OXFORD AQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL AS

FURTHER MATHEMATICS

FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

June 2024

Version: 1.0 Final

246XFM02/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqa.com

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$h\mathrm{f}(1,-3) = 0.2\times\sqrt{1^2+2(-3)^2}$	M1	Correct substitution into RHS of this expression
	$= 0.2\sqrt{19}$	A1	**PI AWRT** 0.872
	$y_2 = -3 + 0.2\sqrt{19} = -2.128220...$	M1	-3 + their value of $h\mathrm{f}(-2,3)$
	$y_3 = -2.128220 + 0.2\times 3.24016....$ $y_3 = -2.128220 + 0.648032...$	M1	Correct substitution using their x_2 and their y_2
	$[= -1.480188035]$ –		
	1.4802	A1	Correct answer given to 4 dp

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2(a)	Shear with x-axis invariant	**B1**	Allow shear parallel to or along or in direction of x-axis Ignore any additional descriptions
		1	

Q	Answer	Marks	Comments
2(b)(i)	$\mathbf{QP} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 2 & 2k+1 \\ 4 & 4k+3 \end{bmatrix}$	**M1** **A1**	For a 2 × 2 matrix with 3 correct elements **CAO**
		2	

Q	Answer	Marks	Comments
2(b)(ii)	$\det(\mathbf{QP}) = 2(4k+3) - 4(2k+1) = 2$	**B1**	Obtains $\det(\mathbf{QP}) = 2$ or $\det(\mathbf{Q}) = 2$ **oe** **PI**
	4.5×2	**M1**	Multiplies 4.5 by their $\det(\mathbf{QP})$ or multiplies 4.5 by their $\det(\mathbf{Q})$ and states that $\det(\mathbf{P}) = 1$
	= 9 [square units]	**A1ft**	**ft** their $\det(\mathbf{QP})$ or $\det(\mathbf{Q})$
		3	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$\mathrm{f}(1) = -3$ and $\mathrm{f}(2) = 3$	**M1**	Correct evaluation of a suitable interval
	sign change **&** continuous function, so the root α lies in the interval $1 < \alpha < 2$	**A1**	Must state or show that that there is a change of sign and state that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	

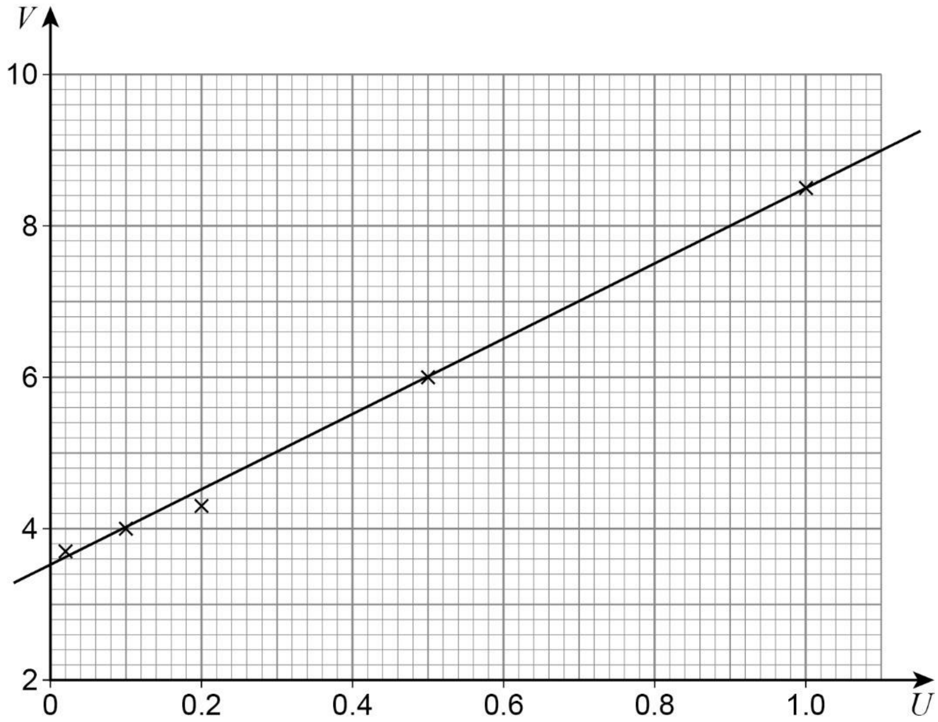
Q	Answer	Marks	Comments
3(b)	$\mathrm{f}(1.5) = -2$; [negative] so $1.5 < \alpha$ $[< 2]$	**M1**	Evaluation of $\mathrm{f}(1.5)$ and selects correct range. Interval **PI** by subsequent calculation of $\mathrm{f}(1.75)$
	$\mathrm{f}(1.75) = -0.375$; [negative] so $1.75 < \alpha$ $[< 2]$	**m1**	Evaluation of $\mathrm{f}(1.75)$ which may be rounded or truncated and selects correct range. Interval **PI** by subsequent calculation of $\mathrm{f}(1.875)$
	$\mathrm{f}(1.875) = 1.046875$	**m1**	Evaluation of $\mathrm{f}(1.875)$ which may be exact $\left(\frac{67}{64}\right)$, rounded or truncated
	[positive] so $1.75 < \alpha < 1.875$	**A1**	**CSO**
		4	

	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4(a)	$\frac{1}{V} = \frac{\frac{1}{U}}{a + \frac{b}{U}}$	**M1**	Obtains a correct equation in terms of U and V or $\frac{1}{u}$ and $\frac{1}{v}$ **oe**
	$\frac{1}{V} = \frac{1}{aU + b}$ $V = \frac{aU + b}{1}$ $V = aU + b$	**A1**	Obtains correct linear relationship **CSO**
		2	

Q	Answer	Marks	Comments
4(b)(i)	<table><tr><td>U</td><td>1</td><td>0.5</td><td>0.2</td><td>0.1</td><td>0.02</td></tr><tr><td>V</td><td>8.5</td><td>**6.0**</td><td>**4.3**</td><td>**4.0**</td><td>**3.7**</td></tr></table>	**M1 A1**	**M1**: Three correct, condone 6 for 6.0 or 4 for 4.0 **A1**: All correct
		2	

Q	Answer	Marks	Comments
4(b)(ii)		B1ft	Their points plotted within half a square tolerance
		B1	Line of best fit seen
		2	



Q	Answer	Marks	Comments
4(c)	y – intercept = 3.5 gradient = 5.0 $a = 5.0$ $b = 3.5$	B1 B1	Sight of y–intercept or gradient values for their straight line of best fit **AWFW** [4.7, 5.3] for a, must be 1, 2 or 3 sf **AWFW** [3.4, 3.6] for b, must be 2 or 3 sf
		2	

Q	Answer	Marks	Comments
4(d)	$\frac{5.0}{u} + 3.5 = \frac{1}{0.26}$ $u = 14$	M1 A1ft	Substitutes $v = 0.26$ and their a, b into a correct equation for u **ft** their $\dfrac{a}{\frac{1}{0.26} - b}$ Must be given to at least 2 sf
		2	

	Question 4 Total	**10**	

Q	Answer	Marks	Comments
5(a)(i)	$[x'] = -\frac{\sqrt{3}}{2}x + \frac{1}{2}(mx[+c])$ $[y'] = \frac{1}{2}x + \frac{\sqrt{3}}{2}(mx[+c])$	M1	Valid attempt to find x', y' Condone sign errors Condone using x for x' and y for y' May only consider mx
	$\frac{1}{2}x + \frac{\sqrt{3}}{2}(mx[+c])$ $= m\left(-\frac{\sqrt{3}}{2}x + \frac{1}{2}(mx[+c])\right)[+c]$	m1	**ft** their $y' = m$(their x') $[+c]$
	$\frac{1}{2} + \frac{\sqrt{3}}{2}m = -\frac{\sqrt{3}}{2}m + \frac{1}{2}m^2$		
	$\frac{1}{2}m^2 - \sqrt{3}m - \frac{1}{2} = 0$	m1	Attempt to find m by comparing coefficients of x or setting x coefficients = 0. Rearrangement of quadratic not necessary for this mark **PI**
	$m = \sqrt{3} + 2$, $m = \sqrt{3} - 2$	A1	Correct values of m **CSO**
		4	

Q	Answer	Marks	Comments
5(a)(ii)	$c\left(-\frac{\sqrt{3}}{2} + 1 + \frac{1}{2}m\right) = 0$	M1	Forms correct equation in terms of c and m May be seen in part **(a)(i)** **Not** implied by $y = \left(\sqrt{3} - 2\right)x + c$
	$m = \sqrt{3} - 2$ gives $c \times 0 = 0$ hence $y = \left(\sqrt{3} - 2\right)x + c$ [where c is real]	A1	With no restrictions on c
	$y = \left(\sqrt{3} + 2\right)x$	B1	
		3	

Q	Answer	Marks	Comments
5(b)	$\mathbf{AB} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	B1	States the matrix for an anticlockwise rotation of $\frac{\pi}{2}$ about the origin
	$\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{-1} = \frac{1}{-1}\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$ $= \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$	M1	Finds the inverse of **A** **PI** by sight of $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ or $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
	$\mathbf{B} = \mathbf{A}^{-1}\mathbf{AB}$	M1	Sets up correct equation for **B** **PI** by a correct expression for **B** later
	$= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$	A1	Finds **B** or finds the value of $\cos 2\theta$, $\sin 2\theta$ or $\tan 2\theta$
	$\cos 2\theta = \frac{1}{2}$ or $\sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$	M1	Uses their value of $\cos 2\theta$ or $\sin 2\theta$ to attempt to find $\tan\theta$
	so $y = \frac{\sqrt{3}}{3}x$	A1	**oe** eg $y = \left(\tan\frac{\pi}{6}\right)x$ **CSO**
		6	

	Question 5 Total	**13**	

Q	Answer	Marks	Comments
6	$\dfrac{(1-0.35)\times(1-0.42)}{0.35\times(1-0.79)+(1-0.35)\times(1-0.42)}$	M1 M1	**M1**: Correct calculation for the numerator **PI** by sight of 0.377 **M1**: Correct calculation for the denominator **PI** by sight of 0.4505
	$=\dfrac{754}{901}$	A1	**AWRT** 0.837

	Question 6 Total	**3**	

Q	Answer	Marks	Comments
7(a)	0.653	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
7(b)(i)	$\mathrm{G}_H(t) = (1 - p) + pt$ $\mathrm{G}_H(t) = 0.347 + 0.653t$	**B1ft**	**oe** **ft** their part **(a)**
		1	

Q	Answer	Marks	Comments
7(b)(ii)	$\mathrm{G}'_H(t) = 0.653$	**M1**	Differentiates their $\mathrm{G}_H(t)$
	$\mathrm{G}''_H(t) = 0$	**A1ft**	Award so long as $\mathrm{G}_H(t)$ is in the form $(1 - p) + pt$
	$\mathrm{Var}(H) = \mathrm{G}''_H(1) + \mathrm{G}'_H(1) - (\mathrm{G}'_H(1))^2$ $= 0 + 0.653 - 0.653^2$	**M1**	Must see either the formula or all the values substituted into the formula
	$= 0.226591$	**A1**	**oe** **AWRT** 0.2266
		4	

	Question 7 Total	**6**	

Q	Answer	Marks	Comments
8(a)	$1 - (1-p)^3 = \frac{61}{125}$	**M1**	Forms correct equation **oe** e.g. $3p - 3p^2 + p^3 = \frac{61}{125}$
	$p = \frac{1}{5}$	**A1**	**oe**
		2	

Q	Answer	Marks	Comments
8(b)(i)	$\frac{n-4}{n} = \frac{7}{66}\left(\frac{n+1}{2}\right)$	**M1**	Forms correct equation **oe**
	$7n^2 - 125n + 528 = 0$	**A1**	Forms correct quadratic equation equal to zero **oe PI** Missing = 0 can be implied by attempts to solve the quadratic
	$(7n - 48)(n - 11) = 0$	**M1**	Valid attempt to solve their quadratic equation **PI** By sight of 11 or $\frac{48}{7}$
	$n = 11$	**A1**	If $\frac{48}{7}$ seen, it must be rejected
		4	

Q	Answer	Marks	Comments
8(b)(ii)	$\text{Var}(X) = \dfrac{1-0.2}{0.2^2} = 20$	B1ft	**oe** **ft** their p which must be between 0 and 1
	$\text{Var}(Y) = \dfrac{11^2-1}{12} = 10$	B1ft	**oe** **ft** their n which must be a positive integer
	$\text{Var}(X-Y)$ $= \text{Var}(X) + \text{Var}(Y) - 2\,\text{cov}(X,Y)$ $13 = 20 + 10 - 2\,\text{cov}(X,Y)$ $\text{cov}(X,Y) = 8.5$	M1	Uses correct equation with their $\text{Var}(X)$ and $\text{Var}(Y)$ to find $\text{cov}(X,Y)$ **PI**
	$\rho = \dfrac{\text{cov}(X,Y)}{\sqrt{\text{Var}(X)\,\text{Var}(Y)}} = \dfrac{8.5}{\sqrt{20\times10}}$	M1	Applies correct formula for ρ
	$\rho = 0.601$	A1	**AWRT** 0.601
		5	

	Question 8 Total	**11**	

Q	Answer	Marks	Comments
9(a)	$v_{BA} = \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix} - \begin{bmatrix} 2.5 \\ -1 \end{bmatrix}$	M1	Difference of the given velocities, either way round
	$v_{BA} = \begin{bmatrix} -2 \\ 2.5 \end{bmatrix}$ [m s^{-1}]	A1	Correct relative velocity Condone missing units
		2	

Q	Answer	Marks	Comments
9(b)	$\begin{bmatrix} 20 \\ d \end{bmatrix} + \begin{bmatrix} -2 \\ 2.5 \end{bmatrix} t = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $20 - 2t = 0$	M1	Equation to find t or a proportionality argument **oe**
	$t = 10$	A1	Correct t
	$d + 2.5 \times 10 = 0$ $d = -25$	A1	Correct d
		3	

	Question 9 Total	**5**	

Q	Answer	Marks	Comments
10(a)	$[P] = MLT^{-2} \times L^{-2}$ $= ML^{-1}T^{-2}$	B1	**AG** Must be convincingly shown
		1	

Q	Answer	Marks	Comments
10(b)	$[\rho] = ML^{-3}$ $[g] = LT^{-2}$ $[h] = L$	M1	Finds correct dimensions of at least two of the quantities on RHS Condone use of units
	$[\rho gh] = ML^{-3} \times LT^{-2} \times L$	m1	Multiplies their three dimensions Condone use of units
	$= ML^{-1}T^{-2}$ $\therefore$ dimensionally consistent	A1	Correct conclusion from correct working Accept $[P] = [\rho gh]$ in place of a conclusion
		3	

	Question 10 Total	**4**	

Q	Answer	Marks	Comments
11(a)	$5\times3+3\times(-2) = 5\times0.375+3v_Q$	M1 A1	**M1**: Uses conservation of momentum to obtain equation of form $\pm5\times3\pm3\times2 = \pm5\times0.375\pm3v_Q$ **A1**: Correct equation
	$v_Q = \dfrac{15-6-1.875}{3}$		
	$v_Q = 2.375\ \left[\text{m s}^{-1}\right]$	A1	Correct speed
		3	

Q	Answer	Marks	Comments
11(b)	$I = 5\times0.375-5\times3=-13.125$	M1	Uses impulse equation to obtain calculation of form $\pm5\times0.375\pm5\times3$ or $\pm3\times2.375\pm3\times2$
	Magnitude of impulse = 13.125 [N s]	A1	Correct magnitude of impulse
		2	

	Question 11 Total	**5**	

Q	Answer	Marks	Comments
12(a)	$I = 1800\int_0^{0.1}\left(t - 10t^2\right)\mathrm{d}t$	**M1**	Forms correct integral to find the impulse
	$=\left[900t^2 - 6000t^3\right]_0^{0.1}$		
	$= 3$ Ns	**A1**	Correct impulse with correct units **oe**
		2	

Q	Answer	Marks	Comments
12(b)	$3 = 0.2(u+v)$	**M1**	Uses impulse equation to obtain their $\pm I = \pm 0.2u \pm 0.2v$ or their $I = 0.2\times 2u$ and their $I = 0.2\times u$
	$v = 15 - u$	**A1ft**	**PI** Correct expression for speed v with their I For vector solutions, implied by correct lower and upper bound for u
	$0 \le v \le u$ $0 \le 15 - u \le u$	**m1**	**PI** Forms inequality $0 \le$ their v in terms of $u \le u$ **oe** or finds a correct lower or upper bound for u Condone use of strict inequalities
	$7.5 \le u \le 15$	**A1**	Correct range for u Score **SC1** for $0 \le u \le 15$ or $0 < u \le 15$ or $u \le 15$ if otherwise no marks would be scored
		4	

Q	Answer	Marks	Comments
12(b) ALT	Initial velocity = $-u$ Final velocity = eu $3 = 0.2eu - 0.2(-u)$	**M1**	Uses impulse equation to obtain their $I = \pm 0.2eu \pm 0.2u$ **oe** May use a different letter for e
	$e = \dfrac{15}{u} - 1$	**A1ft**	**PI** Correct expression for e for their I
	$0 \le \dfrac{15}{u} - 1 \le 1$	**m1**	**PI** Forms inequality $0 \le$ their e in terms of $u \le 1$ or finds a correct lower or upper bound for u Condone use of strict inequalities
	$1 \le \dfrac{15}{u} \le 2$ $7.5 \le u \le 15$	**A1**	Correct range for u Score **SC1** for $0 \le u \le 15$ or $0 < u \le 15$ if otherwise no marks would be scored
		4	

	Question 12 Total	**6**	