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MATHEMATICS

MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

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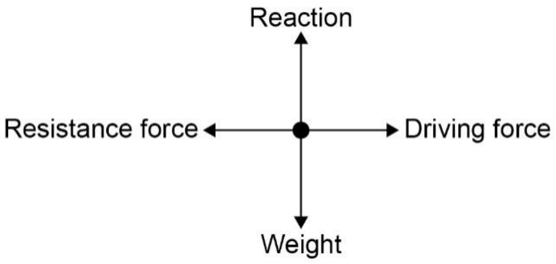
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)		**M1 A1**	**M1**: At least two arrows/forces on the diagram with at least one vertical force, with correct names **A1**: All four arrows/forces shown on the diagram (in the correct directions), with correct names Allow interchange of the directions of the two horizontal forces
		2	

Q	Answer	Marks	Comments
1(b)	Resultant force: $D - 0.91v^{\frac{5}{3}} = ma$	**M1**	Forming an equation of motion Allow one error or omission **PI**
	$D = ma + 0.91v^{\frac{5}{3}}$		
	$D = 280 \times 3.5 + 0.91 \times (15)^{\frac{5}{3}}$	**m1**	Correct substitution into their equation for the driving force $280 \times 3.5 = 980,\ 0.91 \times (15)^{\frac{5}{3}} = 83.02...$
	$D = 1100$ [N]	**A1**	Correct value for the driving force **Note** Unrounded value is 1063.022057 [N]
		3	

Q	Answer	Marks	Comments
1(c)	When in equilibrium $\frac{P}{v} - 0.91v^{\frac{5}{3}} = 0$ $v = \left(\frac{P}{0.91}\right)^{\frac{3}{8}}$ $v = \left(\frac{50 \times 10^3}{0.91}\right)^{\frac{3}{8}}$ $v = 60 \ \left[\mathrm{m\,s^{-1}}\right]$	M1 A1	Use of $P = Fv$ with equation of motion **Note** Unrounded answer is $59.906\ldots\left[\mathrm{m\,s^{-1}}\right]$
		2	

Q	Answer	Marks	Comments
1(d)	[The maximum speed would be] lower [than $60\ \mathrm{m\,s^{-1}}$] as a component of the motorcycle's weight now acts down the slope [so the resultant force becomes zero at a lower speed]	B1 E1	**oe**, such as decreases A valid explanation
		2	

	Question 1 Total	**9**	

Q	Answer	Marks	Comments
2(a)(i)	$[\mathbf{F} =]\ 4\cos(2t)\,\mathbf{i} - 8e^{t}\,\mathbf{j} + 24t\,\mathbf{k}$	**B1**	**ACF**
		1	

Q	Answer	Marks	Comments
2(a)(ii)	Equilibrium requires the resultant force to be zero [so all three components of **F** would need to be simultaneously zero]	**E1**	States the condition for equilibrium or states the acceleration is zero
	[However] $8e^{t} > 0$ for $t \geq 0$	**E1**	Explains that the **j** component [of the resultant force] or e^{t} can never equal zero
		2	

Q	Answer	Marks	Comments
2(b)(i)	$\mathbf{F}_2 = \left(4\cos(2t)\mathbf{i} - 8e^{t}\mathbf{j} + 24t\mathbf{k}\right) - \left(3\cos(2t)\mathbf{i} - 2e^{t}\mathbf{j} - 8t\mathbf{k}\right)$	**M1**	their resultant force – $\mathbf{F}_1$
	$\mathbf{F}_2 = \cos(2t)\mathbf{i} - 6e^{t}\mathbf{j} + 32t\mathbf{k}$	**A1**	
		2	

Q	Answer	Marks	Comments
2(b)(ii)	$\mathbf{F}_2 = \cos\left(2\times\frac{\pi}{4}\right)\mathbf{i} - 6e^{\frac{\pi}{4}}\mathbf{j} + \left(32\times\frac{\pi}{4}\right)\mathbf{k}$	**M1**	Substitutes $t = \frac{\pi}{4}$ into their $\mathbf{F}_2$ **PI**
	$\mathbf{F}_2 = -6e^{\frac{\pi}{4}}\mathbf{j} + 8\pi\mathbf{k}$		
	$\lvert\mathbf{F}_2\rvert = \sqrt{\left(6e^{\frac{\pi}{4}}\right)^2 + (8\pi)^2}$	**m1**	Note: $6e^{\frac{\pi}{4}} = 13.159...$, $8\pi = 25.132...$
	$\lvert\mathbf{F}_2\rvert = 28$ [N]	**A1ft**	**AWRT** 28 **ft** an $\mathbf{F}_2$ of the form $\pm\cos(2t)\mathbf{i} \pm 6e^{t}\mathbf{j} \pm 32t\mathbf{k}$
		3	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3(a)	(3, 3) The centre of mass of the uniform circular lamina is at its geometric centre	**B1** **E1**	 Allow any mention of symmetry of the circle
		2	

Q	Answer	Marks	Comments
3(b)	$(2+3+7+11+5)\bar{X}$ $= [2\times 0+]3\times 3+7\times 3+11\times 6+5\times 3$	**M1**	Forming equation for x-coordinate of the centre of mass Condone one error
	$\bar{X} = \dfrac{111}{28}$	**A1**	Any correct exact form
	$(2+3+7+11+5)\bar{Y}$ $= 2\times 3+3\times 6[+7\times 0]+11\times 3+5\times 3$	**M1**	Forming equation for y-coordinate of the centre of mass Condone one error
	$\bar{Y} = \dfrac{18}{7}$	**A1**	Any correct exact form
	$\left(\dfrac{111}{28}, \dfrac{18}{7}\right)$		If **M1 A0 M1 A0** awarded, then allow **SC1** for (3.96, 2.57)
		4	

Q	Answer	Marks	Comments
3(c)	$\tan\theta = \dfrac{\frac{111}{28}-3}{6-\frac{18}{7}}$ $\left[= \dfrac{\left(\frac{27}{28}\right)}{\left(\frac{24}{7}\right)} = \dfrac{9}{32}\right]$ $\theta = \tan^{-1}\left(\dfrac{\frac{111}{28}-3}{6-\frac{18}{7}}\right)$ $\theta = 16^\circ$	**M1 A1ft** **A1**	**M1**: At least numerator or denominator correct, **ft** their coordinates for the COM **A1ft**: Both numerator and denominator correct, **ft** their coordinates for the COM Note: unrounded answer is 15.7086…° **CAO**
		3	

	Question 3 Total	**9**	

Q	Answer	Marks	Comments
4(a)(i)	$\left[\frac{1}{2}mv^2=\right]$ $0.5\times\left(80\times10^{-3}\right)\times45^2=81\ [\text{J}]$	B1	**AG** Must be convincingly shown
		1	

Q	Answer	Marks	Comments
4(a)(ii)	At maximum height, speed of arrow is $45\cos\left(10^\circ\right)$ $\left[\frac{1}{2}mv^2=\right]$ $0.5\times\left(80\times10^{-3}\right)\times\left(45\cos\left(10^\circ\right)\right)^2$ $=78.6\ [\text{J}]$	M1 A1	**CAO** to 3 sf **AWRT** 78.6 [J]
		2	

Q	Answer	Marks	Comments
4(b)	Loss in KE = Gain in GPE $81-78.6=mg\Delta h$ $\Delta h=\dfrac{81-78.6}{\left(80\times10^{-3}\right)\times9.8}$ $\Delta h=3.06122...\ [\text{m}]$ Maximum height above the ground $=3.06122...+1.6$ $=4.7\ [\text{m}]$	M1 A1 A1ft	At least LHS correct, **ft** their 78.6 Allow 3.11536… [m] from unrounded value for minimum KE **PI** by correct final answer **ft** their Δh value + 1.6
		3	

Q	Answer	Marks	Comments
4(c)(i)	Time to cover the horizontal displacement of 70 metres $T = \dfrac{70}{45\cos(10^\circ)}$ $T = 1.58$ [s]	**B1**	**AWRT** 1.58 Note: unrounded answer is 1.57955… [s]
		1	

Q	Answer	Marks	Comments
4(c)(ii)	Vertical displacement when horizontal displacement is 70 metres $[s =]\ 45\sin(10^\circ)\,T - 0.5\times 9.8\times T^2 + 1.6$	**M1 A1**	**M1**: Use of $s = ut + \frac{1}{2}at^2$ with $u = 45\sin(10^\circ)$ and $a = \pm 9.8$ Condone +1.6 omitted (May be recovered later in solution) **PI** **A1**: **AWFW** [0.114, 0.117] or **AWFW** [1.714, 1.717] **PI**
	$[s =]\ 45\sin(10^\circ)\times 1.57955\ldots - 0.5\times 9.8\times(1.57955\ldots)^2 + 1.6$		Note: $45\sin(10^\circ)\times 1.57955\ldots - 0.5\times 9.8\times(1.57955\ldots)^2 = 0.117...$
	$[s =]$ 1.717... [m]	**A1**	Expect 1.714… if $T = 1.58$ used **AWRT** 1.7 from correct working
	As $0.8 < 1.717... < 1.8$, the arrow does hit the target	**B1**	Comparison of their vertical displacement with the height(s) of the target and correct conclusion
		4	

	Question 4 Total	**11**	

Q	Answer	Marks	Comments
5(a)	$\lvert\mathbf{r}\rvert = \sqrt{\left(e^{-3t}\cos(3t)\right)^2 + \left(e^{-3t}\sin(3t)\right)^2}$	M1	
	$\lvert\mathbf{r}\rvert = \sqrt{e^{-6t}\cos^2(3t) + e^{-6t}\sin^2(3t)}$		
	$\lvert\mathbf{r}\rvert = \sqrt{e^{-6t}\left(\cos^2(3t) + \sin^2(3t)\right)}$		
	$\lvert\mathbf{r}\rvert = \sqrt{e^{-6t}}$	m1	Use of $\cos^2(3t) + \sin^2(3t) = 1$
	$\lvert\mathbf{r}\rvert = e^{-3t}$	A1	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
5(b)	$\mathbf{v} = \begin{bmatrix} -3e^{-3t}\left(\sin(3t)+\cos(3t)\right) \\ -3e^{-3t}\left(\sin(3t)-\cos(3t)\right) \end{bmatrix}$	**M1 A1**	**M1**: At least one component correct Condone written in any vector form **A1**: Both components correct, written as a column vector
		2	

Q	Answer	Marks	Comments
5(c)(i)	$\lvert\mathbf{v}\rvert^2 = 9e^{-6t}\left(\sin(3t)+\cos(3t)\right)^2 + 9e^{-6t}\left(\sin(3t)-\cos(3t)\right)^2$	**M1**	**oe**
	$\lvert\mathbf{v}\rvert^2 = 9e^{-6t}\left(\sin^2(3t)+\cos^2(3t) + 2\sin(3t)\cos(3t) + \sin^2(3t)+\cos^2(3t) - 2\sin(3t)\cos(3t)\right)$	**M1**	Both brackets expanded
	$\lvert\mathbf{v}\rvert^2 = 18e^{-6t}$		
	$\lvert\mathbf{v}\rvert = \sqrt{18}\ e^{-3t}$	**A1**	**CAO** Condone $\lvert\mathbf{v}\rvert = 3\sqrt{2}\ e^{-3t}$
		3	

Q	Answer	Marks	Comments
5(c)(ii)	$\left[\text{KE} = \frac{1}{2}mv^2\right] = 0.5\times 6\times 18$	**M1**	Substitution of $m=6$ and their b into $\frac{1}{2}mv^2$ Condone appearance of exponential term
	$[\text{KE} =]\ 54\,[\text{J}]$	**A1ft**	**ft** their b
		2	

	Question 5 Total	**10**	

Q	Answer	Marks	Comments
6	Kinetic energy at *A* $\left[\frac{1}{2}mv^2 =\right]$ $0.5 \times 16 \times 35^2$ $= 9800$ [J]	**B1**	**PI**
	Friction experienced by particle $\mu mg\cos(28^\circ) = 0.64 \times 16 \times 9.8\cos(28^\circ)$ $= 88.60555...$ [N]	**B1**	**PI** by correct friction force
	Let *AB* = d Change in height between *A* and *B* $[\Delta h =] d\sin(28^\circ)$	**B1**	Seen or used
	Conservation of Energy $9800 = 88.60555... \times d + 16 \times 9.8 \times d\sin(28^\circ)$	**M1**	Initial KE = 'Work Done' term (in terms of a distance) + 'Change in GPE' term (in terms of a distance) **PI** by correct value distance
	$d = \dfrac{9800}{88.60555... + 16 \times 9.8\sin(28^\circ)}$ $d = 60$ [m]	**A1**	Unrounded answer is 60.4122… [m]

Q	Answer	Marks	Comments
6 ALT	Friction experienced by particle		
	$\mu mg\cos(28^\circ) = 0.64\times 16\times 9.8\cos(28^\circ)$	B1	**PI** by correct friction force
	$= 88.60555...$ [N]		
	Resultant force experienced by particle		
	$-88.60555...-16\times 9.8\times\sin(28^\circ)$ $[=-162.218...]$	B1	
	$-162.218... = 16a$		
	$a = -10.1386...$ $[\mathrm{m\,s^{-1}}]$	B1	
	Let $AB = d$		
	$v^2 = u^2 + 2ad \quad \Rightarrow \quad d = \dfrac{v^2-u^2}{2a}$		
	$d = \dfrac{0^2-35^2}{2\times -10.1386..}$	M1	**ft** their acceleration
	$d = 60$ [m]	A1	Unrounded answer is 60.4122… [m]

	Question 6 Total	**5**	

Q	Answer	Marks	Comments
7(a)(i)	$x=(v\cos\theta)t$	**B1**	**oe**
	$y=(v\sin\theta)t-\frac{1}{2}gt^2$	**B1**	
	$t=\frac{x}{v\cos\theta}$		
	$y=(v\sin\theta)\times\frac{x}{v\cos\theta}-\frac{1}{2}g\times\left(\frac{x}{v\cos\theta}\right)^2$	**M1 A1**	**M1**: Eliminates t in both terms with at least one term correct **A1**: All correct
	$(v\sin\theta)\times\frac{x}{v\cos\theta}=x\tan\theta$		
	$-\frac{1}{2}g\times\left(\frac{x}{v\cos\theta}\right)^2=-\frac{gx^2}{2v^2}\times\frac{1}{\cos^2\theta}$		
	$-\frac{gx^2}{2v^2}\times\frac{1}{\cos^2\theta}=-\frac{gx^2}{2v^2}\sec^2\theta$		
	$y=x\tan\theta-\frac{gx^2}{2v^2}\sec^2\theta$	**A1**	**AG** Must be convincingly shown
		5	

Q	Answer	Marks	Comments
7(a)(ii)	The particle does not experience air resistance	**E1**	Any valid assumption
		1	

Q	Answer	Marks	Comments
7(b)	$10 = 25\tan\alpha - \dfrac{9.8\times 25^2}{2\times 30^2}\sec^2\alpha$	M1	Substitution of values into the result given in **part (a)(i)** **PI**
	$10 = 25\tan\alpha - \dfrac{9.8\times 25^2}{2\times 30^2}\times\left(1+\tan^2\alpha\right)$	M1	Use of $1+\tan^2\alpha = \sec^2\alpha$ **PI**
	$245\tan^2\alpha - 1800\tan\alpha + 965 = 0$	M1	Forms a three-term quadratic equation in $\tan\alpha$
	$\tan\alpha = \dfrac{1800 \pm \sqrt{1800^2 - 4\times 245\times 965}}{2\times 245}$		
	$\tan\alpha = 0.5822..., 6.7646$		
	$\alpha = 30, 82$	A1 A1	**A1**: At least one correct value **A1**: Both values correct and no others Note: unrounded values are 30.210… and 81.591…
		5	

	Question 7 Total	**11**	

Q	Answer	Marks	Comments
8(a)	The length of the string does not increase	**E1**	Allow '[length] does not change'
		1	

Q	Answer	Marks	Comments
8(b)	$T\sin\theta = m\omega^2 r$ $r = l\sin\theta$ $T\sin\theta = m\omega^2 l\sin\theta$ $T = m\omega^2 l$	**M1** **A1**	
		2	

Q	Answer	Marks	Comments
8(c)	$T\cos\theta = mg$ $\frac{mg}{\cos\theta} = m\omega^2 l$ $\frac{g}{l\cos\theta} = \omega^2$ $\sqrt{\frac{g}{l\cos\theta}} = \frac{2\pi}{t}$ $[t =]\ 2\pi\sqrt{\frac{l\cos\theta}{g}}$	**B1** **M1** **A1**	**PI** Use of $\omega = \frac{2\pi}{t}$, where t is the time period **ACF**
		3	

Q	Answer	Marks	Comments
8(d)	$1.2 < t < 1.5$	**M1**	**oe**, both time periods seen or used
	$t = 2\pi\sqrt{\dfrac{l\cos\theta}{g}}$		
	$\cos\theta = \dfrac{g}{l}\left(\dfrac{t}{2\pi}\right)^2$		
	When t = 1.2		
	$\cos\theta = \dfrac{9.8}{0.71}\left(\dfrac{1.2}{2\pi}\right)^2$		
	θ = 59.770...	**A1**	One correct value for θ
	When t = 1.5		
	$\cos\theta = \dfrac{9.8}{0.71}\left(\dfrac{1.5}{2\pi}\right)^2$		
	θ = 38.124...		
	$38 < \theta < 60$	**A1**	$38.124... < \theta < 59.770...$ Fully correct strict inequalities for θ
		3	

	Question 8 Total	**9**	

Q	Answer	Marks	Comments
9(a)	Taking moments about *B* $12g\cos(75^\circ)\times 3.5 = 15\sin(75^\circ)\times 7 + \mu_1 \times 15\cos(75^\circ)\times 7$ $106.529... = 101.422... + 27.175...\mu_1$ $\mu_1 = 0.188$	**M1 A1** **A1**	**M1**: At least one side correct **PI** **A1**: Both sides fully correct **PI** **Note** Unrounded value is $\mu_1 = 0.187949...$
		3	

Q	Answer	Marks	Comments
9(b)	Forces in equilibrium (vertical direction) $12g = \mu_1 \times 15 + R_G$ $R_G = 12\times 9.8 - 0.187949...\times 15$ $R_G = 114.78...$ [N] Forces in equilibrium (horizontal direction) $\mu_2 R_G = 15$ $\mu_2 = \dfrac{15}{114.78...}$ $\mu_2 = 0.131$	**M1 m1** **A1** **M1** **A1**	$R_G =$ normal reaction on rod from ground **M1**: At least one side correct **PI** **m1**: Both sides fully correct **PI** **ft** their μ_1 **AWRT** 115 [N] **PI** **CAO** to 3 sf **Note** Unrounded value is $\mu_2 = 0.13068...$
		5	

	Question 9 Total	**8**	