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MA01

(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)(i)	$\lceil \lfloor x_1 = \rfloor \rfloor$ 15	B1	
		1	

Q	Answer	Marks	Comments
1(a)(ii)	$[n =]$ 10	B1	
		1	

Q	Answer	Marks	Comments
1(b)	$L = pL + 12$ and $L = 2pL + 3$	M1	**oe** Forms two equations relating L and p **PI** by $pL + 12 = 2pL + 3$ **oe**
	$\frac{12}{1-p} = \frac{3}{1-2p}$ or $12(1-2p) = 3(1-p)$ or $\frac{L-12}{L} = \frac{L-3}{2L}$ or $2L(L-12) = L(L-3)$ or $L = 2(L-12) + 3$ or $L = \frac{L-3}{2} + 12$	M1	**oe** Eliminates L or p to form a single equation. **PI** by both correct final answers or $pL = 9$
	$[L =]$ 21	A1	**CAO**
	$[p =] \frac{3}{7}$	A1	**CAO**
		4	

	Question 1 Total	6	

Q	Answer	Marks	Comments
2(a)	$729x^{42}y^{4}$ or $3x^{2}y^{\frac{3}{4}}$ or $\frac{1}{3}x^{-2}y^{-\frac{3}{4}}$	**M1**	Correct application of index rules. Condone one error in the coefficient or a power.
	$\left[ax^{b}y^{c} =\right] 243x^{40}y^{\frac{13}{4}}$	**M1 A1**	**oe** **M1**: Answer in correct form with at least two of a, b and c correct. **A1**: Correct answer in the correct form. Condone $a = 3^{5}$
		3	

Q	Answer	Marks	Comments
2(b)	$25 + 40\sqrt{w} + 16w$ or $25 - 80\sqrt{w} + 64w$ or $\left(10 - 4\sqrt{w}\right)\left(12\sqrt{w}\right)$	**M1**	**oe** Correct expansion of a term from the numerator. Simplified or unsimplified. or Uses the difference of two squares to write the numerator as the correct product of two terms.
	$\frac{25 + 40\sqrt{w} + 16w - 25 + 80\sqrt{w} - 64w}{8\sqrt{w}}$ or $\frac{120\sqrt{w} - 48w}{8\sqrt{w}}$	**M1**	**oe** Correct fraction with numerator expanded simplified or unsimplified. **PI** by $3\left(5 - 2\sqrt{w}\right)$ or $\frac{3\left(10 - 4\sqrt{w}\right)}{2}$
	$15 - 6\sqrt{w}$	**A1**	**CAO** Condone $15 + (-6)\sqrt{w}$ or $-6\sqrt{w} + 15$
		3	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$\left[\text{Gradient of } l_1 =\right] \quad \frac{-4-(-3)}{6-(-1)}$	M1	**oe** Correct method for finding the gradient of l_1
	$-\frac{1}{7}$ and 7	M1	Sight of both gradients
	$y-(-3)=7(x-(-1))$ and $y=7x+4$	A1	**oe** Uses gradient and coordinates of Q to form equation leading to the required result May see use of $y=7x+c$ and substitution of coordinates of Q to find c but must be a complete method **A1** dependent on at least **M1** awarded
		3	

Q	Answer	Marks	Comments
3(b)(i)	$(a-(-1))^2+(7a+4-(-3))^2 \quad [=360a]$ or $(a+1)^2+(7a+7)^2 \quad [=360a]$ or $\sqrt{(a-(-1))^2+(7a+4-(-3))^2} \quad [=6\sqrt{10a}]$ or $\sqrt{(a+1)^2+(7a+7)^2} \quad [=6\sqrt{10a}]$	M1	Correct expression for QR^2 or QR
	$a^2+2a+1+49a^2+98a+49 \quad [=360a]$	M1	Expands their brackets of the form $(a+m)^2+(7a+n)^2$ **oe** correctly
	$50a^2+100a+50=360a$ or $50a^2-260a+50=0$ and $5a^2-26a+5=0$	A1	Set equal to $360a$ before **AG** Must be convincingly shown.
		3	

Q	Answer	Marks	Comments
3(b)(ii)	$\left[5a^2 - 26a + 5 = 0 \Rightarrow\right]$ $(5a-1)(a-5)$ $[=0]$ $\left(\frac{1}{5}, \frac{27}{5}\right)$ and $(5, 39)$	**M1** **M1 A1**	Correct attempt to solve the quadratic equation. May see quadratic formula used but must be a correct substitution. **PI** by the correct x-coordinates of the possible positions of R **M1**: At least one correct pair of coordinates. **A1**: Both correct pairs of coordinates.
		3	

	Question 3 Total	**9**	

Q	Answer	Marks	Comments
4(a)	$(5p-67)-(2p-18)=2((2p-18)-(p-4))$ $3p-49=2(p-14)$ or $3p-49=2p-28$ and $p=21$	**M1** **A1**	**oe** Correct equation in p **oe** Second line of working before **AG**
		2	

Q	Answer	Marks	Comments
4(b)	$[d=24-17=]$ 7	**B1**	**PI** Correct value for the common difference.
	$[a=17-14=]$ 3	**B1**	**PI** Correct value for the first term.
	$3+(m-1)\times 7=990$ or $7m-4=990$	**M1**	**oe** Correct equation equating an expression for the mth term to 990
	$[m=]$ 142	**A1**	**CAO**
		4	

Q	Answer	Marks	Comments
4(c)	$[142\div 2=]$ 71	**B1ft**	Number of terms in the series of even terms. **ft** their $142\div 2$ provided that it is even.
	$[S=]\frac{1}{2}\times 71\times(10+990)$	**M1**	**oe** Correct use of a formula for the sum of an arithmetic series. **ft** their 71 provided that it is a positive integer.
	$[S=]$ 35500	**A1ft**	**ft** their 71 provided that it is a positive integer.
		3	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)	$\left[\lfloor \mathrm{f}(-5) = \rfloor\right]$ $3(-5)^2 + (4k+5)(-5) + 10k + a \left[= 59 - 17k\right]$ $75 - 20k - 25 + 10k + a = 59 - 17k$ or $50 - 10k + a = 59 - 17k$ or $-20k + 10k + a = 9 - 17k$ and $a = 9 - 7k$	**M1** **A1**	**oe** Correct substitution of $x = -5$ into $\mathrm{f}(x)$ **oe** Additional line of working with power evaluated and brackets expanded set equal to $59 - 17k$ before **AG** Must be convincingly shown Remainder theorem not used scores **M0 A0**
		2	

Q	Answer	Marks	Comments
5(b)	$y = 7x - 3k$	**B1**	**PI** Correct equation of *L*
	$3x^2 + (4k+5)x + 10k + 9 - 7k = 7x - 3k$	**M1**	**oe** Substitutes $a = 9 - 7k$ and sets equal to their $7x - 3k$
	$3x^2 + (4k-2)x + 6k + 9 \ [= 0]$	**A1**	Simplified to quadratic equation in x set equal to zero with like terms collected. **PI** by correct discriminant
	$(4k-2)^2 - 4\times 3\times(6k+9) \ [> 0]$	**M1**	**oe** Correct discriminant for their quadratic equation in x **ft** their quadratic equation in x
	$\lfloor\lceil 16k^2 - 16k + 4 - 72k - 108 > 0 \Rightarrow \rfloor\rceil$ $16k^2 - 88k - 104 \ [> 0]$ or $2k^2 - 11k - 13 \ [> 0]$	**M1**	Simplifies their quadratic equation to a correct three-term quadratic equation in k **ft** their discriminant.
	$(2k-13)(k+1) \ [> 0]$	**M1**	**oe** Correct attempt to solve the correct quadratic equation May see quadratic formula used but must be a correct substitution **PI** by both correct critical values.
	$[k =] -1$ and $[k =] \frac{13}{2}$	**A1**	**oe** Correct critical values. Accept $\frac{13}{2}$ unsimplified.
	$k < -1$ or $k > \frac{13}{2}$	**A1**	**oe CAO** Correct inequalities Condone $k < -1$, $k > \frac{13}{2}$ but not $k < -1$ and $k > \frac{13}{2}$ Accept $\frac{13}{2}$ unsimplified.
		8	

	Question 5 Total	**10**	

Q	Answer	Marks	Comments
6(a)	$(0, p)$	**B1**	
		1	

Q	Answer	Marks	Comments
6(b)	$(x-4)^2 \ldots$ or $x^2 - 8x + 16 \ldots$	**M1**	
	$6(x-4)^2 - 96 + 103$	**A1**	Allow $6\left\lfloor\left\lceil(x-4)^2 - 16\right\rceil\right\rfloor + 103$ $6\left\lceil(x-4)^2 - 16 + \frac{103}{6}\right\rceil$ $6\left\lfloor\left\lceil(x-4)^2 + \frac{7}{6}\right\rceil\right\rfloor$ $6(x-4)^2 - 6\times 16 + 103$
	$6(x-4)^2 + 7$	**A1**	**CAO** Allow $6(x-4)^2 + p + (7-p)$
	Translation	**E1**	
	$\begin{bmatrix} 4 \\ 7-p \end{bmatrix}$	**E1ft**	If incorrect **ft** their completed square form and their answer to **part (a)** Must be given as a vector.
		5	

	Question 6 Total	**6**	

Q	Answer	Marks	Comments
7(a)	$\left[\left(12x^{\frac{4}{5}}+1\right)\left(x^{-\frac{4}{5}}-12\right)=\right]$ $12-144x^{\frac{4}{5}}+x^{-\frac{4}{5}}-12$ and $x^{-\frac{4}{5}}-144x^{\frac{4}{5}}$	**B1**	**oe** Correct expansion unsimplified and **AG**
		1	

Q	Answer	Marks	Comments
7(b)	$\left[\int\left(12x^{\frac{4}{5}}+1\right)\left(x^{-\frac{4}{5}}-12\right)\mathrm{d}x\right]$ $=5x^{\frac{1}{5}}-\frac{5}{9}\times144x^{\frac{9}{5}}\ [+c]=$		
	$5x^{\frac{1}{5}}-80x^{\frac{9}{5}}\ [+c]$	**M1 A1**	**M1**: Correct integration of a term in x simplified or unsimplified. **A1**: Correct integration simplified or unsimplified.
	$5x^{\frac{1}{5}}\left(1-16x^{\frac{8}{5}}\right)+c$	**A1**	**CAO** Must have $'+c'$
		3	

	Question 7 Total	**4**	

Q	Answer	Marks	Comments
8(a)	$\left[\mathrm{f}(x)=\right]\ kx^{-\frac{1}{2}}-6+5x^{\frac{1}{2}}+\frac{9}{2}$	**B1**	**oe** Correct equation for *C* with at least one term in index form. **PI** by correct differentiation.
	$\left[\mathrm{f}'(x)=\right]\ -\frac{k}{2}x^{-\frac{3}{2}}+\frac{5}{2}x^{-\frac{1}{2}}$	**M1 A1ft**	**oe ft** their equation of *C* provided there is at least one term that is a fractional power of x Can be simplified or unsimplified. Allow terms not given in index form. **M1**: At least one correct term for their $\mathrm{f}(x)$ **A1ft**: Correct derivative.
	$\left[x=\frac{1}{4}\Rightarrow \mathrm{f}'\left(\frac{1}{4}\right)=\right]$ $-\frac{k}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}+\frac{5}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ or $-\frac{k}{2}\times 8+\frac{5}{2}\times 2$ and $5-4k$	**A1**	**oe** $x=\frac{1}{4}$ substituted into the correct derivative and **AG**
		4	

Q	Answer	Marks	Comments
8(b)(i)	$-43 = 5 - 4k$ and $k = 12$	**B1**	$\frac{dy}{dx} = -43$ substituted and **AG**
		1	

Q	Answer	Marks	Comments
8(b)(ii)	$[y =] \dfrac{12 - 6\sqrt{\frac{1}{4}} + 5 \times \frac{1}{4}}{\sqrt{\frac{1}{4}}} + \frac{9}{2}$ or $[y =] \dfrac{12 - 6 \times \frac{1}{2} + 5 \times \frac{1}{4}}{\frac{1}{2}} + \frac{9}{2}$ $[= 25]$	**M1**	**oe** $k = 12$ and $x = \frac{1}{4}$ substituted into the equation of C **ft** their equation of C in index form from **part (a)** if used. **PI** by correct y-coordinate of P
	$y - 25 = -43\left(x - \frac{1}{4}\right)$ or $y = -43x + \frac{143}{4}$	**m1**	**oe** **ft** their y-coordinate of P Correct equation of tangent. May see use of $y = -43x + c$ and coordinates of P substituted but must be a complete method.
	$172x + 4y = 143$	**A1**	**oe CAO** Correct equation in the correct form.
		3	

Q	Answer	Marks	Comments
8(c)	$\left[\frac{dy}{dx} = -6x^{-\frac{3}{2}} + \frac{5}{2}x^{-\frac{1}{2}} \Rightarrow\right]$ $\left[\frac{d^2y}{dx^2} = \right] \ 9x^{-\frac{5}{2}} - \frac{5}{4}x^{-\frac{3}{2}}$	**B1ft**	**oe** **ft** their first derivative from **part (a)** provided there is at least one term that is a fractional power of x
	$\left[x = \frac{12}{5} \Rightarrow \frac{d^2y}{dx^2} = \right] \ 9\left(\frac{12}{5}\right)^{-\frac{5}{2}} - \frac{5}{4}\left(\frac{12}{5}\right)^{-\frac{3}{2}}$ $[= 0.67239...]$	**M1**	Correct attempt to evaluate their second derivative at $x = \frac{12}{5}$ **PI** by **AWRT** 0.7 Must be working with the correct second derivative for this mark.
	$\left[x = \frac{12}{5} \Rightarrow \frac{d^2y}{dx^2} = \right]$ 0.67239... and since $\frac{d^2y}{dx^2} > 0$ then Q is a minimum.	**E1**	Correct value for the second derivative at $x = \frac{12}{5}$ **AWRT** 0.7 and Indication that second derivative is positive at Q with the correct conclusion.
		3	

Q	Answer	Marks	Comments
8(d)	$x > \frac{12}{5}$	**B1ft**	**oe** **ft** their answer to **part (c)**
		1	

	Question 8 Total	**12**	

Q	Answer	Marks	Comments
9(a)(i)	1.0208, [0.5148], [0.4167], 0.4855, 0.6736	**B2,1**	**CAO** **B1**: Two correct values. **B2**: All values correct.
		2	

Q	Answer	Marks	Comments
9(a)(ii)	$h = 0.25$	**B1**	**PI**
	$\left[I \approx \frac{h}{2}\{...\}\right]$ $\left[\frac{h}{2}\{...\} = \right] 1.0208 + 0.6736 + 2(0.5148 + 0.4167 + 0.4855)$	**M1**	**ft** Their values from **part (a)(i)** **PI** by 0.56605
	$[I \approx 0.125 \times 4.5284 =]$ 0.566	**A1**	**CAO**
		3	

Q	Answer	Marks	Comments
9(b)(i)	$\left[\int\left(\frac{1}{4x^2} + \frac{x^3}{6}\right)\mathrm{d}x = \right] -\frac{1}{4x} + \frac{x^4}{24}\ [+c]$	**B2,1**	**B1**: One correct term, simplified or unsimplified **B2**: Correct integration with both terms simplified.
		2	

Q	Answer	Marks	Comments
9(b)(ii)	$\left[\int_{0.5}^{1.5}\left(\frac{1}{4x^2}+\frac{x^3}{6}\right)\mathrm{d}x=\right]$ $\left(-\frac{1}{4(1.5)}+\frac{(1.5)^4}{24}\right)-\left(-\frac{1}{4(0.5)}+\frac{(0.5)^4}{24}\right)$ $=\left(-\frac{1}{6}+\frac{27}{128}\right)-\left(-\frac{1}{2}+\frac{1}{384}\right)$ $=\left(\frac{17}{384}\right)-\left(-\frac{191}{384}\right)$ $=\frac{13}{24}$	**M1** **A1**	**ft** their answer to **part (b)(i)** Must see limits substituted **PI** by at least 3 of $-\frac{1}{6}, \frac{27}{128}, -\frac{1}{2}$ and $\frac{1}{384}$ **oe** used or seen **CAO** Integral correctly evaluated from correct working. If **M0** then allow **SC1** for $\frac{13}{24}$
		2	

Q	Answer	Marks	Comments
9(c)	$\dfrac{0.566-\frac{13}{24}}{\frac{13}{24}}\times 100 \quad [=4.49230...]$ or $\dfrac{0.566-\frac{13}{24}}{\frac{13}{24}} \quad [=0.044923...]$ Since the percentage difference is 4.5% the student is correct. or As the difference as a fraction is 0.045 the student is correct.	**B1ft** **E1ft**	Correct unsimplified expression for the percentage difference. **ft** Their 0.566 and $\frac{13}{24}$ **PI** by **AWRT** 4 or 4.5 or Correct unsimplified expression for the difference as a fraction of the exact value. **ft** Their 0.566 and $\frac{13}{24}$ **PI** by **AWRT** 0.04 or 0.045 **ft** their 0.566 and 0.5417 Correct value for the percentage difference and statement that the student is correct or correct conclusion based upon their percentage difference provided **B1** awarded. **AWRT** 4.5% or 4% Condone omission of the % sign. or **ft** their 0.566 and 0.5417 Correct value for the difference as a fraction of the exact value and statement that the student is correct or correct conclusion based upon their fraction provided **B1** awarded. **AWRT** 0.04 or 0.045
		2	

Q	Answer	Marks	Comments
9(c) ALT	Either $[5\%\text{ of exact value} =]\ \frac{13}{480}$ or 0.02708... and $\left[\text{Difference} = 0.566 - \frac{13}{24} = \right]$ $\frac{73}{3000}$ or 0.02433... or $\left[105\%\text{ of exact value} = \frac{13}{24} \times 1.05 = \right]$ $\frac{91}{160}$ or 0.56875 The student is correct and $0.02433... < 0.02708...$ or $[0.51458 <]\ 0.566 < 0.56875$	**B1ft** **E1ft**	Either Correctly calculates 5% of the exact value and the difference in the approximation. **ft** their 0.566 and $\frac{13}{24}$ Accept decimal equivalents to 2 sf or better. or Correctly calculates 105% of the exact value. **ft** their $\frac{13}{24}$ Accept decimal equivalent rounded or truncated to 3sf or better. Correct statement based upon their values and a comparison of values provided **B1** awarded. **ft** their 5% of the exact value and the difference in the approximation, or their 105% of the exact value.
		2	

Q	Answer	Marks	Comments
9(d)	Increase the number of strips (or ordinates)	**E1**	Correct, valid reason.
		1	

	Question 9 Total	**12**	

Q	Answer	Marks	Comments
10	$[r =] x^2$	**B1**	Correct common ratio for the series of areas. **PI** in later working.
	$\frac{x^2}{1-x^2} = \frac{49}{32}$	**M1**	**oe** Forms a correct equation using the formula for S_∞ equated to $\frac{49}{32}$
	$32x^2 = 49 - 49x^2$ or $81x^2 = 49$ or $x^2 = \frac{49}{81}$	**M1**	**oe** Clears the fractions obtaining a correct equation in x^2 **PI** by the correct value for x
	$[x =] \frac{7}{9}$	**A1**	Condone $x = -\frac{7}{9}$ seen.
	$[P =] 4 \times \frac{\frac{7}{9}}{1-\frac{7}{9}}$ or $[P =] 4 \times \frac{7}{2}$	**M1**	**oe** Substitutes their positive value for x into the formula for S_∞ and forms an expression for the sum of the perimeters.
	$[P =] 14$	**A1**	**CAO** Ignore units if included.

	Question 10 Total	**6**	