OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Monday 13 May 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 The complex number z is given by

$$z = 2 + i\sqrt{5}$$

1 (a) Express z in the form $r(\cos\theta + i\sin\theta)$ where $r > 0$ and $-\pi < \theta \leq \pi$

Give your value of θ to three significant figures.

[2 marks]

Answer _____

1 (b) Write down the complex number z^* in the form $r(\cos\theta + i\sin\theta)$ where $r > 0$ and $-\pi < \theta \leq \pi$

[2 marks]

Answer _____

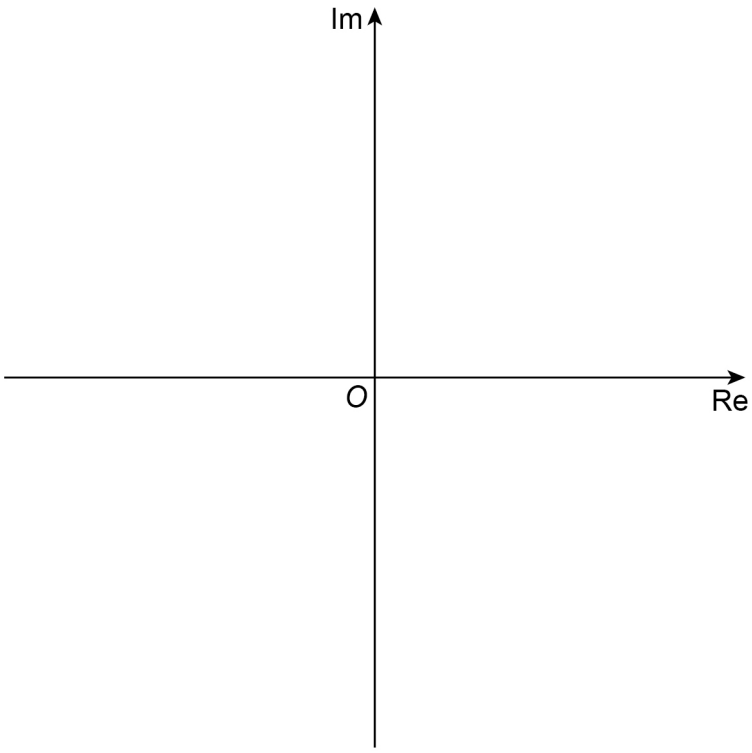
Do not write outside the box

1 (c) On an Argand diagram, the complex number z is represented by the point P and the complex number z^* is represented by the point Q

The point R is such that $OPRQ$ is a rhombus, where O is the origin.

1 (c) (i) Sketch the rhombus $OPRQ$ on the Argand diagram below.

[2 marks]



1 (c) (ii) Find the number represented by point R

[1 mark]

Answer _____

1 (c) (iii) Find the area of rhombus $OPRQ$

Give your answer in exact form.

[2 marks]

Answer _____

9

Turn over ►

Do not write outside the box

2 (a) Expand and simplify $(4+h)^3$

[2 marks]

Answer _____

2 (b) A curve has equation $y = x^3 + 7x$

2 (b) (i) A line passes through two points on the curve, one where $x = 4$ and the other where $x = 4 + h$

Find the gradient of this line in the form $a + bh + h^2$ where a and b are constants.

[3 marks]

Answer _____

Do not write outside the box

2 (b) (ii) Use your answer to **part (b) (i)** to find the gradient of the curve at the point where $x = 4$

Show the limiting process used.

[2 marks]

Answer _____

7

Turn over for the next question

Do not write outside the box

3 It is given that

$$\sum_{r=1}^{n}\left(ar^3+br^2\right) = \frac{1}{12}n(n+1)(3n+4)(5n+1)$$

is true for all integers $n \geq 1$

Find the value of a and the value of b

[4 marks]

$a =$ _____ $b =$ _____

4

Do not write outside the box

4 The quadratic equation

$$z^2 + 4z + w = 0$$

has roots $5 - \mathrm{i}$ and α

Find the complex constant w and the root α

Give your answers in the form $a + \mathrm{i}b$

[4 marks]

$w =$ _____ $\alpha =$ _____

4

Turn over ►

Do not write outside the box

5 The volume V cm^3 of sand in a container is given by

$$V = 4h^3$$

where h cm is the height of the sand in the container.

Sand is flowing out of the container through a small hole in the base at a constant rate of 16 cm^3 per second.

Find the rate at which the height of the sand is decreasing when the height of the sand in the container is 2.5 cm

Give your answer in centimetres per second to two significant figures.

[5 marks]

Do not write outside the box

Answer _____

5

Turn over for the next question

Do not write outside the box

6 (a) (i) Show that

$$\frac{1}{r} - \frac{1}{r+1} = \frac{1}{r(r+1)}$$

[1 mark]

6 (a) (ii) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)}$$

can be written as a single simplified fraction in terms of n

[4 marks]

Do not write outside the box

6 (b) Write down the value of

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)}$$

[1 mark]

Answer _____

6 (c) Find the exact value of

$$\sum_{r=1001}^{2000} \frac{1}{r(r+1)}$$

[2 marks]

Answer _____

8

Turn over ►

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7 The curve C has equation

$$y = \frac{x^2}{x^2 + ax + 3}$$

where a is a constant such that C has exactly one asymptote.

7 (a) Write down the equation of the asymptote of C

[1 mark]

Answer _____

7 (b) Show that

$$a^2 < p$$

where p is an integer.

[2 marks]

7 (c) Show that if the line $y = k$ does **not** intersect C then

$$k^2\left(12 - a^2\right) - 12k > 0$$

[4 marks]

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7 (d) In the case when $a = \sqrt{5}$, use the result from **part (c)** to find the y-coordinate of each of the stationary points of C

[3 marks]

Answer _____

10

Do not write outside the box

8 (a) Expand and simplify $(\alpha+\beta)^4$

[2 marks]

Answer _____

8 (b) Hence, or otherwise, express $\alpha^4+\beta^4$ in terms of $\alpha+\beta$ and $\alpha\beta$

[3 marks]

Answer _____

Do not write outside the box

8 (c) The quadratic equation $2x^2 - x + 6 = 0$ has roots α and β

8 (c) (i) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$

[2 marks]

$\alpha + \beta =$ _____ $\alpha\beta =$ _____

8 (c) (ii) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha^2}{\beta^2}$ and $\frac{\beta^2}{\alpha^2}$

[5 marks]

Answer _____

12

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9 The line L is the locus of points on an Argand diagram such that

$$|z-2| = |z-4\mathrm{i}|$$

9 (a) Line L passes through the point representing the complex number $z = 1 + c\,\mathrm{i}$ where c is a real constant.

Find the value of c

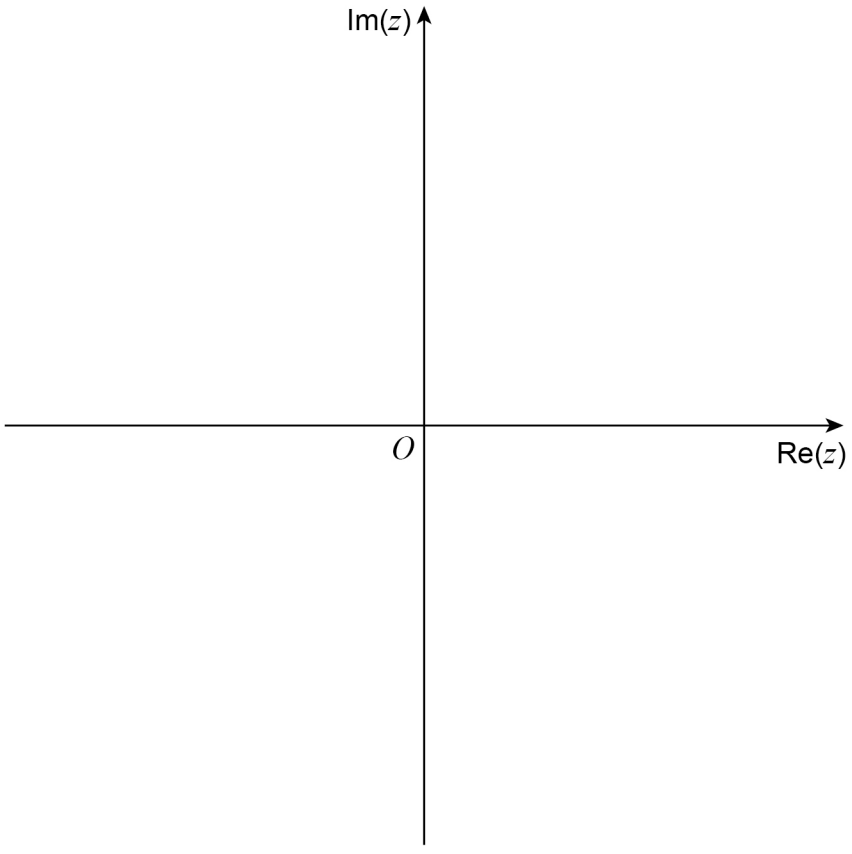
[1 mark]

Answer _____

9 (b) Sketch L on the Argand diagram below.

Include the values where L intersects the real and imaginary axes.

[4 marks]



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9 (c) The half-line H is the locus of points on an Argand diagram such that

$$\arg(z+a) = \tan^{-1}(b)$$

where a and b are real constants.

Every point on H also lies on L

9 (c) (i) Find the value of a

[1 mark]

Answer _____

9 (c) (ii) Find the value of b

[1 mark]

Answer _____

7

Turn over for the next question

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10 The locus of a point $P(x, y)$ is such that the distance from P to the point $(0, 9)$ is equal to the distance from P to the line with equation $y = -3$

The locus of P is the curve C_1

10 (a) Find the equation of C_1 in the form

$$x^2 = ay + b$$

where a and b are constants.

[4 marks]

Answer _____

10 (b) A reflection in the line $y = x$ maps curve C_1 onto curve C_2

10 (b) (i) Write down the equation of C_2

[1 mark]

Answer _____

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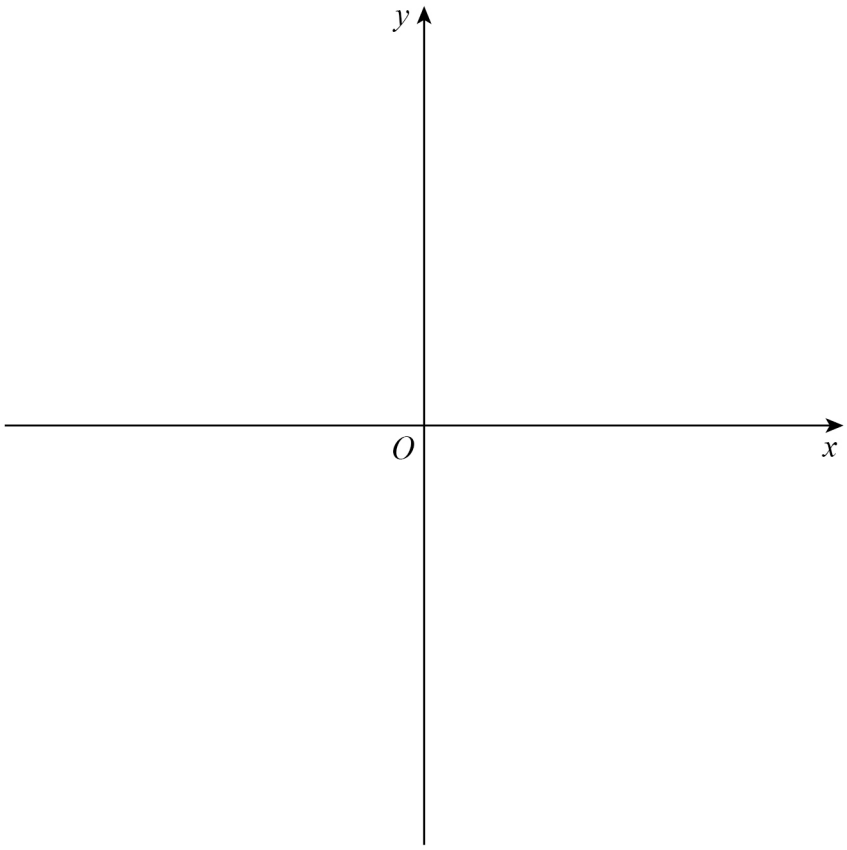
10 (b) (ii) Sketch the graph of C_2

Include the value of any axis intercepts.

[2 marks]



10 (b) (iii) The line with equation $y = mx$ intersects C_2 twice.

Find the range of possible values for m

[5 marks]

Turn over ►

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Answer _____

10 (b) (iv) Write down the equations of the two tangents to C_2 which pass through the origin.

[2 marks]

Answer _____

14

END OF QUESTIONS

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