OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

# INTERNATIONAL AS
# MATHEMATICS

## (9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Monday 20 May 2024 07:00 GMT Time allowed: 1 hour 30 minutes

**Materials**

For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).

- You may use a graphical calculator.

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

**Information**

- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| **TOTAL** | |

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

*Do not write outside the box*

## Section A

### Pure Mathematics

Answer **all** questions in the spaces provided.

**1 (a)** Solve the equation

$$4^{5x} = 7$$

Give your answer to three decimal places.

**[2 marks]**

$x =$ _____

**1 (b)** Show that

$$\log_6 180 = a + \log_6 b$$

where $a$ and $b$ are prime numbers.

**[2 marks]**

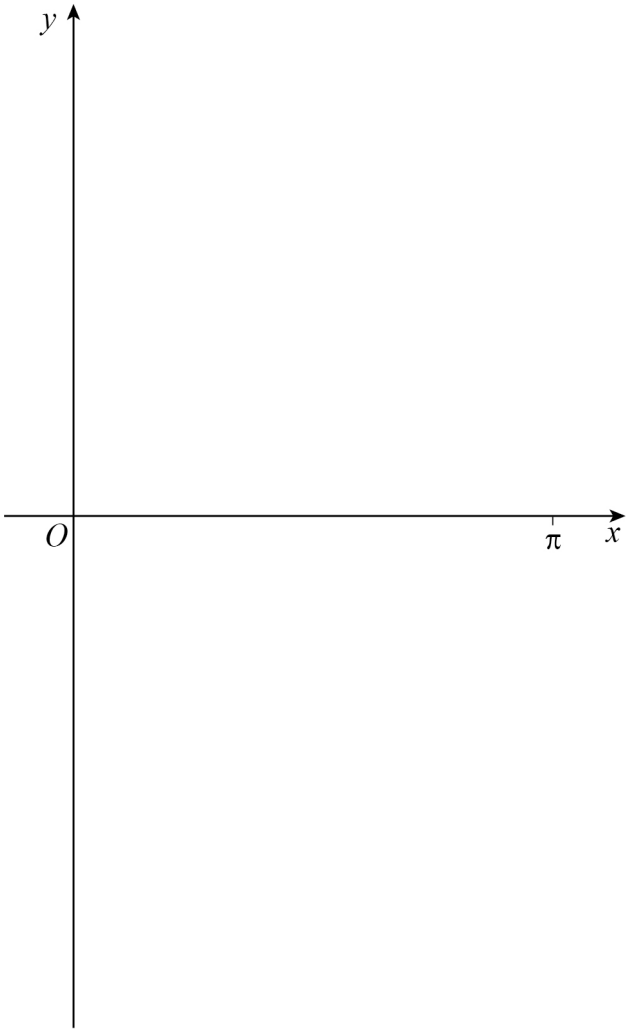
4

*Do not write outside the box*

**2 (a)** On the axes sketch the graph of $y = \tan\left(x - \frac{\pi}{2}\right)$ for $0 < x < \pi$ radians.

Show on your graph the values of any intercepts with the axes.

**[2 marks]**




**2 (b)** The function $\mathrm{f}$ is defined by

$$\mathrm{f}(x) = \tan\left(x - \frac{\pi}{2}\right)$$

Write down the period of $\mathrm{f}$

**[1 mark]**

Answer _____

3

**Turn over ►**

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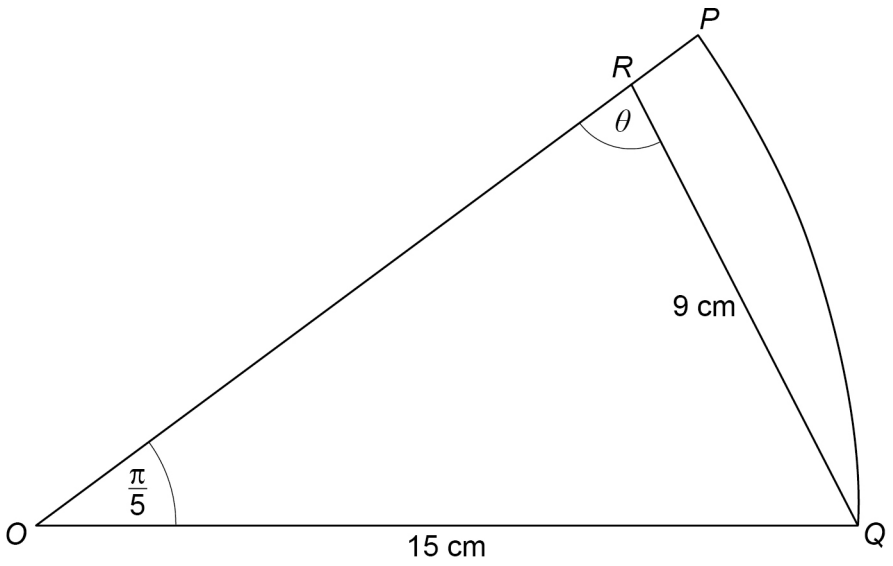
**3** The diagram shows the sector $OPQ$ of a circle with centre $O$, and the line segment $QR$ where $R$ lies on $OP$




The length $OQ = 15$ cm

The length $QR = 9$ cm

The angle $POQ = \frac{\pi}{5}$ radians

The angle $ORQ = \theta$ radians

**3 (a)** Find the possible values of $\theta$

Give your answers to three decimal places.

**[3 marks]**

$\theta =$ _____

*Do not write outside the box*

**3 (b)** The finite region bounded by the line segments $PR$ and $QR$, and the arc $PQ$ has area $A$ cm$^2$

Find the greatest possible value of $A$

Give your answer to three significant figures.

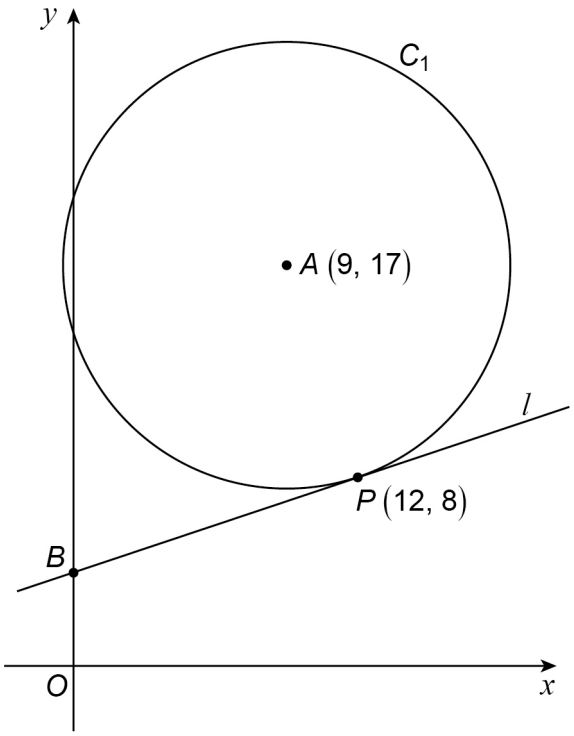
**[5 marks]**

$A =$ _____

8

**Turn over for the next question**

**4** The diagram shows the circle $C_1$ with centre at $A(9, 17)$ and the line $l$




The line $l$ is the tangent to $C_1$ at $P(12, 8)$ and intersects the $y$-axis at the point $B$

**4 (a)** Find the coordinates of $B$

**[4 marks]**

Answer _____

*Do not write outside the box*

**4 (b)** The circles $C_2$ and $C_3$ both have their centres at $B$ and each circle touches the circle $C_1$ at exactly one point.

The radius of $C_3$ is greater than the radius of $C_2$

Find the equation of $C_2$ and the equation of $C_3$

Give your answers in the form $x^2 + (y - n)^2 = k$ where $n$ and $k$ are integers.

**[5 marks]**

Answer _____ and _____

9

**Turn over for the next question**

*Do not write outside the box*

**5 (a)** Show that the equation

$$\frac{6\sin\theta}{\cos\theta - 3} = \frac{\cos\theta + 3}{3 - \sin\theta}$$

can be written as

$$5\sin^2\theta - 18\sin\theta - 8 = 0$$

**[3 marks]**

**5 (b)** Use the result in **part (a)** to solve the equation

$$\frac{6\sin 3x}{\cos 3x - 3} = \frac{\cos 3x + 3}{3 - \sin 3x}$$

in the interval $0^\circ \leq x \leq 180^\circ$

Give your answers to the nearest $0.1^\circ$

**[5 marks]**

Answer _____

**5 (c)** Use your answers in **part (b)** to solve the equation

$$\frac{6\sin(3y+15^\circ)}{\cos(3y+15^\circ)-3}=\frac{\cos(3y+15^\circ)+3}{3-\sin(3y+15^\circ)}$$

in the interval $0^\circ \leq y \leq 180^\circ$

Give your answers to the nearest $0.1^\circ$

**[1 mark]**

Answer _____

9

*Do not write outside the box*

**6** It is given that $x$ satisfies the equation

$$\log_3\left[2\log_3(4x-1)-\log_3(2x+7)\right]=1$$

**6 (a)** Show that

$$8x^2-31x-94=0$$

**[5 marks]**

*Do not write outside the box*

**6 (b)** Use the result in **part (a)** to solve the equation

$$\log_3\left[2\log_3(4x-1)-\log_3(2x+7)\right]=1$$

**[2 marks]**

Answer _____

7

**Turn over for the next section**

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## Section B

## Statistics

Answer **all** questions in the spaces provided.

**7** Huang is choosing fruit to eat.

He can choose an apple, a banana, both or neither.

$A$ represents the event that Huang chooses an apple.

$B$ represents the event that Huang chooses a banana.

It is given that $\mathrm{P}(A) = 0.37$, $\mathrm{P}(B) = 0.16$ and $\mathrm{P}(A \cap B) = 0.07$

**7 (a)** Determine whether $A$ and $B$ are independent.

**[2 marks]**

*Do not write outside the box*

**7 (b)** Find $P(A \cup B)$, the probability that Huang chooses an apple or a banana.

**[2 marks]**

Answer _____

**7 (c)** Find the probability that Huang chooses an apple given that he chooses a banana.

**[2 marks]**

Answer _____

6

**Turn over for the next question**

*Do not write outside the box*

**8** The discrete random variable $X$ has the probability distribution

| $x$ | $-2$ | 1 | 3 | $a$ |
|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | $p$ | 0.20 | 0.41 | 0.23 |

where $a$ and $p$ are constants.

**8 (a)** Show that $p = 0.16$

**[1 mark]**

**8 (b)** The mean of $X$ is 4.1

Find the value of $a$

**[2 marks]**

Answer _____

*Do not write outside the box*

**8 (c)** Find the standard deviation of $X$

Give your answer to three significant figures.

**[3 marks]**

Answer _____

6

**Turn over for the next question**

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**9** The probability of winning a game $A$ is 0.4

The outcome of each game is independent.

**9 (a)** Kaley plays game $A$ 9 times.

For game $A$, the random variable $X$ represents the number of times she wins.

Kaley also plays a game $B$ which she plays 100 times.

For game $B$, the random variable $Y$ represents the number of times she wins.

The mean of $Y$ is 16.2 and $Y$ is independent of $X$

**9 (a) (i)** Find $\mathrm{E}(Y - X)$

**[2 marks]**

Answer _____

**9 (a) (ii)** It is given that $\mathrm{Var}(X + Y) = 5$

Find $\mathrm{Var}(Y)$

**[2 marks]**

Answer _____

*Do not write outside the box*

**9 (b)** In a competition, Kaley plays game $A$ $n$ times.

The probability that Kaley wins at least once is greater than 0.92

Find the smallest possible value of $n$

**[4 marks]**

Answer _____

8

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## Section C

## Mechanics

Answer **all** questions in the spaces provided.

**10** **The acceleration due to gravity, $g$, should be taken as $9.8\ \text{m s}^{-2}$**

A ball is thrown vertically upwards with initial speed $7\ \text{m s}^{-1}$ from a position 1.4 metres above the ground.

Assume that the ball experiences no air resistance and can be modelled as a particle.

Find the time for which the ball is at least 3 metres above the ground.

**[5 marks]**

Answer _____

5

**Turn over ►**

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**11** **The acceleration due to gravity, $g$, should be taken as 9.8 m s$^{-2}$**

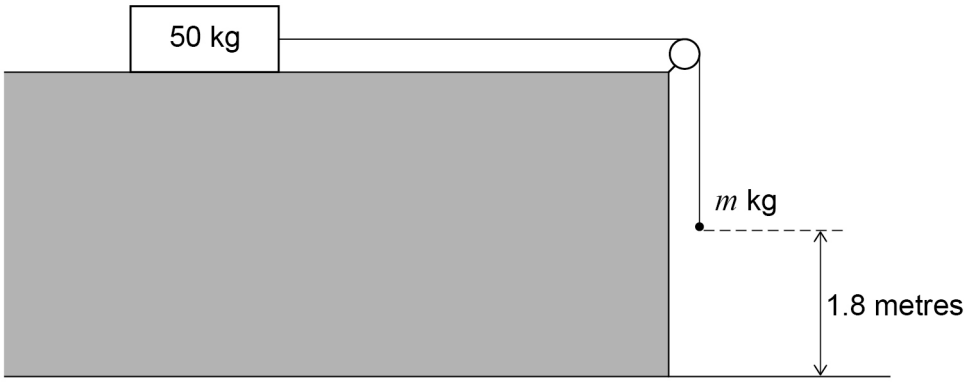
A block is attached to one end of a light inextensible string. The other end of the string is attached to a particle.

The string passes over a smooth light pulley.

The block has mass 50 kg and lies on a rough horizontal surface.

The block is held at rest.

The particle has mass $m$ kg and hangs freely below the pulley 1.8 metres above horizontal ground, as shown in the diagram.




The block is released.

The block does not reach the pulley in its subsequent motion.

The coefficient of friction between the block and the surface is 0.3

The particle accelerates at 0.4 m s$^{-2}$ until it reaches the ground.

**11 (a)** Find the value of $m$

**[5 marks]**

*Do not write outside the box*

Answer _____

**11 (b)** Show that the greatest speed reached by the particle is $1.2\ m\,s^{-1}$

**[2 marks]**

**11 (c)** The block is released from rest at a point *O*

After the particle reaches the ground, the block decelerates uniformly due to friction before coming to rest at the point *A*

Find the distance *OA*

**[3 marks]**

Answer _____

**10**

**Turn over ►**

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**12** Two particles $A$ and $B$ are moving directly towards each other on a straight line on a smooth horizontal surface.

Particle $A$ has mass 3 kg and particle $B$ has mass 2 kg

The particles collide.

Before the collision, particle $A$ has speed 4 $m\,s^{-1}$ and particle $B$ has speed 7 $m\,s^{-1}$

After the collision, particle $B$ has speed 1 $m\,s^{-1}$

Determine whether the direction of motion of particle $B$ has been reversed by the collision.

Fully justify your answer.

**[5 marks]**

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5

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