OXFORD AQA

INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

June 2024

Version: 1.0 Final

246XFM04/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A > \mu_B$	**B1**	Both hypotheses
	$z = \dfrac{8.2 - 6.5}{\sqrt{\dfrac{2.1^2}{7} + \dfrac{2.2^2}{9}}}$	**M1**	Applies formula
	$= 1.57$	**A1**	**AWRT** 1.57
	$z(95\%) = 1.6449$	**B1**	Finds correct critical value **AWRT** 1.645 or 1.64 or finds correct probability **AWRT** 0.058
	$1.57 < 1.65$ Do not reject H_0	**A1ft**	Correctly compares their z or t test statistic and their critical value or their p value and 0.05 (0.025 if a two-tailed test is attempted) and concludes that the null hypothesis is not rejected
	Insufficient evidence to suggest that the mean number of days a patient takes to recover from the illness is less for medicine *B* than medicine *A*	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value **oe** Condone definite conclusion

	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2(a)	$\mathrm{E}(R) = \dfrac{\mathrm{E}(X_1) + 2\mathrm{E}(X_2) + 3\mathrm{E}(X_3)}{6}$ $= \dfrac{\lambda + 2\lambda + 3\lambda}{6}$	**M1**	Finds $\mathrm{E}(R) = \dfrac{\lambda + 2\lambda + 3\lambda}{6}$ or unsimplified equivalent
	$= \lambda$ therefore unbiased estimator	**A1**	Must see conclusion and no errors
		2	

Q	Answer	Marks	Comments
2(b)(i)	$\mathrm{Var}(R) = \dfrac{\mathrm{Var}(X_1)}{6^2} + \dfrac{\mathrm{Var}(X_2)}{3^2} + \dfrac{\mathrm{Var}(X_3)}{2^2}$	**M1**	Finds $\mathrm{Var}(R)$ in terms of $\mathrm{Var}(X_1)$, $\mathrm{Var}(X_2)$ and $\mathrm{Var}(X_3)$ **PI**
	$= \dfrac{\lambda}{36} + \dfrac{\lambda}{9} + \dfrac{\lambda}{4} = \dfrac{7\lambda}{18}$	**A1**	**oe**
	Relative Efficiency $= \dfrac{\frac{1}{\mathrm{Var}(R)}}{\frac{1}{\mathrm{Var}(X_1)}} = \dfrac{18/7\lambda}{1/\lambda}$	**M1**	Applies relative efficiency formula either way round with their $\mathrm{Var}(R)$ and $\mathrm{Var}(X_1) = \lambda$
	$= \dfrac{18}{7}$	**A1**	**AWRT** 2.57
		4	

Q	Answer	Marks	Comments
2(b)(ii)	Estimator R is more efficient than estimator X_1 as $\dfrac{18}{7} > 1$	**E1ft**	Compares their relative efficiency with 1 and makes correct conclusion
		1	

	Question 2 Total	**7**	

<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td>3</td><td>
<table>
<tr><th>Outcomes</th><th>Mode</th><th>Probability</th></tr>
<tr><td>(0, 0, 0)</td><td>0</td><td>0.8^3
[= 0.512]</td></tr>
<tr><td>(0, 0, 20)
(0, 20, 0)
(20, 0, 0)</td><td>0</td><td>$3 \times 0.8^2 \times 0.2$
[= 0.384]</td></tr>
<tr><td>(0, 20, 20)
(20, 0, 20)
(20, 20, 0)</td><td>20</td><td>$3 \times 0.8 \times 0.2^2$
[= 0.096]</td></tr>
<tr><td>(20, 20, 20)</td><td>20</td><td>0.2^3
[= 0.008]</td></tr>
</table>
<table>
<tr><td>m</td><td>0</td><td>20</td></tr>
<tr><td>$\mathrm{P}(M = m)$</td><td>0.896</td><td>0.104</td></tr>
</table>
</td><td>M1

A1

M1

A1

A1</td><td>Identifies a possible outcome
PI

Identifies all the outcomes for an associated mode
PI

Finds at least one probability from the table
PI

Finds all probabilities in the table
PI

oe</td></tr>
<tr><td></td><td>Question 3 Total</td><td>5</td><td></td></tr>
</table>

Q	Answer	Marks	Comments
4	H_0: There is not an association between region lived in and holiday location H_1: There is an association between region lived in and holiday location	**B1**	Both hypotheses, variables must be stated in at least the null hypothesis **oe**
	<table><tr><td>Expected</td><td>A</td><td>NA</td></tr><tr><td>N</td><td>53.148</td><td>49.852</td></tr><tr><td>S</td><td>75.852</td><td>71.148</td></tr></table>	**M1** **A1**	At least two expected values correct to at least 2 d.p. All correct to at least 2 d.p.
	$\sum \frac{(\lvert O-E\rvert -0.5)^2}{E} = \frac{(\lvert 44-53.148\rvert -0.5)^2}{53.148} + \frac{(\lvert 85-75.852\rvert -0.5)^2}{75.852} + \frac{(\lvert 59-49.852\rvert -0.5)^2}{49.852} + \frac{(\lvert 62-71.148\rvert -0.5)^2}{71.148}$	**M1**	Attempts to calculate test statistic Condone use of $\sum \frac{(O-E)^2}{E}$
	= 4.94	**A1**	**AWRT** 4.9
	$\chi^2_1(95\%) = 3.841$	**B1**	Finds correct critical value **AWRT** 3.8 or finds correct probability **AWRT** 0.026
	4.94 > 3.841 Reject H_0	**A1ft**	Correctly compares their χ^2 test statistic and their critical value or their p value and 0.05 and rejects null hypothesis
	Sufficient evidence to suggest that there is an association between region lived in and holiday location	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value **oe** Condone definite conclusion

	Question 4 Total	**8**	

Q	Answer	Marks	Comments
5(a)	X and Y are normally distributed	**E1**	States that X and Y are normally distributed or X and Y are independent
		1	

Q	Answer	Marks	Comments
5(b)	$H_0 : \sigma_X = \sigma_Y$ $H_1 : \sigma_X \neq \sigma_Y$	**B1**	Both hypotheses, **oe**
	$s_X{}^2 = 0.009995$	**B1**	
	$s_Y{}^2 = 0.004096$	**B1**	
	$\frac{s_X{}^2}{s_Y{}^2} = \frac{0.009995}{0.004096}$	**M1**	Either way round **PI**
	$= 2.44$	**A1**	**AWRT** 2.44 or **AWRT** 0.41
	$F_{40,25}(99\%) = 2.453$	**B1**	Finds correct critical value corresponding to their test statistic **AWRT** 2.45 or **AWRT** 0.408 or finds correct probability **AWRT** 0.0103
	$2.44 < 2.453$ Do not reject H_0	**A1ft**	Correctly compares their F test statistic and their critical value or their probability and 0.01 (0.02 if a one tailed test is attempted) and does not reject the null hypothesis
	Insufficient evidence to suggest that the population standard deviation of male reaction times is different from the population standard deviation of female reaction times	**E1ft**	Gives a conclusion in context based on a comparison of their F test statistic and their corresponding critical value **oe** Must not be definite (eg use of "suggest", "support")
		8	

	Question 5 Total	**9**	

Q	Answer	Marks	Comments
6(a)(i)	$\mathrm{M}_X(t) = \mathrm{e}^{\mu t}\,\mathrm{M}_Z(\sigma t)$	**M1**	Applies formula
	$\mathrm{M}_X(t) = \mathrm{e}^{\mu t}\,\mathrm{e}^{\frac{1}{2}\sigma^2 t^2} = \mathrm{e}^{\mu t+\frac{1}{2}\sigma^2 t^2}$	**A1**	
		2	

Q	Answer	Marks	Comments
6(a)(ii)	$\mathrm{M}_{X+Y}(t) = \mathrm{M}_X(t)\mathrm{M}_Y(t)$ $= \mathrm{e}^{\mu t+\frac{1}{2}\sigma^2 t^2}\,\mathrm{e}^{\lambda\left(\mathrm{e}^t-1\right)}$	**M1**	Applies formula
	$\mathrm{M}_{X+Y}(t) = \mathrm{e}^{\mu t+\frac{1}{2}\sigma^2 t^2+\lambda\left(\mathrm{e}^t-1\right)}$	**A1**	
		2	

Q	Answer	Marks	Comments
6(b)(i)	$\mathrm{M}_W(t) = \int_0^{\ln 2} \mathrm{e}^{tw}\times\frac{2}{3}\mathrm{e}^{2w}\,\mathrm{d}w$	**M1**	Applies mgf formula Condone missing or incorrect limits
	$\mathrm{M}_W(t) = \int_0^{\ln 2} \frac{2}{3}\mathrm{e}^{(t+2)w}\,\mathrm{d}w$ $\mathrm{M}_W(t) = \left[\frac{2\mathrm{e}^{(t+2)w}}{3(t+2)}\right]_0^{\ln 2}$	**A1**	Correctly integrates Condone missing or incorrect limits
	$\mathrm{M}_W(t) = \frac{2\left(\mathrm{e}^{(t+2)\ln 2}-1\right)}{3(t+2)}$	**A1**	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
6(b)(ii)	$\mathrm{M}'_W(t) = \dfrac{6(t+2)\mathrm{e}^{(t+2)\ln 2}\ln 2 - 6\left(\mathrm{e}^{(t+2)\ln 2}-1\right)}{9(t+2)^2}$	**M1**	Applies quotient rule **oe** Condone one sign error
	$\mathrm{M}'_W(t) = \dfrac{2(t\ln 2 + 2\ln 2 - 1)\mathrm{e}^{(t+2)\ln 2}+2}{3(t+2)^2}$	**A1**	**CSO** **ACF** provided numerator is $3(t+2)^2$ **ISW** once a correct form is seen
		2	

Q	Answer	Marks	Comments
6(b)(iii)	Mean $= \mathrm{M}'_W(0) = \dfrac{2(2\ln 2 - 1)\mathrm{e}^{2\ln 2}+2}{3(2)^2}$	**M1**	Attempts to find their $\mathrm{M}'_W(0)$
	Mean = 0.424	**A1ft**	**AWRT** 0.424 **ft** their $\mathrm{M}'_W(t)$ if full marks not awarded in **part (b)(ii)**
		2	

	Question 6 Total	**11**	

Q	Answer	Marks	Comments
7(a)	$z(96\%) = 1.7507$	**B1**	**AWRT** 1.75
	$115 \pm 1.7507 \times \frac{18}{\sqrt{200}}$	**M1**	Attempts to calculate one of the limits
	(112.77, 117.23)	**A1**	Correct upper and lower limits **AWRT** 112.77 and 117.23
	Power = $P\left(Z < \frac{112.77-116}{\frac{18}{\sqrt{200}}}\right) + P\left(Z > \frac{117.23-116}{\frac{18}{\sqrt{200}}}\right)$	**M1**	Identifies correct standardised probabilities for power or Type II error corresponding to their limits **PI**
	$= 0.0056 + 0.1673$	**A1**	Correctly calculates one of the probabilities **AWRT** 0.006 or 0.167 or calculates P(Type II error) **AWFW** [0.825, 0.828]
	$= 0.173$	**A1**	**AWFW** [0.172, 0.175] May be calculated using 1 – P(Type II error)
		6	

Q	Answer	Marks	Comments
7(b)	The power of the test decreases	**E1**	**oe**
		1	

	Question 7 Total	**7**	

Q	Answer	Marks	Comments
8(a)	$z(99\%) = 2.3263$	**B1**	**AWRT** 2.33
	$z(96.5\%) = 1.8119$	**B1**	**AWRT** 1.81
	$\bar{x} + 2.3263\dfrac{\sigma}{\sqrt{40}} = 5243.65$	**M1 A1ft**	**M1**: Forms at least one correct equation using their $z(99\%)$ or $z(96.5\%)$
	$\bar{x} - 1.8119\dfrac{\sigma}{\sqrt{40}} = 4858.26$		**A1ft**: Forms both correct equations using their $z(99\%)$ and $z(96.5\%)$
	$\sigma = \$589$	**A1**	Condone omission of \$ sign
	$\bar{x} = \$5027$	**B1**	Condone omission of \$ sign
		6	

Q	Answer	Marks	Comments
8(b)	$z(98\%) = 2.0537$	**B1**	**AWRT** 2.05
	$2 \times 2.0537 \times \dfrac{589}{\sqrt{n}} = 300$	**M1**	Set up an equation of the form $k \times \text{their } z \times \dfrac{\text{their } \sigma}{\sqrt{n}} = 300$ **oe** where k = 1 or 2
	$n = 65$	**A1**	Allow $n = 66$
		3	

	Question 8 Total	**9**	

Q	Answer	Marks	Comments
9(a)	H_0 : N has a binomial distribution B(5, 0.5) H_1 : N does not have a binomial distribution with B(5, 0.5)	B1	Both hypotheses B(5, 0.5) **oe** must be stated
	Number of heads / Expected frequency 0 – 6.25 1 – 31.25 2 – 62.5 3 – 62.5 4 – 31.25 5 – 6.25	M1 A1	**M1**: Finds one correct expected frequency **A1**: Finds all correct expected frequencies
	$\sum \frac{(O-E)^2}{E} = \frac{(11-6.25)^2}{6.25} + \frac{(40-31.25)^2}{31.25}$ $+ \frac{(75-62.5)^2}{62.5} + \frac{(48-62.5)^2}{62.5}$ $+ \frac{(22-31.25)^2}{31.25} + \frac{(4-6.25)^2}{6.25}$	M1	Attempts to calculate test statistic
	= 15.472	A1	**AWRT** 15.5 from correct work
	$\chi^2_5(99\%) = 15.086$	B1	Finds correct critical value **AWRT** 15.1 or correct probability **AWRT** 0.009
	15.472 > 15.086 Reject H_0	A1ft	Correctly compares their χ^2 test statistic and their critical value or their probability and 0.01 and rejects the null hypothesis
	Sufficient evidence to suggest that Aeryn's model is not a good fit	E1	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value **oe** Condone definite conclusion
		8	

Q	Answer	Marks	Comments
9(b)	4	**B1ft**	**ft** Their degrees of freedom –1
		1	

	Question 9 Total	**9**	

Q	Answer	Marks	Comments
10(a)	$\left[s_p^2 =\right] \dfrac{(n-1)\times 5.4^2 + (n+2)\times 5.2^2}{2n+1}$ $= 27.933$	**M1**	Forms correct equation **oe**
	$56.2n + 24.92 = 55.866n + 27.933$ $0.334n = 3.013$	**m1**	Rearranges equation to reach the form $an + b = cn + d$ **PI** By sight of 9.02
	$n = 9.02 \Rightarrow n = 9$	**A1**	**CSO**
		3	

Q	Answer	Marks	Comments
10(b)	$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A < \mu_B$	**B1**	Both hypotheses
	$t = \dfrac{43.2 - 39.4}{\sqrt{27.933\left(\frac{1}{9} + \frac{1}{12}\right)}}$	**M1**	Applies formula
	$= 1.63$	**A1ft**	**AWRT** 1.63 **ft** their positive integer n
	$t_{19}(95\%) = 1.729$	**B1ft**	Finds critical value **AWRT** 1.73 or finds probability **AWRT** 0.0597 **ft** their positive integer n, if $n > 30$ allow use of normal
	$1.63 < 1.729$ Do not reject H_0	**A1ft**	Correctly compares their t or z test statistic and their critical value or their probability and 0.05 (0.025 if a two tailed test is attempted) and concludes that the null hypothesis is not rejected
	Insufficient evidence to suggest that the mean salaries in town *B* are higher than the mean salaries in town *A*	**E1ft**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value for their positive integer n Condone definite conclusion
		6	

	Question 10 Total	**9**	