
	$\theta_1 = 77[.0]°$	**A1**	One correct angle Allow 13°
	$\tan\theta_2 = \dfrac{3.5}{2.181}$		
	$\theta_2 = 58[.1]°$	**A1**	Second correct angle Allow 32°
		**A1**	Correct diagram
		4	

	Question 9 Total	**12**	