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MATHEMATICS

MA02

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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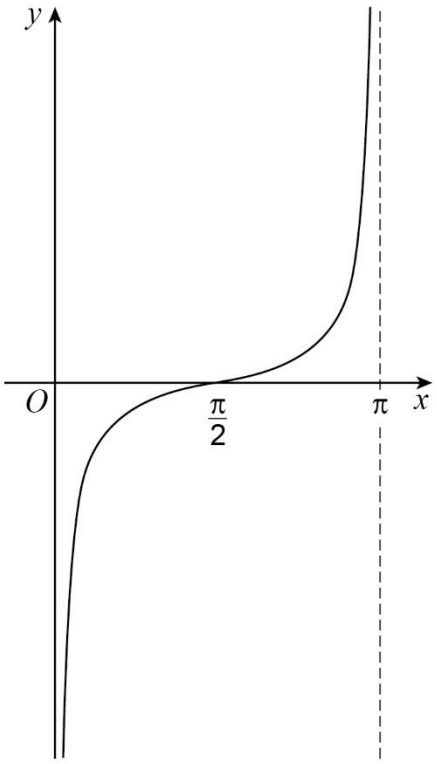
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$5x = \dfrac{\log_a 7}{\log_a 4}$ or $5x = \log_4 7$ $[= 1.40367...]$ or $10x = \log_2 7$ $[= 2.80735...]$ or $x = \log_{1024} 7$ $[= 0.28073...]$	M1	Correctly expresses x, $5x$ or $10x$ in terms of logarithms. **PI** by correct final answer or correct value of x, $5x$ or $10x$ rounded or truncated to at least 3dp.
	$\left[x = \frac{1}{5}\log_4 7 =\right]$ 0.281	A1	**CAO**
		2	

Q	Answer	Marks	Comments
1(b)	$\left[\log_6 180 =\right]$ $\log_6 36 + \log_6 5$ or $\log_6 6^2 + \log_6 5$ or $\log_6 6 + \log_6 6 + \log_6 5$ or $\log_6 2 + \log_6 90$ or $\log_6 6 + \log_6 30$	M1	**oe** Use of addition property of logarithms to form a correct numerical expression. Condone omission of base of logarithm
	$\left[\log_6 180 =\right]$ $2 + \log_6 5$	A1	Correct answer in the correct form, including base of logarithm. **M1** not scored or working seen in relation to a and b scores **M0A0**
		2	

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)		**B1** **B1**	Curve in the correct form and only in the first and fourth quadrants. Must appear to be asymptotic to the y-axis and the line $x = \pi$ Line $x = \pi$ need not be seen. Correct value of the x-intercept marked at point where their curve intersects the x-axis. Condone coordinates given or value given in degrees.
		2	

Q	Answer	Marks	Comments
2(b)	π [radians]	**B1**	Condone given in degrees.
		1	

	Question 2 Total	**3**	

Q	Answer	Marks	Comments
3(a)	$\frac{\sin\theta}{15} = \frac{\sin\frac{\pi}{5}}{9}$	**M1**	**oe** Correct use of Sine Rule. Allow 36° for $\frac{\pi}{5}$ radians.
	$\left[\theta = \sin^{-1}(0.97964...) = \right]$ 1.369	**A1**	**AWRT** 1.369 Condone answer given in degrees. **AWRT** 78.4°
	$\left[\theta = \pi - 1.36867... = \right]$ 1.773	**A1ft**	**ft** their acute angle. **AWRT** 1.773 Condone answer given in degrees. **AWRT** 101.6° If **M1 A0 A0** awarded, allow **SC1** for 1.37 and 1.77
		3	

Q	Answer	Marks	Comments
3(b)	[Area of Sector *OPQ*] $\frac{1}{2}\times 15^2\times\frac{\pi}{5}$	**M1**	Use of Area $=\frac{1}{2}r^2\theta$ Condone use of degrees.
	$\left[\text{Area of Sector } OPQ =\right]$ 70.68583... $\left[\text{cm}^2\right]$ or $\frac{45\pi}{2}$ $\left[\text{cm}^2\right]$	**A1**	**AWRT** 70.7 **PI** by correct final answer.
	[Area of Triangle *OQR*] $\frac{1}{2}\times 9\times 15\times\sin\left(\pi-\frac{\pi}{5}-1.773\right)$ or $\frac{1}{2}\times 9\times 15\times\sin 0.74035...$	**M1**	Use of Area $=\frac{1}{2}ab\sin C$ **ft** their obtuse or acute angle from **part (a)** Condone use of degrees.
	$\left[\text{Area of Triangle } OQR =\right]$ 45.53199... $\left[\text{cm}^2\right]$	**A1ft**	**AWRT** 45.5 **ft** their obtuse angle from **part (a)** **PI** by correct final answer.
	$[A =]$ 70.68583... – 45.53199...		
	$[A =]$ 25.2	**A1ft**	**AWRT** 25.2 **ft** their area of sector *OPQ* – their area of triangle *OQR* provided at least both M1 marks awarded and obtuse angle used Ignore units if included.
		5	

	Question 3 total	**8**	

Q	Answer	Marks	Comments
4(a)	$\frac{17-8}{9-12}$ $[=-3]$	**M1**	Method for finding the gradient of *AP* **PI** by correct gradient of *AP*
	$[\text{Gradient of } l =]$ $\frac{1}{3}$	**A1ft**	**ft** their gradient of *AP* May see in later working.
	$y-8=\frac{1}{3}(x-12)$ or $y=\frac{1}{3}x+4$	**M1**	**oe** Correct equation of l **ft** their gradient of l **PI** by correct final answer.
	$(0,4)$	**A1**	Correct coordinates of *B* Condone value of *y*-intercept given only.
		4	

Q	Answer	Marks	Comments
4(b)	$\sqrt{9^2+(17-4)^2}$	**M1**	**oe PI** **ft** their coordinates of *B* Method for finding the length of *AB* using their *y*-coordinate of *B*
	[Length of *AB* =] $5\sqrt{10}$	**A1**	Accept $\sqrt{250}$ **PI** by $2\sqrt{10}$, $8\sqrt{10}$, 40 or 640
	$x^2+(y-4)^2$ [$=k$] or $\left[k=\left(5\sqrt{10}-3\sqrt{10}\right)^2=40\Rightarrow\right]$ $x^2+(y-n)^2=40$ or $\left[k=\left(5\sqrt{10}+3\sqrt{10}\right)^2=640\Rightarrow\right]$ $x^2+(y-n)^2=640$	**M1**	Correct LHS of equation. **ft** their coordinates of *B* or Equation of the correct form set equal to 40 or 640
	[C_2] $x^2+(y-4)^2=40$ [C_3] $x^2+(y-4)^2=640$	**A2**	**A1**: One correct equation. **A2**: Both correct equations. Equations do not need to be identified.
		5	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)	$\left[6\sin\theta = \dfrac{\cos^2\theta - 9}{3 - \sin\theta}\right]$		
	$6\sin\theta(3 - \sin\theta) = \cos^2\theta - 9$ or $18\sin\theta - 6\sin^2\theta = \cos^2\theta - 9$	**M1**	Forms a correct equation containing $\cos^2\theta$ with fractions removed
	$18\sin\theta - 6\sin^2\theta = (1 - \sin^2\theta) - 9$	**M1**	Use of $\sin^2\theta + \cos^2\theta = 1$ to eliminate $\cos^2\theta$ eg $6\sin\theta = \dfrac{(1 - \sin^2\theta) - 9}{3 - \sin\theta}$
	$18\sin\theta - 6\sin^2\theta = -8 - \sin^2\theta$ $5\sin^2\theta - 18\sin\theta - 8 = 0$	**A1**	A correct unsimplified equation in $\sin\theta$ and $\sin^2\theta$ with brackets removed before **AG** Must be convincingly shown.
		3	

Q	Answer	Marks	Comments
5(b)	$\left[5\sin^2 3x - 18\sin 3x - 8 = 0 \Rightarrow\right]$ $(5\sin 3x + 2)(\sin 3x - 4)$ $[= 0]$	**M1**	Attempt to solve the quadratic equation. May see use of the quadratic formula but must be a correct substitution. Condone θ for $3x$ **PI** by $[\sin 3x =] -\frac{2}{5}$ and $[\sin 3x =] 4$ seen.
$\sin 3x = -\frac{2}{5}$ and $\sin 3x = 4$ rejected		**A1**	Both correct values for $\sin 3x$ with $\sin 3x = 4$ rejected. Rejection **PI** in later working Condone θ for $3x$
$[3x =]$ $203.5[78178...]^\circ$ or $[3x =]$ $336.4[21821...]^\circ$		**A1**	Anything that rounds or truncates correctly to 1 dp. **PI** by **AWRT** 67.9° or **AWRT** 112.1° Allow $[3x =]$ $-23.5[78178...]^\circ$ Condone more accurate values: $x = 67.85939...^\circ$ $x = 112.14060...^\circ$
$[x =]$ $67.9^\circ, 112.1^\circ$		**B2,1**	Ignore values outside the given interval. If both correct answers given deduct 1 mark for each extra incorrect value in the given interval to a minimum of **B0** If one correct answer only given then **B1** if there is no more than one incorrect answer in the given interval.
		5	

Q	Answer	Marks	Comments
5(c)	$[3y + 15^\circ = 3x \quad \Rightarrow \quad y = x - 5^\circ]$ $[y =]$ $62.9^\circ, 107.1^\circ$	**B1ft**	**ft** their answer(s) to **part (b)** provided their value(s) for y are in the given interval.
		1	

	Question 5 Total	**9**	

Q	Answer	Marks	Comments
6(a)	$\left[2\log_3(4x-1)-\log_3(2x+7)=\right]\ 3$ or $\log_3 27$	**B1**	Elimination of $\log_3$ from LHS **PI** in later working.
	$\left[2\log_3(4x-1)=\right]\ \log_3(4x-1)^2$	**M1**	Use of logarithm property seen inside or outside of an equation.
	$\left[\log_3(4x-1)^2-\log_3(2x+7)=\right]$ $\log_3\frac{(4x-1)^2}{2x+7}$	**M1**	Use of a second logarithm property seen inside or outside of an equation.
	$\left[\log_3\frac{(4x-1)^2}{2x+7}=3\Rightarrow\right]$ $\frac{(4x-1)^2}{2x+7}=27$	**M1**	**oe** Correct equation with logarithms eliminated.
	$16x^2-8x+1=54x+189$ or $16x^2-62x-188=0$ and $8x^2-31x-94=0$	**A1**	**oe** Brackets expanded and fraction cleared before **AG** Must be convincingly shown.
		5	

Q	Answer	Marks	Comments
6(b)	$\left[8x^2-31x-94=0\Rightarrow\right]$ $(8x-47)(x+2)\ [=0]$	**M1**	Attempt to solve the quadratic equation. May see use of the quadratic formula but must be a correct substitution. **PI** by $x=\frac{47}{8}$ and $x=-2$
	$[x=]\ \frac{47}{8}$	**A1**	**oe CAO**
		2	

	Question 6 Total	**7**	

Q	Answer	Marks	Comments
7(a)	$\left[\mathrm{P}(A)\times\mathrm{P}(B)=\right]$ $0.37\times0.16 \quad [=0.0592]$ Not independent as $\mathrm{P}(A)\times\mathrm{P}(B)=0.0592\neq0.07=\mathrm{P}(A\cap B)$	**M1** **A1**	Correct method to find $\mathrm{P}(A)\times\mathrm{P}(B)$ **PI** by 0.0592 Correct conclusion from correct working
		2	

Q	Answer	Marks	Comments
7(b)	$\left[\mathrm{P}(A\cup B)=\right]$ $0.37+0.16-0.07$	**M1**	
	$\left[\mathrm{P}(A\cup B)=\right]$ 0.46	**A1**	**CAO**
		2	

Q	Answer	Marks	Comments
7(c)	$\left[\mathrm{P}(A\mid B)=\right]$ $\frac{0.07}{0.16}$	**M1**	
	$\left[\mathrm{P}(A\mid B)=\right]$ 0.4375 or $\frac{7}{16}$	**A1**	**CAO**
		2	

	Question 7 Total	**6**	

Q	Answer	Marks	Comments
8(a)	$[p =]\ 1 - 0.2 - 0.41 - 0.23 = 0.16$	**B1**	**AG** Must be convincingly shown. Condone: $p + 0.20 + 0.41 + 0.23 = 1$ so $p = 0.16$
		1	

Q	Answer	Marks	Comments
8(b)	$-2 \times 0.16 + 1 \times 0.2 + 3 \times 0.41 + 0.23a = 4.1$ $[a =\ ]\ 13$	**M1** **A1**	Correct equation. **oe**
		2	

Q	Answer	Marks	Comments
8(c)	$\left[\mathrm{E}\left(X^2\right) =\right]\ (-2)^2 \times 0.16 + 1^2 \times 0.2 + 3^2 \times 0.41 + 13^2 \times 0.23$ $[= 43.4]$ $\left[\mathrm{Var}(X) =\right]\ 43.4 - 4.1^2\quad [= 26.59]$ $\left[\text{Standard deviation} = \sqrt{26.59} =\right]\ 5.16$	**M1** **M1** **A1**	**oe** Correct expression for $\mathrm{E}\left(X^2\right)$ simplified or unsimplified. **ft** their a **PI** by correct value or correct variance. Correct expression for $\mathrm{Var}(X)$ simplified or unsimplified. **ft** their $\mathrm{E}\left(X^2\right)$ **PI** by correct variance. **AWRT** 5.16
		3	

	Question 8 Total	**6**	

Q	Answer	Marks	Comments
9(a)(i)	$\left[\mathrm{E}(X) = \right]\ 9\times0.4\quad [=3.6]$	**M1**	**oe** Correct expression for $\mathrm{E}(X)$ **PI**
	$\left[\mathrm{E}(Y-X)\ =\ 16.2-3.6\ =\right]\ 12.6$	**A1**	**oe**
		2	

Q	Answer	Marks	Comments
9(a)(ii)	$\left[\mathrm{Var}(X) = \right]\ 9\times0.4\times0.6\quad [=2.16]$	**M1**	**oe** Correct expression for $\mathrm{Var}(X)$ **PI**
	$\left[\mathrm{Var}(Y)=5-2.16=\right]\ 2.84$	**A1**	**oe**
		2	

Q	Answer	Marks	Comments
9(b)	$\left[\mathrm{P}(X\geq1)>0.92\ \ \Rightarrow\right]\ \mathrm{P}(X=0)<0.08$	**M1**	Use of 0.08 **PI**
	$0.6^n < 0.08$	**m1**	Forms a relationship involving only 0.6^n and 0.08 **PI** by 0.1296 or **AWRT** 0.0778 (0.07776)
	$\left[\begin{array}{l}\log 0.6^n < \log 0.08\\ \Rightarrow n\log 0.6 < \log 0.08\end{array}\right]$ $\Rightarrow n > \dfrac{\log 0.08}{\log 0.6}\ [=4.94\ldots]$	**m1**	Attempt at solving their relationship between 0.6^n and 0.08 **PI** by 4.94… **oe**
	$[n=]\ 5$	**A1**	**CAO**
		4	

	Question 9 Total	**8**	

Q	Answer	Marks	Comments
10	$1.6 = 7t + \frac{1}{2} \times (-9.8) \times t^2$	**M1 A1**	**M1**: Use of $s = ut + \frac{1}{2}at^2$ Condone ± 9.8, and 3 instead of 1.6 **A1**: Fully correct.
	$4.9t^2 - 7t + 1.6 = 0$	**M1**	Arranges to three-term quadratic equation and attempts to solve.
	$t_1 = \frac{2}{7}$, $t_2 = \frac{8}{7}$	**A1**	Both values correct. **AWRT** 0.29, 1.14
	$[t_2 - t_1 =]$ $\frac{6}{7}$ [seconds]	**A1ft**	**AWRT** 0.86 Allow 0.85 **ft** their t_1 and t_2 (fractions, 2 sf or better) Condone $\frac{2}{7} \le t \le \frac{8}{7}$ but not $t = \frac{2}{7}$, $\frac{8}{7}$
10 Alt	$0^2 = 7^2 + 2(-9.8)s$ or $v^2 = 7^2 + 2(-9.8)(1.6)$	**M1**	Correct method to find maximum height above point of projection **PI** in later working or by $s = 3.9$ or $[\pm]0.9$ or correct method to find the speed at a height of 1.6 metres above point of projection
	$s = 2.5$ [metres] or $v = 4.2$ [m s^{-1}]	**A1**	Correct maximum height above point of projection **PI** in later working or by $s = 3.9$ or $[\pm]0.9$ or correct speed at a height of 1.6 metres above point of projection
	$-0.9 = \frac{1}{2}(-9.8)t^2$ or $0 = 4.2 - 9.8t$	**M1**	**oe** Correct method to find time taken to move from 3 metres above ground to maximum height or reverse
	$[t =]$ $\frac{3}{7}$	**A1**	**oe** Correct time. **PI** **AWRT** 0.43
	[Time =] $\frac{6}{7}$ [seconds]	**A1ft**	**ft** their $t = \frac{3}{7}$ **AWRT** 0.86 Allow 0.85

	Question 10 Total	**5**	

Q	Answer	Marks	Comments
11(a)	$\left[F =\right]\ 0.3\times 50g$	**B1**	A correct expression for friction seen or used. **PI** by $\left[F =\right] 15g$ or 147
	$T - F = 50\times 0.4$ and $mg - T = m\times 0.4$ or $mg - 0.3\times 50g = 0.4(50 + m)$ or $m(g - 0.4) = 20 + 15g$ or $mg - 167 = m\times 0.4$	**M1 A1**	**oe** **ft** their F for **M1** only. **M1**: Forms at least one correct equation of motion for the block/particle or forms an equation of motion for the system with at least two terms correct **A1**: Both equations of motion correct for the block and the particle, or the correct equation of motion for the system
	$m = \dfrac{167}{g - 0.4}$	**m1**	**oe** Attempt at solving for m using their equation(s) **PI** by $m = 18$
	$m = 18$	**A1**	**AWRT** 18 (17.76…)
		5	

Q	Answer	Marks	Comments
11(b)	$v^2 = 0^2 + 2(0.4)(1.8)$ or $1.8 = 0.5(0.4)t^2$	**M1**	Attempt at an appropriate equation with values substituted. Allow sign errors.
	$v^2 = 1.44$ and $[v =]\ 1.2\ [\text{m s}^{-1}]$ or $t = 3$ and $[v =]\ 0.4 \times 3 = 1.2\ [\text{m s}^{-1}]$	**A1**	**AG** Must be convincingly shown.
		2	

Q	Answer	Marks	Comments
11(c)	$[-0.3 \times 50g = 50a \Rightarrow]$		
	$a = -2.94$	**B1**	
	$0^2 = 1.2^2 + 2 \times (-2.94) \times s$	**M1**	Ignore signs using their a Allow if a value rounded.
	$[s = 0.24]$		
	$[0.24 + 1.8 =]$ $2.04\ [\text{metres}]$	**A1ft**	Sums 1.8 and s **ft** their s provided **M1** scored and final answer is greater than 1.8 **AWRT** 2.0 Condone 2.05
		3	

	Question 11 Total	**10**	

Q	Answer	Marks	Comments
12	$3\times 4+2\times(-7)=3v_A+2\times(-1)$	**M1**	Considers *B* not being reversed v_A not required.
	$[v_A =]\ 0\ [\text{m s}^{-1}]$	**A1**	Allow a clear rejection of this case instead of $[v_A =]\ 0\ [\text{m s}^{-1}]$
	$3\times 4+2\times(-7)=3v_A+2\times 1$	**M1**	Considers *B* being reversed. **PI** by a full correct explanation after considering situation where *B* is not reversed
	$[\lvert v_A\rvert =]\ \frac{4}{3}\ [\text{m s}^{-1}]$ or $[v_A =]\ -\frac{4}{3}\ [\text{m s}^{-1}]$	**A1**	**PI** by a full correct explanation after considering situation where *B* is not reversed
	Direction of particle *B* **has** been reversed by the collision	**E1**	If they progress to 2 values for v_A, the valid value should be highlighted or the invalid situation rejected.

	Question 12 Total	**5**	