OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Friday 31 May 2024 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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ANSWER IN THE SPACES PROVIDED

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Answer **all** questions in the spaces provided.

1 (a) Express $2\cos\theta + \sqrt{5}\sin\theta$ in the form $R\cos(\theta - \alpha)$
where $R > 0$ and $0^\circ < \alpha < 90^\circ$

Give the value of α to the nearest degree.

[3 marks]

Answer _____

1 (b) (i) Hence write down the minimum value of $2\cos\theta + \sqrt{5}\sin\theta$

[1 mark]

Answer _____

1 (b) (ii) Find the smallest possible positive value of θ at which the minimum value in **part (b)(i)** occurs.

Give your answer to the nearest degree.

[1 mark]

Answer _____

5

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2 The functions f and g are defined by

$$f(x) = \sqrt{18 - 2x^2} \quad \text{for } 0 \leq x \leq 3$$

$$g(x) = \frac{3}{2x - 1} \quad \text{for } x > 0.5$$

2 (a) Determine the range of the function f

[1 mark]

Answer _____

2 (b) (i) Find $fg(x)$

[1 mark]

Answer _____

2 (b) (ii) Find the domain of the function fg

[2 marks]

Answer _____

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2 (b) (iii) Solve the equation $\mathrm{fg}(x) = 4$

[3 marks]

Answer _____

2 (c) The inverse of the function fg is h

Find $\mathrm{h}(x)$

[3 marks]

Answer _____

10

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3 Describe a sequence of **two** geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 4x^2 + 12x + 9$

[5 marks]

5

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4 (a) It is given that $X = A\mathrm{e}^{-kt}$ where A and k are constants.

Show that $\dfrac{\mathrm{d}X}{\mathrm{d}t} = -kX$

[1 mark]

4 (b) The temperature X °C of water in a cup at time t minutes decreases at a rate that is directly proportional to the temperature of the water.

When $t = 0$ the temperature of the water is 90 °C

When $t = 5$ the temperature of the water is 80 °C

Use the differential equation in **part (a)** to find the value of t when the temperature of the water is 22.5 °C

Give your answer correct to one decimal place.

[5 marks]

Answer _____

6

Turn over ►

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5 (a) Use Simpson's rule with 4 strips to find an estimate for

$$\int_0^2 3 - \tan^{-1}(2x - 0.5)\,\mathrm{d}x$$

Give your answer to three decimal places.

[4 marks]

Answer _____

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5 (b) A curve has equation

$$y = 0.5\tan(3 - x) + 0.25$$

5 (b) (i) In the interval $\frac{\pi}{2} < x < \pi$, the curve intersects the line $y = x$ at a single point where $x = \alpha$

Show that α lies between 1.7 and 1.8

[2 marks]

5 (b) (ii) Show that the equation $0.5\tan(3 - x) + 0.25 - x = 0$ can be rearranged into the form

$$x = 3 - \tan^{-1}(2x - 0.5)$$

[2 marks]

Question 5 continues on the next page

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5 (b) (iii) Use the iterative formula

$$x_{n+1} = 3 - \tan^{-1}\left(2x_n - 0.5\right)$$

with $x_1 = 1.7$ to find the value of x_2 and the value of x_3

Give your answers to three decimal places.

[2 marks]

$x_2 =$ _____ $x_3 =$ _____

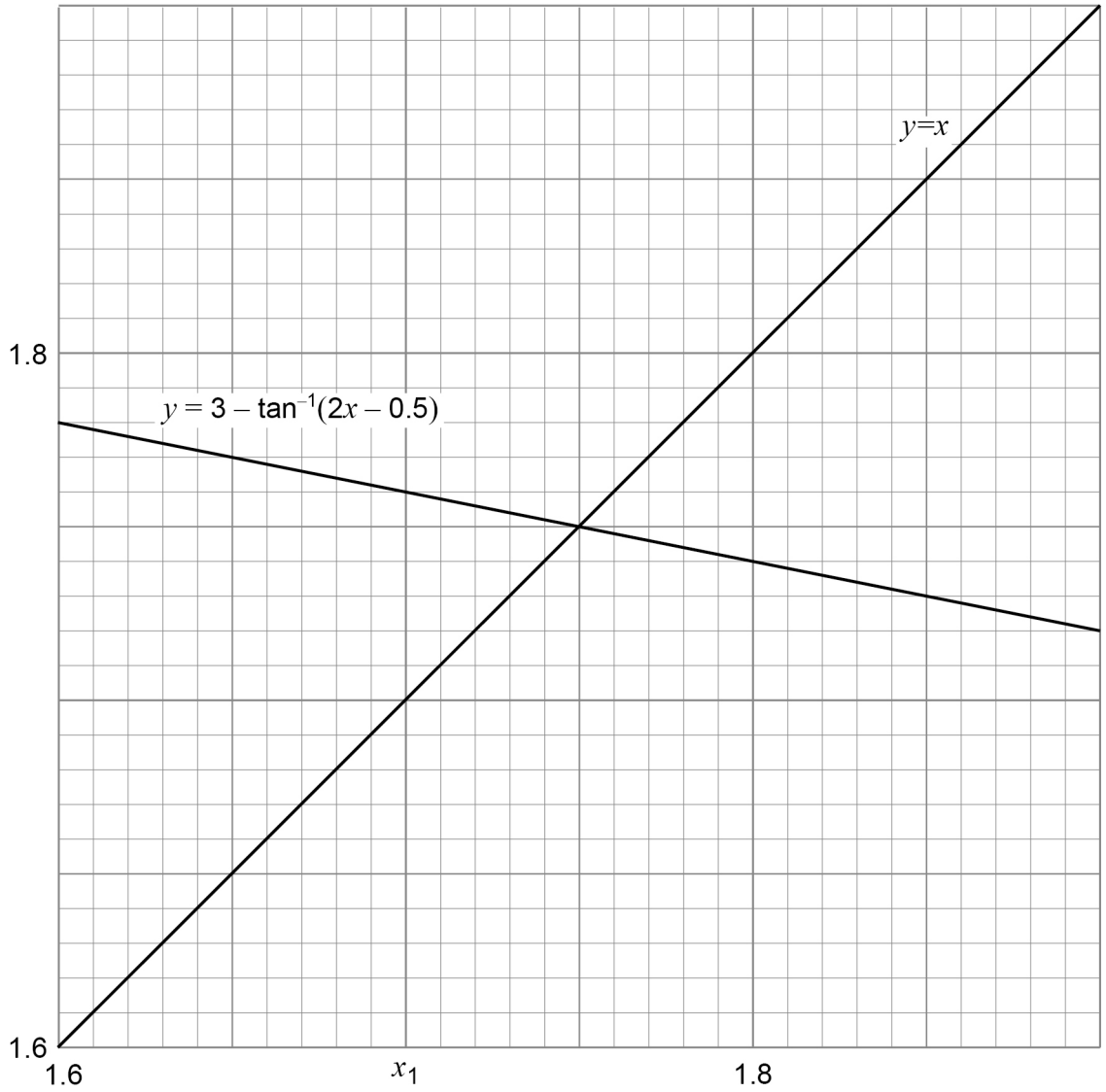
Do not write outside the box

5 (b) (iv) The figure below shows parts of the graphs of

$$y = 3 - \tan^{-1}(2x - 0.5) \text{ and } y = x, \text{ and the position of } x_1$$

On the figure, draw a cobweb or staircase diagram to show how convergence takes place, indicating the position of x_2 and the positions of x_3

[2 marks]



12

Turn over for the next question

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6 (a) Show that

$$\left(\sin\theta - \operatorname{cosec}\theta\right)\left(\cos\theta - \sec\theta\right) = 0.5\sin 2\theta$$

[3 marks]

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6 (b) Hence solve the equation

$$\left(\sin\left(1.5x+0.1\right)-\text{cosec}\left(1.5x+0.1\right)\right)\left(\cos\left(1.5x+0.1\right)-\sec\left(1.5x+0.1\right)\right)=0.4$$

Give your answers in radians to three decimal places in the interval $0 < x < \pi$

[3 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

7 (a) Find $\int \frac{x-2}{2x^2-8x+3}\,\mathrm{d}x$

[2 marks]

Answer _____

7 (b) Find $\int \frac{5x+1}{5x-1}\,\mathrm{d}x$

[3 marks]

Answer _____

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7 (c) Find $\int \sin^2 3x \, dx$

[3 marks]

Answer _____

8

Turn over for the next question

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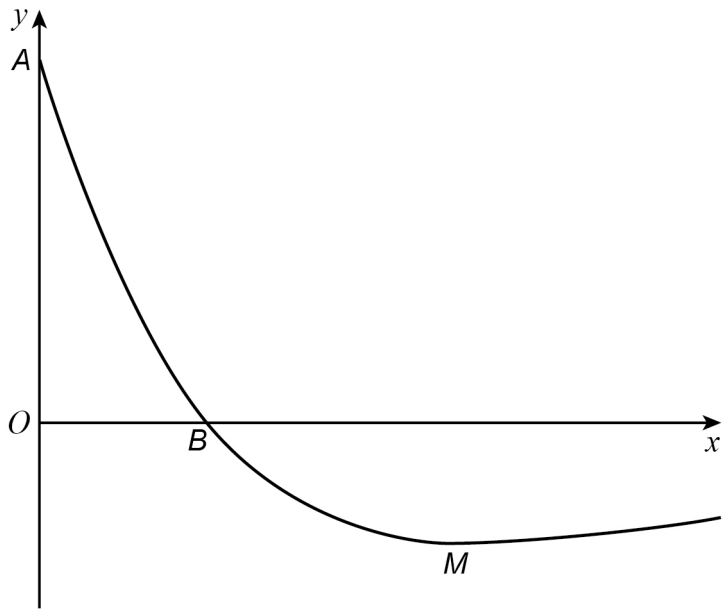
8 The diagram shows part of the curve

$$y = (5 - 4x)e^{-0.5x}$$



The curve intersects the axes at A and B and has a stationary point at M

8 (a) Write down the coordinates of A and B

[1 mark]

A _____ B _____

8 (b) (i) Find $\frac{dy}{dx}$

Give your answer in the form $(ax + b)e^{-0.5x}$ where a and b are constants.

[3 marks]

$\frac{dy}{dx} =$ _____

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8 (b) (ii) Find the coordinates of M

Give your answer in an exact form.

[3 marks]

Answer _____

8 (c) Find the area of the finite region bounded by the curve $y = (5 - 4x)e^{-0.5x}$ and the line segment AB

Give your answer in an exact form.

[6 marks]

Answer _____

13

Turn over ►

Do not write outside the box

9 A curve is defined by the parametric equations

$$x = a(2\theta - \sin 2\theta) \quad \text{and} \quad y = a(1 - \cos 2\theta) \qquad \text{for} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

where a is a non-zero constant.

9 (a) Find an equation of the tangent to the curve at the point P where $\theta = \frac{\pi}{4}$

[6 marks]

Answer _____

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9 (b) Find the equation of the normal to the curve at P

Give your answer in the form $y = mx + c$

[2 marks]

Answer _____

9 (c) The tangent to the curve at P intersects the y-axis at the point Q

The normal to the curve at P intersects the y-axis at the point R

Show that the area of the triangle PQR is $\frac{1}{4}a^2(\pi - 2)^2$

[2 marks]

10

Turn over for the next question

Do not write outside the box

10 A curve C has equation

$$y = 4x - (x - y)^2$$

10 (a) The line $x = 4$ intersects the curve C at the points A and B

Find the length of the line segment AB

[2 marks]

Answer _____

Do not write outside the box

10 (b) Find the coordinates of the stationary point of C

Give your answer in an exact form.

[4 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

11 (a) Express $\dfrac{75}{(5-x)(5+2x)^2}$ in the form $\dfrac{A}{5-x}+\dfrac{B}{5+2x}+\dfrac{C}{(5+2x)^2}$

[4 marks]

Answer _____

11 (b) Find the binomial expansion of $(5+2x)^{-1}$ up to and including the term in x^2

[2 marks]

Answer _____

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11 (c) Use your answers to **parts (a)** and **(b)** to show that

$$\frac{75}{(5-x)(5+2x)^2} = D + Ex + Fx^2$$

for small values of x, where D, E and F are constants.

[5 marks]

Answer _____

11

Turn over ►

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12 It is given that $y = \mathrm{f}(x)$ satisfies the differential equation

$$x\left(16 - y^2\right) = 2\left(x^2 + 5\right)\frac{\mathrm{d}y}{\mathrm{d}x}$$

Find the solution of the differential equation given that $y = 1$ when $x = 0$

Give your answer in the form $y = \mathrm{f}(x)$

[9 marks]

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Answer _____

9

Turn over for the next question

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13 The point A has coordinates $(2, -2, 6)$

The point B has coordinates $(8, 6, -4)$

13 (a) Find a vector equation of the line AB

[1 mark]

Answer _____

13 (b) The line l has equation $\mathbf{r} = \begin{bmatrix} -2 \\ 3 \\ c \end{bmatrix} + \mu \begin{bmatrix} 4 \\ -5 \\ 3 \end{bmatrix}$

13 (b) (i) It is given that the line l intersects the line AB

Show that $c = 3$

[3 marks]

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13 (b) (ii) Show that the shortest distance from the point $(2, 2, 6)$ to the line l is $q\sqrt{q}$, where q is a constant.

[5 marks]

9

Turn over ►

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14 (a) (i) It is given that $\cot x = \dfrac{\cos x}{\sin x}$

Use the quotient rule to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\mathrm{cosec}^2 x$$

[2 marks]

14 (a) (ii) Use the substitution $u = 1 + \cot x$ to find the value of $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\mathrm{cosec}^2 x}{(1+\cot x)^2}\,\mathrm{d}x$

Give your answer in the form $p + q\sqrt{3}$ where p and q are rational numbers.

[5 marks]

Answer _____

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14 (b) Find

$$\int \frac{1}{\sqrt{(2x+1)} + \sqrt{(2x-1)}}\,\mathrm{d}x$$

[3 marks]

Answer _____

10

END OF QUESTIONS

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