OXFORD AQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Wednesday 5 June 2024   07:00 GMT   Time allowed: 1 hour 30 minutes

Materials
* For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
* You may use a graphical calculator.

Instructions
* Use black ink or black ball-point pen. Pencil should only be used for drawing.
* Fill in the boxes at the top of this page.
* Answer **all** questions.
* You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
* If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
* Do all rough work in this book. Cross through any work you do not want to be marked.

Information
* The marks for questions are shown in brackets.
* The maximum mark for this paper is 80.

Advice
* Unless stated otherwise, you may quote formulae, without proof, from the booklet.
* Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Answer **all** questions in the spaces provided.

1 The number of days a patient takes to recover from a particular illness after using medicine *A* has a normal distribution with population standard deviation 2.1 days.

The number of days a patient takes to recover from the same illness after using medicine *B* has a normal distribution with population standard deviation 2.2 days.

For medicine *A*, a random sample of 7 patients has a sample mean for the number of days to recover of 8.2 days.

For medicine *B*, a random sample of 9 patients has a sample mean for the number of days to recover of 6.5 days.

It is claimed that the population mean for the number of days to recover after using medicine *B* is less than the population mean for the number of days to recover after using medicine *A*

Test the claim using the 5% level of significance.

[6 marks]

6

2 The discrete random variables X_1, X_2 and X_3 follow independent Poisson distributions with the same mean λ

The random variable $R = \dfrac{X_1 + 2X_2 + 3X_3}{6}$

2 (a) Show that R is an unbiased estimator of λ

[2 marks]

2 (b) (i) Find the efficiency of R relative to X_1

[4 marks]

Answer _____

2 (b) (ii) Hence, explain which of R and X_1 is the more efficient estimator of λ

[1 mark]

7

Turn over ▶

3   A game consists of three independent identical repeated events.

In each event, a player can only score either 0 or 20

The probability of scoring 20 is 0.2

Neroon plays the game 3 times.

Find the sampling distribution of his modal score M

[5 marks]

Answer _____

5

Turn over for the next question

4 Cordelia is investigating if there is an association between the region a person lives in and their chosen holiday location.

The data she collects is summarised in the table below.

Holiday location

		Abroad	Not abroad	Total
Region lived in	**North**	44	59	103
	South	85	62	147
	Total	129	121	250

She claims that there is an association between the region a person lives in and their chosen holiday location.

Test Cordelia's claim, using the 5% level of significance.

[8 marks]

Turn over for the next question

8

Turn over ▶

5 Stanley investigates the reaction time in seconds for males and females for the start of a race.

He records the reaction time X of a random sample of 41 males.

He records the reaction time Y of a random sample of 26 females.

His summarised results are

$$\sum(x-\bar{x})^2 = 0.3998 \quad \text{and} \quad \sum(y-\bar{y})^2 = 0.1024$$

Stanley claims that the population standard deviation is equal for both males and females.

5 (a) Stanley wants to carry out a hypothesis test using the *F*-distribution.

State an assumption that Stanley will need to make.

[1 mark]

5 (b) Test Stanley's claim using the 2% level of significance.

[8 marks]

Turn over for the next question

Turn over ▶

6 **(a)** The normal distribution $Z \sim N(0, 1)$ has moment generating function

$$M_Z(t) = e^{\frac{1}{2}t^2}$$

6 **(a)(i)** Find the moment generating function of the random variable

$$X = \mu + \sigma Z$$

where μ and σ are constants.

Give your answer in the form $M_X(t) = e^{f(t)}$ where $f(t)$ is a function of t

[2 marks]

Answer

6 **(a)(ii)** The random variable Y is independent of X and has moment generating function

$$M_Y(t) = e^{\lambda\left(e^t - 1\right)}$$

where λ is a constant.

Find the moment generating function of $X + Y$

Give your answer in the form $M_{X+Y}(t) = e^{g(t)}$ where $g(t)$ is a function of t

[2 marks]

Answer _____

Question 6 continues on the next page

Turn over ▶

6 (b) The continuous random variable W has probability density function

$$f(w) = \begin{cases} \dfrac{2}{3}e^{2w} & 0 \le w \le \ln 2 \\ 0 & \text{otherwise} \end{cases}$$

6 (b)(i) Show that the moment generating function of W is given by

$$M_W(t) = \frac{2\left(e^{(t+2)\ln 2} - 1\right)}{3(t+2)}$$

[3 marks]

6 (b)(ii) Find $M'_W(t)$

Give your answer in the form $M'_W(t) = \dfrac{h(t)}{3(t+2)^2}$ where $h(t)$ is a function of t

[2 marks]

Answer _____

6 (b)(iii) Hence find the mean of W

Give your answer to three decimal places.

[2 marks]

Answer _____

11

Turn over ▶

7 Scores on a test have a normal distribution with population mean μ and population standard deviation 18

A hypothesis test is carried out using the 8% level of significance and a random sample of 200 students.

The hypotheses are

$$H_0 : \mu = 115$$
$$H_1 : \mu \neq 115$$

It is given that the actual value of μ is 116

7 (a) Find the power of the test.

Give your answer to three decimal places.

[6 marks]

_____

_____

_____

_____

_____

Answer _____

7 (b) The test is repeated using the same random sample but with a smaller level of significance.

Explain how the power of the test is affected.

[1 mark]

_____

_____

_____

7

Turn over for the next question

Turn over ▶

8 The expenses that a company pays each day has a normal distribution with known population variance σ^2

A random sample of expenses paid on 40 different days is taken and has sample mean $\bar{x}$

The upper limit of a 98% confidence interval calculated using the sample is $\$5243.65$

The lower limit of a 93% confidence interval calculated using the sample is $\$4858.26$

8 (a) Find the value of $\bar{x}$ and the value of σ

Give your answers to the nearest dollar.

[6 marks]

$\overline{x} =$ _____ $\sigma =$ _____

8 (b) A random sample of size n is taken.

A 96% confidence interval is calculated from the sample and has width $300 to the nearest dollar.

Find the value of n

[3 marks]

Answer _____

Turn over for the next question

9

Turn over ▶

9 In a game, a coin is tossed 5 times and N, the number of times it lands with heads facing upwards, is counted.

Aeryn models N with the binomial distribution $N \sim \text{B}(5, 0.5)$

She observes the game being played 200 times.

Her summarised results are shown in the table.

n	0	1	2	3	4	5
Frequency	11	40	75	48	22	4

9 (a) Investigate the goodness of fit of Aeryn's model for N, using the 1% level of significance.

[8 marks]

9 (b) State the number of degrees of freedom for the test in **part (a)** if Aeryn's model for N was $N \sim \text{B}(5, p)$ where p is unknown.

[1 mark]

Answer _____

Turn over for the next question

Turn over ▶

10 The salaries paid to people working in town *A* are known to be normally distributed.

The salaries paid to people working in town *B* are known to be normally distributed and with the same population variance as town *A*

The table below shows details of a random sample for town *A* and a random sample for town *B*

Town	Size of Random Sample	Sample Mean (thousand dollars)	Unbiased Estimator of Population Standard Deviation (thousand dollars)
A	n	$\overline{x}_A = 39.4$	$s_A = 5.4$
B	$n + 3$	$\overline{x}_B = 43.2$	$s_B = 5.2$

The pooled estimate of variance s_p^2 is calculated to be 27.933, correct to three decimal places in units of $(\text{thousand dollars})^2$
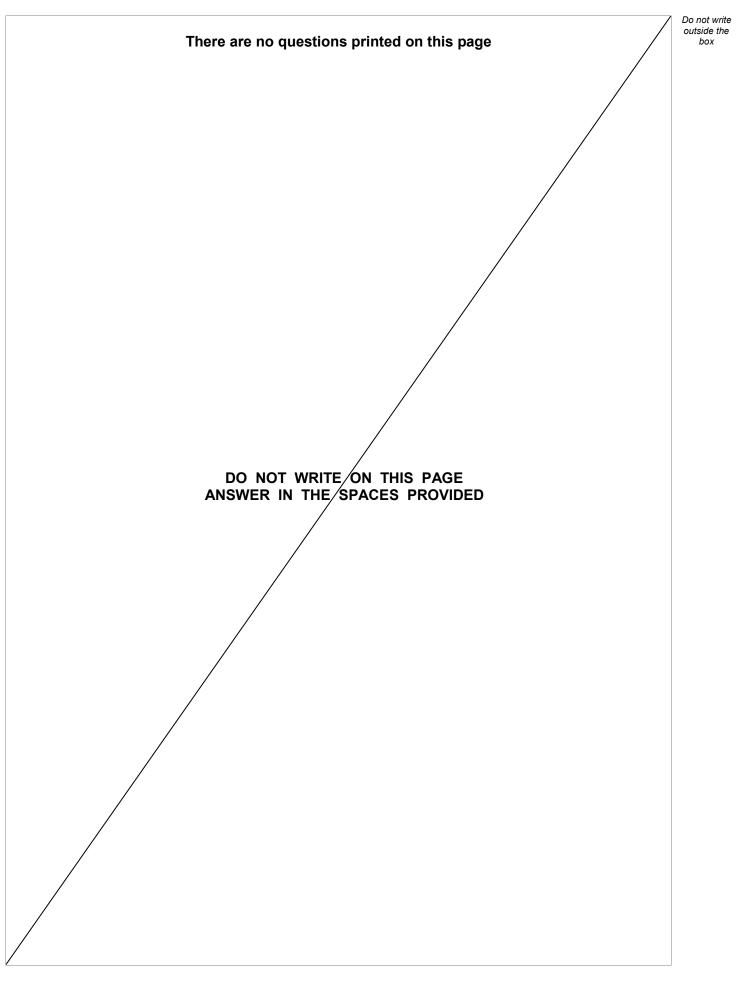
10 (a) Find the value of n

[3 marks]

Answer _____

10 (b) A newspaper claims that the mean salary in town *B* is greater than the mean salary in town *A*

Test the newspaper's claim using the 5% level of significance.

[6 marks]

9

END OF QUESTIONS

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