OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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Candidate signature _____

I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Thursday 30 May 2024 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Answer **all** questions in the spaces provided.

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1 The matrix $\mathbf{M}_R = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ represents the single transformation R

The matrix $\mathbf{M}_S = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ represents the single transformation S

1 (a) (i) Describe fully the single transformation R represented by the matrix $\mathbf{M}_R$

[2 marks]

1 (a) (ii) Describe fully the single transformation S represented by the matrix $\mathbf{M}_S$

[2 marks]

1 (b) The composite transformation T is obtained by carrying out S followed by R

Find the 3×3 matrix $\mathbf{M}_T$ which represents the transformation T

[2 marks]

$\mathbf{M}_T =$

6

Turn over ►

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2 The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by

$$\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \qquad \mathbf{b} = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} \qquad \mathbf{c} = -3\mathbf{i} + t\,\mathbf{j} + 9\mathbf{k}$$

where t is a real number.

2 (a) Find $\mathbf{a} \times \mathbf{b}$

[1 mark]

Answer _____

2 (b) In the case when $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar, use a scalar triple product to find the value of t

[3 marks]

Answer _____

2 (c) In the case when $\mathbf{c}$ is parallel to $\mathbf{a} \times \mathbf{b}$ find the value of t

[2 marks]

Answer _____

6

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3 The roots of the quartic equation

$$x^4 - px^2 + qx - r = 0$$

are α, α, α and β

It is given that p, q and r are positive real numbers.

3 (a) Show that $p^2 = 12r$

[4 marks]

3 (b) Express the product pr in terms of q

[2 marks]

Answer _____

6

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4 A curve C has equation $y^2 = 4x$

The arc of C between the points $\left(3, 2\sqrt{3}\right)$ and $\left(4, 4\right)$ is rotated through 2π radians about the x-axis.

Show that the area of the curved surface generated is $\frac{8\pi}{3}\left(5\sqrt{5} - 8\right)$

[5 marks]

5

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5 Use an integrating factor to find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{1}{x}y = \frac{1}{x}\sin^{-1}x$$

Give your answer in the form $y = \mathrm{f}(x)$

[5 marks]

$y =$ _____

5

Turn over ►

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6 It is given that

$$z = \cos\theta + \mathrm{i}\sin\theta$$

6 (a) (i) Use de Moivre's theorem to show that $z^n + \frac{1}{z^n} = 2\cos n\theta$

[3 marks]

6 (a) (ii) Write down an expression for $z - \frac{1}{z}$ in terms of $\sin\theta$

[1 mark]

$$z - \frac{1}{z} =$$

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6 (b) **Hence** express $\cos^3\theta \sin^4\theta$ in the form

$$p\cos\theta + q\cos 3\theta + r\cos 5\theta + s\cos 7\theta$$

where p, q, r and s are rational numbers.

[5 marks]

$\cos^3\theta \sin^4\theta =$ _____

9

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7 It is given that $y = (1+x)^2 \ln(1+x)$

7 (a) Show that $\frac{d^2y}{dx^2} = 2\ln(1+x)+3$

[2 marks]

7 (b) Prove by induction that for all integers $n \geq 3$

$$\frac{d^n y}{dx^n} = (-1)^{n-1} \frac{2(n-3)!}{(1+x)^{n-2}}$$

[5 marks]

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7 (c) **Hence** find the exact value for the sum of the coefficients of x^{10} and x^{11} in the Maclaurin series expansion for $(1+x)^2 \ln(1+x)$

[2 marks]

Answer _____

9

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8 Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} = 4 + 12x + 10\sin x$$

given that $y = 1$ when $x = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 5$ when $x = 0$

Give your answer in the form $y = \mathrm{f}(x)$

[10 marks]

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$y =$ _____

10

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9 Find the exact value of the improper integral

$$\int_0^{\frac{\pi}{4}} \left(\frac{1+2x}{x+x^2} - \frac{\sin 2x}{1-\cos 2x} \right) \mathrm{d}x$$

showing the limiting process used.

Give your answer in the form $\ln k$ where k is an expression in terms of π

[8 marks]

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Answer _____

8

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10 The plane Π_1 has vector equation $\mathbf{r} \cdot \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = 2$

The line L has Cartesian equation $\frac{x-7}{2} = 1-y = \frac{z-3}{4}$

10 (a) Find the acute angle between the line L and the plane Π_1

Give your answer to the nearest $0.1°$

[4 marks]

Answer _____

10 (b) The point P has coordinates $(4, 1, 15)$

Find the shortest distance from P to the line L

Give your answer in an exact form.

[4 marks]

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Answer _____

10 (c) The plane Π_2 has Cartesian equation $x - 3y + 2z = 1$

Find a vector equation for the line of intersection of the planes Π_1 and Π_2

Give your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

[5 marks]

Answer _____

13

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11 The matrix $\mathbf{M}$ is defined as

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & c & -11 \\ 2 & -1 & 1 \end{bmatrix}$$

where c is an integer.

11 (a) Find $\mathbf{M}^{-1}$ in terms of c

[5 marks]

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Answer _____

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11 (b) It is given that the distinct eigenvalues of $\mathbf{M} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & c & -11 \\ 2 & -1 & 1 \end{bmatrix}$ are λ_1, λ_2 and λ_3

where

$$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = -4$$

11 (b) (i) Show that $c = 5$

[5 marks]

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11 (b) (ii) Find the three eigenvalues.

[2 marks]

Answer _____

11 (b) (iii) Find an eigenvector corresponding to the least eigenvalue.

[3 marks]

Answer _____

15

Turn over ►

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12 The five roots of the equation $z^5 = 4 - 4\mathrm{i}$ when plotted on an Argand diagram form the vertices of a pentagon.

12 (a) Find the root that is closest to the Imaginary axis.

Give your answer in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$

[3 marks]

Answer _____

12 (b) Find the product of the roots that lie above the Real axis.

Give your answer in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$

[2 marks]

Answer _____

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12 (c) Find the perimeter of the pentagon.

Give your answer in the form $a\sin(b\pi)$ where a and b are exact values

[2 marks]

Answer _____

7

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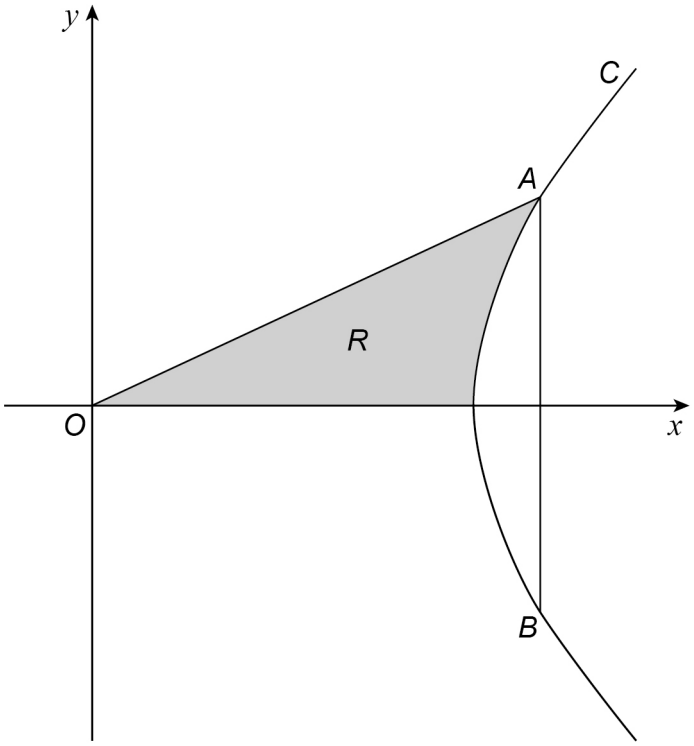
13 The diagram shows the points A and B on the curve C and the lines OA and AB

The point O is the origin and AB is perpendicular to the x-axis.



The Cartesian equation of curve C is $x^2 - y^2 = 8$ where $x \geq 2\sqrt{2}$

The point A has Cartesian coordinates $\left(2\sqrt{3}, 2\right)$

13 (a) Taking the positive x-axis as the initial line and O as the pole, show that the polar equation of curve C is $r = \sqrt{8 \sec 2\theta}$

[3 marks]

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13 (b) **Hence** find the area of the shaded region R bounded by the line OA, the initial line and the curve C

Give your answer in the form $\ln\left(a+b\sqrt{3}\right)$ where a and b are integers.

[6 marks]

Answer _____

13 (c) **Hence** find the area of the region bounded by the curve C and the line AB

Give your answer in an exact form.

[2 marks]

Answer _____

11

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14 A curve C has equation

$$y = 7\cosh x - \sinh x - 4x + 4\tanh^{-1} k$$

where k is a constant and $|k| < 1$

It is given that the line $y = 10$ is a tangent to the curve C

Show that the exact value of k can be expressed in the form $k = \dfrac{\mathrm{e} + p}{\mathrm{e} + q}$

where p and q are real numbers.

[10 marks]

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