OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

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I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Thursday 16 May 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 A curve passes through the point $(1, -3)$ and satisfies the differential equation

$$\frac{dy}{dx} = \sqrt{x^2 + 2y^2}$$

Use Euler's step-by-step method with a step length of 0.2 to estimate the value of y when $x = 1.4$

Give your answer to four decimal places.

[5 marks]

Answer _____

5

Turn over ►

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2 The matrix **P** is defined by $\mathbf{P} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ where k is a positive constant.

The matrix **Q** is defined by $\mathbf{Q} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

2 (a) Describe the transformation represented by matrix **P**

[1 mark]

2 (b) (i) Calculate the matrix **QP** giving your answer in terms of k

[2 marks]

Answer _____

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2 (b) (ii) A triangle T_1 has an area of 4.5 square units.

The triangle T_2 is the image of T_1 under the transformation represented by the matrix **QP**

Find the area of triangle T_2

[3 marks]

Answer _____

6

Turn over for the next question

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3 The function f is given by

$$f(x) = 8x^3 - 28x^2 + 34x - 17$$

The equation $f(x) = 0$ has one positive root α

3 (a) Show that $1 < \alpha < 2$

[2 marks]

3 (b) Starting with the interval $1 < \alpha < 2$ use interval bisection **three times** to find an interval of width 0.125 in which α must lie.

[4 marks]

Answer _____

6

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4 The variables u and v are related by the equation

$$v = \frac{u}{a + bu} \quad \text{for} \quad u \neq 0 \text{ and } v \neq 0$$

where a and b are constants.

4 (a) It is given that $V = \frac{1}{v}$ and $U = \frac{1}{u}$

Show that there is a linear relationship between V and U

[2 marks]

Question 4 continues on the next page

4 (b) The table below shows some values of u and v obtained in an experiment.

u	1	2	5	10	50
v	0.117	0.167	0.230	0.251	0.272

4 (b) (i) Complete the table below.

Give your values of V to one decimal place.

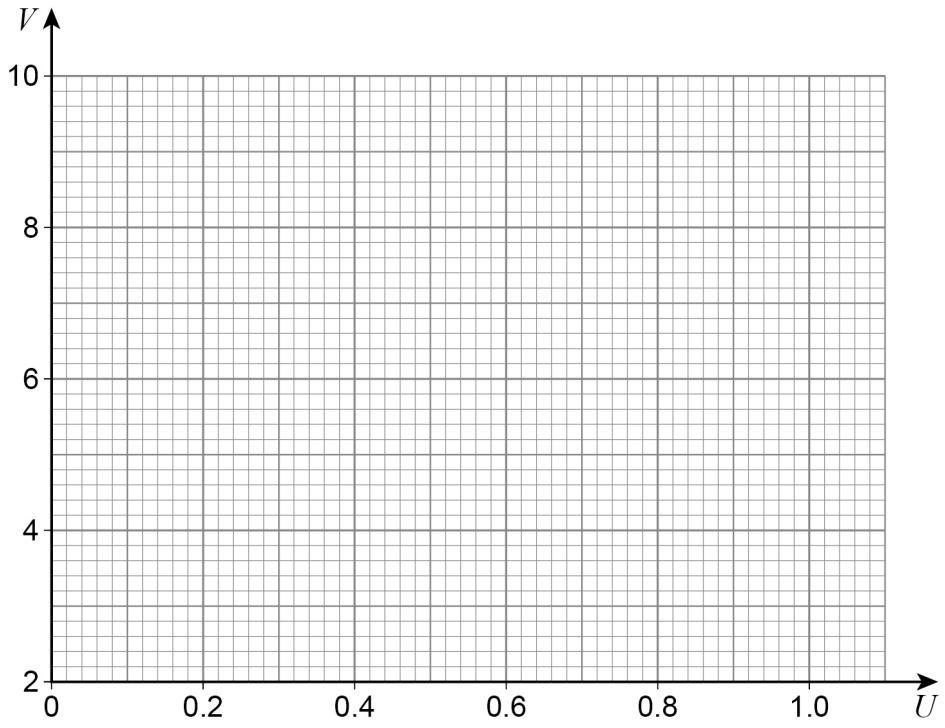
[2 marks]

U	1	0.5	0.2	0.1	0.02
V	8.5				

4 (b) (ii) On the grid plot V against U

Draw a line of best fit for the points you have plotted.

[2 marks]



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4 (c) Use your line of best fit to estimate the value of a and the value of b

Give your values to one decimal place.

[2 marks]

$a =$ _____ $b =$ _____

4 (d) Use your values of a and b to estimate the value of u when $v = 0.26$

Give your answer to two significant figures.

[2 marks]

Answer _____

10

Turn over ►

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5 The matrix **A** is defined by $\mathbf{A} = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

5 (a) (i) It is given that $y = mx + c$ is an invariant line of the transformation represented by **A**

Find the possible values of m

Give your answers in exact form.

[4 marks]

Answer _____

5 (a) (ii) Hence find the equations of the invariant lines of the transformation represented by **A**

[3 marks]

Answer _____

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5 (b) The matrix **B** represents a reflection.

The transformation represented by the matrix **AB** is an anticlockwise rotation of $\frac{\pi}{2}$ radians about the origin.

Find the equation of the mirror line for the reflection represented by **B**

[6 marks]

Answer _____

13

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 The probability that Darius goes for a morning run is 0.35

The probability that he goes to a cafe given that he goes for a morning run is 0.79

The probability that he goes to a cafe given that he does **not** go for a morning run is 0.42

Find the probability that he does **not** go for a morning run given that he does **not** go to a cafe.

[3 marks]

Answer _____

3

Turn over for the next question

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7 In an experiment, a biased coin is tossed 1000 times and lands with heads facing upwards 653 times.

7 (a) Write down an estimate for the probability that the coin lands with heads facing upwards when the coin is tossed once.

[1 mark]

Answer _____

7 (b) The random variable H is defined as

$$H = \begin{cases} 1 & \text{coin lands with heads facing upwards} \\ 0 & \text{otherwise} \end{cases}$$

when the coin is tossed once.

The probability generating function of H is $\mathrm{G}_H(t)$

7 (b) (i) Use your answer to **part (a)** to find $\mathrm{G}_H(t)$

[1 mark]

Answer _____

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7 (b) (ii) Hence find $\text{Var}(H)$

[4 marks]

Answer _____

6

Turn over for the next question

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8 The random variable X has a geometric distribution with parameter p

It is given that $\mathrm{P}(X \leq 3) = \frac{61}{125}$

8 (a) Find the value of p

[2 marks]

Answer _____

8 (b) The random variable Y has a discrete uniform distribution and takes values 1, 2, 3, …, n

It is given that $\mathrm{P}(Y > 4) = \frac{7}{66}\mathrm{E}(Y)$

8 (b) (i) Find the value of n

[4 marks]

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Answer

8 (b) (ii) It is given that $\text{Var}(X - Y) = 13$

Find the value of the product moment correlation coefficient between X and Y

Give your answer to three decimal places.

[5 marks]

Answer

11

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 Two model boats A and B are moving on a lake.

The velocity of A is $\begin{bmatrix} 2.5 \\ -1 \end{bmatrix}$ m s^{-1}

The velocity of B is $\begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}$ m s^{-1}

At time $t = 0$ the displacement of B relative to A is $\begin{bmatrix} 20 \\ d \end{bmatrix}$ metres where d is a constant.

9 (a) Find the velocity of B relative to A

[2 marks]

Answer _____

9 (b) Hence find the value of d given that the two boats collide.

[3 marks]

Answer _____

5

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10 (a) Pressure is a quantity that is measured in units of newtons per square metre, N m^{-2}

Show that the dimensions of pressure are $ML^{-1}T^{-2}$

[1 mark]

10 (b) The pressure P N m^{-2} at a depth of h metres below the surface of a liquid in a tank is given by

$$P = \rho g h$$

where

ρ is the density of the liquid in kg m^{-3}

g is the acceleration due to gravity in m s^{-2}

Show that the formula $P = \rho g h$ is dimensionally consistent.

[3 marks]

4

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11 Two particles P and Q are moving towards each other along a straight line on a smooth horizontal surface.

The particles collide.

Particle P has mass 5 kg and before the collision its speed is 3 m s^{-1}

Particle Q has mass 3 kg and before the collision its speed is 2 m s^{-1}

After the collision the direction of motion of particle P is unchanged and it moves with speed 0.375 m s^{-1}

11 (a) Find the speed of particle Q after the collision.

[3 marks]

Answer _____

11 (b) Find the magnitude of the impulse exerted on particle P by particle Q during the collision.

[2 marks]

Answer _____

5

Turn over ►

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12 A disc of mass 0.2 kg is sliding on a smooth horizontal surface when it collides with a fixed smooth vertical wall.

The wall is perpendicular to the path of the disc.

The magnitude of the force, F newtons, exerted on the disc by the wall at time t seconds after the disc first makes contact with the wall is given by

$$F = 1800\left(t - 10t^2\right)$$

for $0 \leq t \leq 0.1$

The disc is in contact with the wall for 0.1 seconds.

12 (a) Find the magnitude of the impulse exerted on the disc by the wall due to the collision.

State the units of your answer.

[2 marks]

Answer _____ Units _____

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12 (b) The speed of the disc when it hits the wall is u m s^{-1}

Find the range of possible values of u

[4 marks]

Answer _____

6

END OF QUESTIONS

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