OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

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INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA05) Unit M2 Mechanics

Friday 14 June 2024 07:00 GMT Time allowed: 1 hour 30 minutes

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Materials

- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 A motorcycle is moving on a straight, horizontal road.

The mass of the motorcycle is 280 kg

When the motorcycle's speed is v m s^{-1}, the magnitude of the total resistance force experienced by the motorcycle is $0.91v^{\frac{5}{3}}$ newtons.

Assume that the motorcycle can be modelled as a particle.

1 (a) Draw a diagram to show all of the forces acting on the motorcycle when it is moving at a constant speed.

Write down the names of the forces on your diagram.

[2 marks]

1 (b) When the motorcycle's speed is 15 m s^{-1}, its acceleration is 3.5 m s^{-2}

Find the magnitude of the driving force provided by the motorcycle's engine.

[3 marks]

Answer _____

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1 (c) The maximum power output of the motorcycle's engine is 50 kW

Find the maximum possible speed of the motorcycle along this straight, horizontal road.

[2 marks]

Answer _____

1 (d) State, with a reason, how your answer to **part (c)** would change if the motorcycle was moving up a slope of constant gradient.

[2 marks]

9

Turn over for the next question

Do not write outside the box

2 Two variable forces $\mathbf{F}_1$ newtons and $\mathbf{F}_2$ newtons act on a particle of mass 4 kg

At time t seconds the acceleration $\mathbf{a}$ m s^{-2} of the particle is given by

$$\mathbf{a} = \cos(2t)\mathbf{i} - 2e^t\mathbf{j} + 6t\mathbf{k}$$

2 (a) (i) Write down in terms of t the resultant force acting on the particle.

[1 mark]

Answer _____

2 (a) (ii) Explain why the particle is never in equilibrium for $t \geq 0$

[2 marks]

_____

_____

_____

_____

2 (b) At time t seconds the force $\mathbf{F}_1$ newtons is given by

$$\mathbf{F}_1 = 3\cos(2t)\mathbf{i} - 2e^t\mathbf{j} - 8t\mathbf{k}$$

2 (b) (i) Find $\mathbf{F}_2$ in terms of t

[2 marks]

_____

_____

_____

_____

_____

Answer _____

Do not write outside the box

2 (b) (ii) Find the magnitude of $\mathbf{F}_2$ when $t = \frac{\pi}{4}$

[3 marks]

Answer _____

8

Turn over for the next question

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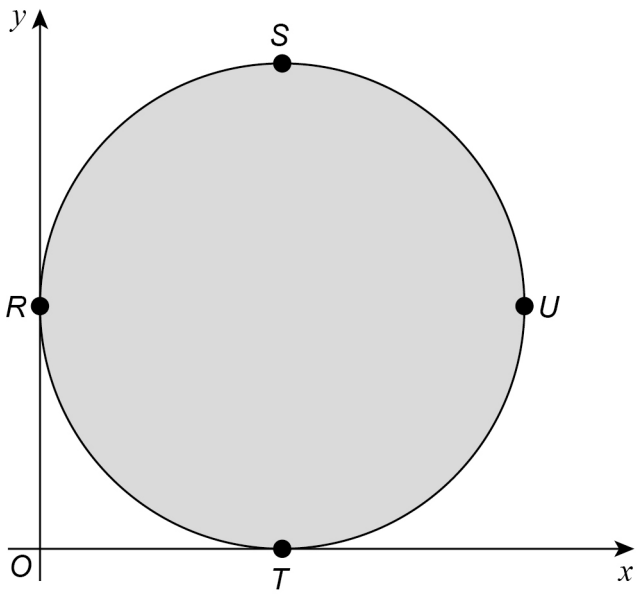
3 Four particles R, S, T and U are fixed on the circumference of a uniform circular lamina to create a system.

The system is shown in the diagram below.



The table below shows the mass of each particle and the coordinates of the points at which they are fixed to the lamina.

Particle	Mass (kg)	Coordinates (metres)
R	2	(0, 3)
S	3	(3, 6)
T	7	(3, 0)
U	11	(6, 3)

The mass of the uniform circular lamina is 5 kg

The diameter of the uniform circular lamina is 6 metres.

3 (a) State, with a reason, the coordinates of the centre of mass of the **uniform circular lamina**.

[2 marks]

Do not write outside the box

3 (b) Find the coordinates of the centre of mass of the **system**.

Give your answer in an exact form.

[4 marks]

Answer _____

3 (c) The system is freely suspended from the point where particle S is fixed to the lamina, and it hangs in equilibrium.

Find the angle between the vertical and the line connecting the points where particles S and T are fixed to the lamina.

Give your answer to the nearest degree.

[3 marks]

Answer _____

9

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4 A person shoots an arrow of mass 80 grams from a bow towards a target.

The arrow leaves the bow with a velocity of 45 m s^{-1} at an angle of 10° above the horizontal.

When the arrow leaves the bow, the horizontal displacement of the target from the tip of the arrow is 70 metres.

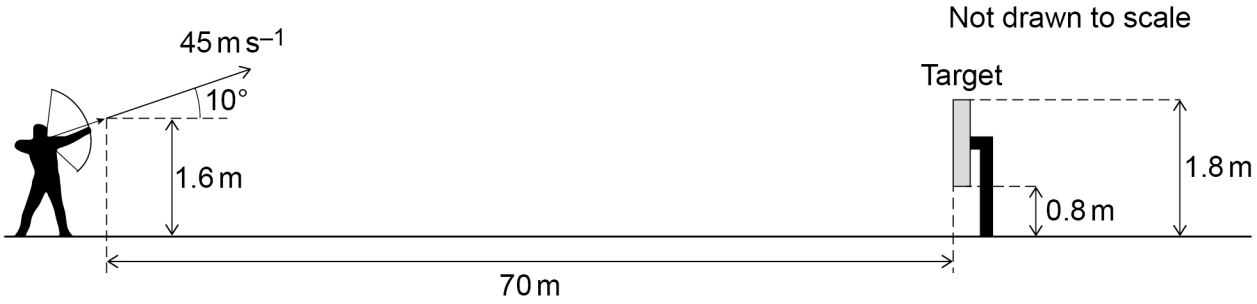
When the arrow leaves the bow, its tip is 1.6 metres above the horizontal ground.

The bottom of the target is 0.8 metres above the horizontal ground.

The top of the target is 1.8 metres above the horizontal ground.

The plane of the target is perpendicular to the horizontal ground.

The arrow moves in a vertical plane that is perpendicular to the target and contains the centre of the target, as shown in the diagram.



During its flight, assume that the arrow does not experience any force except its weight.

4 (a) (i) Show that the kinetic energy of the arrow immediately after it leaves the bow is 81 J

[1 mark]

4 (a) (ii) Find the kinetic energy of the arrow when the arrow is at its maximum height above the ground.

Give your answer to three significant figures.

[2 marks]

Answer _____

Do not write outside the box

4 (b) Use an energy method with your answers to **part (a)** to find the maximum height of the arrow above the ground.

[3 marks]

Answer _____

4 (c) (i) Find the time taken for the arrow to travel 70 metres in the horizontal direction.

Give your answer to three significant figures.

[1 mark]

Answer _____

4 (c) (ii) Hence determine whether the arrow hits the target.

[4 marks]

Answer _____

11

Turn over ►

Do not write outside the box

5 The position vector **r** metres of a body relative to a fixed origin O at time t seconds is given by

$$\mathbf{r} = \begin{bmatrix} e^{-3t}\cos(3t) \\ e^{-3t}\sin(3t) \end{bmatrix}$$

The mass of the body is 6 kg

5 (a) Show that the distance between O and the body at time t seconds is equal to e^{-3t} metres.

[3 marks]

5 (b) The velocity of the body at time t seconds is **v** m s^{-1}

Find **v** in terms of t

Give your answer as a column vector.

[2 marks]

Answer _____

Do not write outside the box

5 (c) (i) Use your answer to **part (b)** to find an expression for the speed of the body at time t seconds.

Give your answer in the form $\sqrt{b}\,e^{ct}$, where b and c are integers.

[3 marks]

Answer _____

5 (c) (ii) Hence find the maximum kinetic energy of the body during its motion.

[2 marks]

Answer _____

10

Turn over ►

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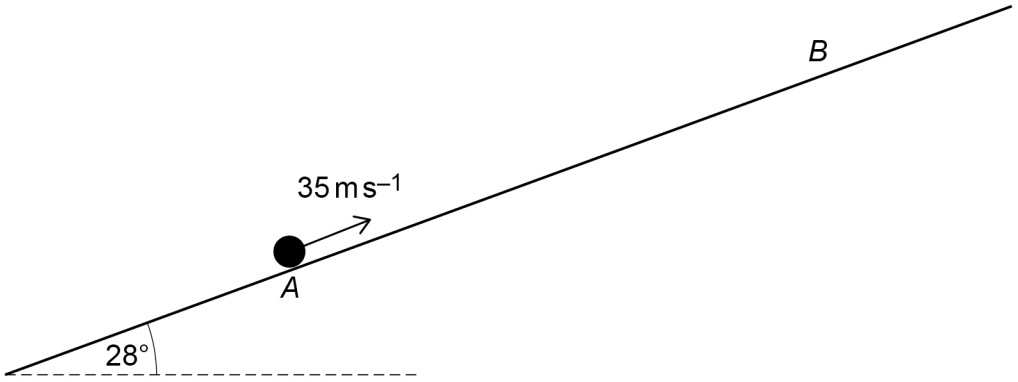
6 A particle of mass 16 kg is moving on a straight line up a rough slope.

The slope makes an angle of 28° with the horizontal, and the particle moves on a line of greatest slope.

The coefficient of friction between the particle and the slope is 0.64

When the particle passes through the point *A* on the slope, it is moving with speed 35 m s^{-1} towards the point *B* on the slope, as shown in the diagram.



The particle comes to rest at *B*

Assume that the particle experiences no air resistance during its motion.

Find the distance *AB*

[5 marks]

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Answer _____

5

Turn over for the next question

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7 A particle is projected with speed v m s^{-1} at an angle θ degrees above the horizontal from a point O on horizontal ground.

The horizontal displacement of the particle from O at time t seconds is x metres.

The vertical displacement of the particle from O at time t seconds is y metres.

7 (a) (i) Show that

$$y = x\tan\theta - \frac{gx^2}{2v^2}\sec^2\theta$$

[5 marks]

7 (a) (ii) State an assumption made in **part (a)(i)**.

[1 mark]

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7 (b) A particle is projected with speed 30 m s^{-1} at an angle α degrees above the horizontal from a point O on horizontal ground.

When the horizontal displacement of the particle from O is 25 metres, the vertical displacement of the particle from O is 10 metres.

Find the possible values for α

[5 marks]

Answer _____

11

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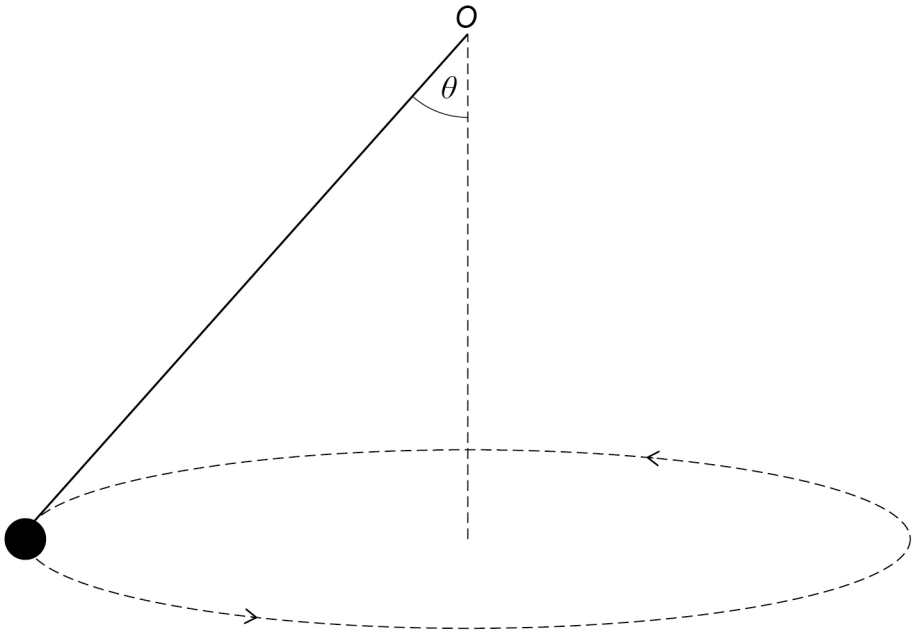
8 A light inextensible string of length l metres has one of its ends attached to a fixed point O

The other end of the string is attached to a small sphere of mass m kg

The small sphere is set into motion so that it moves around the circumference of a horizontal circle with angular speed ω rad s^{-1}

The string makes an angle θ degrees to the vertical, as shown in the diagram.



8 (a) The string is described as being inextensible.

Explain what is meant by inextensible.

[1 mark]

8 (b) Find, in terms of m, ω and l, the tension in the string.

[2 marks]

Answer

Do not write outside the box

8 (c) Find, in terms of g, l and θ, the time taken for the small sphere to move once around the circumference of the horizontal circle.

[3 marks]

Answer _____

8 (d) It is observed that when $l = 0.71$, the small sphere moves around the circumference of the horizontal circle more than four times but less than five times in 6 seconds.

Find the range of possible values of θ

[3 marks]

Answer _____

9

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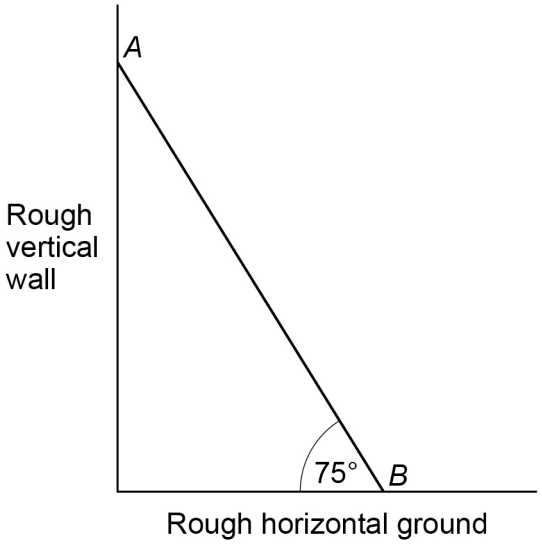
9 A uniform rod of mass 12 kg has length 7 metres.

The rod rests with one end against a rough vertical wall at the point A

The other end of the rod is on rough horizontal ground at the point B

The rod is in equilibrium but it is on the point of slipping at both A and B

The vertical plane containing the rod is perpendicular to the wall and the angle between the rod and the horizontal ground is 75°, as shown in the diagram.



The magnitude of the normal reaction force exerted on the rod due to its contact with the vertical wall at A is 15 newtons.

The coefficient of friction between the wall and the rod is μ_1

The coefficient of friction between the ground and the rod is μ_2

9 (a) By taking moments about the point B, find the value of μ_1

Give your value to three significant figures.

[3 marks]

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$\mu_1 =$

9 (b) Find the value of μ_2

Give your value to three significant figures.

[5 marks]

$\mu_2 =$

8

END OF QUESTIONS

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