OXFORD AQA

INTERNATIONAL QUALIFICATIONS

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA04) Unit S2 Statistics

Thursday 6 June 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 The continuous random variables X_1, X_2 and X_3 are independent.

Their means and variances are shown in the table below:

	Mean	**Variance**
X_1	5	4
X_2	4	1
X_3	7	a

where a is a constant.

1 (a) Find $\mathrm{E}\left(\sum_{i=1}^{3} X_i\right)$

[1 mark]

Answer _____

1 (b) It is given that $\mathrm{Var}\left(\sum_{i=1}^{3} X_i\right) = 6a$

Find the value of a

[2 marks]

Answer _____

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1 (c) Find $\mathrm{E}\left(2X_1 - 3X_2\right)$

[2 marks]

Answer _____

1 (d) Find $\mathrm{Var}\left(2X_1 - 3X_2\right)$

[2 marks]

Answer _____

7

Turn over for the next question

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2 The random variable X represents the number of accidents per day at a ski resort.

Assume that X can be modelled by a Poisson distribution with parameter 15

2 (a) Find the probability of exactly 8 accidents occurring on a given day.

Give your answer to three significant figures.

[2 marks]

Answer _____

2 (b) The mean number of accidents per day is μ

The standard deviation of the number of accidents per day is σ

2 (b) (i) State the value of μ and the value of σ

[1 mark]

$\mu =$ _____ $\sigma =$ _____

Do not write outside the box

2 (b) (ii) Find $P(\mu - \sigma < X < \mu + \sigma)$

Give your answer to three significant figures.

[4 marks]

Answer _____

2 (c) Give a reason why the Poisson distribution for the number of accidents per day may **not** be a good model for the number of people injured per day.

[1 mark]

8

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3 Chen runs a company which uses biofuel to power machinery.

It is known that the biofuel used by Chen provides a mean energy of 25.1 megajoules per litre.

Chen puts an additive in the biofuel with the aim of increasing the mean energy per litre.

Chen tests 60 random samples.

Each sample consists of one litre of biofuel.

He measures the energy provided in megajoules. The results are summarised below.

$$\sum x = 1572 \quad \text{and} \quad \sum x^2 = 41\,850$$

Test whether the mean amount of energy per litre has increased, using the 1% level of significance.

[10 marks]

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10

Turn over for the next question

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4 The continuous random variable X has probability density function $\mathrm{f}(x)$ defined by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{k}x^2 & 1 \leq x < 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is an integer.

4 (a) Show that $k = 21$

[3 marks]

4 (b) Find $\mathrm{P}(X > 3)$

Give your answer in an exact form.

[2 marks]

Answer _____

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4 (c) (i) Find $\mathrm{E}\left(5+2X^{-1}\right)$

Give your answer in an exact form.

[4 marks]

Answer _____

Question 4 continues on the next page

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4 (c) (ii) Find $\text{Var}\left(5+2X^{-1}\right)$

Give your answer in an exact form.

[6 marks]

Answer _____

15

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5 Random samples X_1, X_2 and X_3 are taken from a population with unknown mean μ

5 (a) State, with a reason, whether

$$\frac{1}{2}X_1+\frac{1}{3}X_2+\frac{1}{4}X_3$$

is a statistic.

[2 marks]

5 (b) State, with a reason, whether

$$\frac{1}{3}\sum_{i=1}^{3}X_i^2-\mu^2$$

is a statistic.

[2 marks]

4

Turn over ►

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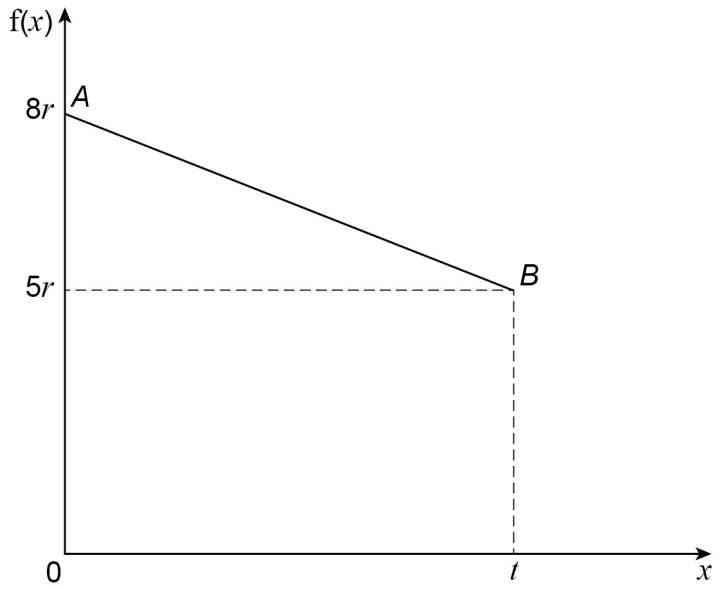
6 A continuous random variable X has probability density function f

The graph of $\mathrm{f}(x)$ is shown in **Figure 1** and consists of a line segment AB for $0 \leq x \leq t$ where t is a positive constant.

It is given that $\mathrm{f}(0) = 8r$ and $\mathrm{f}(t) = 5r$ where r is a positive constant.

For $x < 0$ and $x > t$ $\mathrm{f}(x) = 0$

Figure 1



6 (a) The continuous random variable X has cumulative distribution function F

Show that for $0 \leq x \leq t$

$$\mathrm{F}(x) = \frac{16}{13t}x - \frac{3}{13t^2}x^2$$

[5 marks]

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6 (b) It is given that $F(4) = \frac{1}{13t}$

Find the value of t

Give your answer in an exact form.

[1 mark]

Answer _____

6

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7 The mass, B grams, of each bolt produced by a machine can be modelled by

$$B \sim \mathrm{N}\left(0.8,\ 0.006^2\right)$$

7 (a) Find the probability that a randomly selected bolt has a mass between 0.805 and 0.815 grams.

Give your answer to three significant figures.

[3 marks]

Answer _____

7 (b) The mass, W grams, of each wingnut produced by a machine can be modelled by

$$W \sim \mathrm{N}\left(1,\ 0.005^2\right)$$

A random sample of n wingnuts is taken.

7 (b) (i) In the case when $n = 25$, find $\mathrm{P}\left(\overline{W} > 1.0015\right)$

Give your answer to three significant figures.

[4 marks]

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Answer

7 (b) (ii) In the case when $P\left(\overline{W} > 0.9992\right) > 0.95$, find the minimum possible value of n

[4 marks]

Answer

Question 7 continues on the next page

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7 (c) It is given that B and W are independent normal distributions.

Let M represent the total mass in grams of 100 bolts and 100 wingnuts, such that

$$M \sim \mathrm{N}\left(\mu, \sigma^2\right)$$

7 (c) (i) Find the value of μ and the value of σ^2

[2 marks]

$\mu =$ _____ $\sigma^2 =$ _____

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7 (c) (ii) Find the value of m when $\mathrm{P}(M < m) = 0.98$

Give your answer to four significant figures.

[3 marks]

Answer _____

16

Turn over for the next question

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8 The number of meteors observed in a 15-minute period in a clear night sky is modelled by a Poisson distribution with mean 2

8 (a) Find the probability of at least one meteor being observed in a 30-minute period.

Give your answer to four significant figures.

[3 marks]

Answer _____

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8 (b) A school astronomical society claims that the mean number of meteors observed in a 15-minute period in a clear night sky has changed from 2

A student from the school astronomical society carries out a hypothesis test at the 10% level of significance to determine whether there is evidence to support the school astronomical society's claim.

The student collects the data below over four distinct random intervals of 15 minutes, as shown below.

Interval	*A*	*B*	*C*	*D*
Number of meteors observed in a 15-minute period	2	6	0	4

The student then combines the results to consider the number of meteors observed for a one-hour period.

The first five stages of the student's hypothesis test are shown below.

There is an error in one of the stages.

	Statement
Stage 1	$H_0 : \lambda = 8$ $H_1 : \lambda \neq 8$ where λ is the mean number of meteors observed for a one-hour period
Stage 2	Under H_0 $X \sim \text{Po}(8)$ where X is the number of meteors observed for the one-hour period
Stage 3	Use a two-tailed test at the 10% level of significance
Stage 4	$P(X \leq 3) = 0.0424 \quad (< 0.05)$ $P(X \leq 4) = 0.0996 \quad (> 0.05)$ (correct to four decimal places)
Stage 5	$P(X \geq 13) = 0.0638 \quad (> 0.05)$ $P(X \geq 14) = 0.0342 \quad (< 0.05)$ (correct to four decimal places)
Stage 6	The critical region is $\{0, 1, 2, 3, 13, 14, \ldots\}$ for a 10% level of significance

8 (b) (i) Identify the stage which has the error.

Write down a correct statement for this stage.

[2 marks]

Stage _____

Correct statement _____

_____

Question 8 continues on the next page

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8 (b) (ii) State the student's conclusion for this hypothesis test.

[2 marks]

8 (b) (iii) For this hypothesis test find the probability of a Type I error.

[1 mark]

Answer

8 (c) Let T be the time in minutes between successive meteors being seen.

8 (c) (i) Find $\mathrm{F}(t)$, the cumulative distribution function of T

[2 marks]

$$\mathrm{F}(t) = \begin{cases} \\ \end{cases}$$

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8 (c) (ii) It is given that $\mathrm{P}(T > a) = 0.6$

Find the value of a

Give your answer to the nearest second.

[2 marks]

$a =$ _____

8 (c) (iii) It is given that $\mathrm{P}(T < b + 30 \,|\, T > 30) = 3\mathrm{e}^{-\frac{2}{15}b}$

Find the exact value of b

[2 marks]

$b =$ _____

14

END OF QUESTIONS

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