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FURTHER MATHEMATICS

FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

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246XFM01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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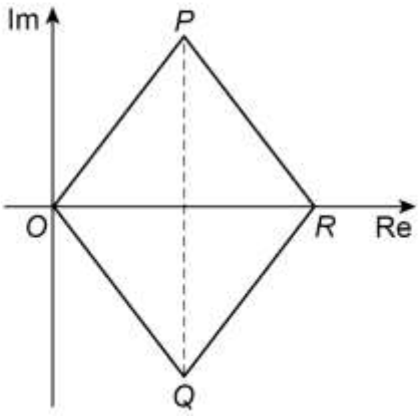
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$r = \sqrt{2^2 + \left(\sqrt{5}\right)^2}$ $\quad [= 3]$ $\theta = \tan^{-1}\left(\dfrac{\sqrt{5}}{2}\right)$ $\quad [= 0.841]$	**M1**	Correct modulus and/or argument May be unsimplified
	$z = 3\left(\cos(0.841) + \mathrm{i}\sin(0.841)\right)$	**A1**	
		2	

Q	Answer	Marks	Comments
1(b)	$z^* = 3\left(\cos(-0.841) + \mathrm{i}\sin(-0.841)\right)$	**M1 A1ft**	**M1**: Accept the correct conjugate written in any form. eg $2 - \mathrm{i}\sqrt{5}$ or $3\left(\cos(0.841) - \mathrm{i}\sin(0.841)\right)$ **A1ft**: Answer in the correct form, **ft** their r and $-\theta$ from part **(a)**
		2	

Q	Answer	Marks	Comments
1(c)(i)		**M1**	Points P and Q drawn as reflections of each other in the real axis – mark intention Condone P and Q swapped or to the left of the imaginary axis
		A1	Correct rhombus
		2	

Q	Answer	Marks	Comments
1(c)(ii)	4	**B1**	Accept $4+0\mathrm{i}$
		1	

Q	Answer	Marks	Comments
1(c)(iii)	area $=\frac{1}{2}\times 4\times 2\sqrt{5}$	**M1**	Full method for the area of *OPRQ*
	$=4\sqrt{5}$	**A1ft**	**ft** Their part **(c)(ii)**
		2	

	Question 1 Total	**9**	

Q	Answer	Marks	Comments
2(a)	$(4+h)^3 = 4^3 + 3\times 4^2 \times h + 3\times 4\times h^2 + h^3$	M1	At least 3 correct terms May be unsimplified
	$= 64 + 48h + 12h^2 + h^3$	A1	
		2	

Q	Answer	Marks	Comments
2(b)(i)	$(4+h)^3 + 7(4+h)$ $= 64 + 48h + 12h^2 + h^3 + 28 + 7h$	M1	Substitutes $4+h$ into $x^3 + 7x$ and expands May be unsimplified **PI**
	$= 92 + 55h + 12h^2 + h^3$		
	Gradient of line $= \dfrac{92 + 55h + 12h^2 + h^3 - (4^3 + 7\times 4)}{4 + h - 4}$	M1	May be unsimplified
	$= \dfrac{55h + 12h^2 + h^3}{h}$		
	$= 55 + 12h + h^2$	A1	
		3	

Q	Answer	Marks	Comments
2(b)(ii)	Gradient of curve $= \lim\limits_{h \to 0}\left(55 + 12h + h^2\right)$ $\left[= 55 + 0 + 0 \right] = 55$	**M1** **A1ft**	Considers their part **(b)(i)** as $h \to 0$ Obtains the correct limit of their part **(b)(i)** as $h \to 0$ **ft** their $a + bh + h^2$ **SC1** for their 55 following $h = 0$
		2	

	Question 2 Total	**7**	

Q	Answer	Marks	Comments
3	$\sum_{r=1}^{n}\left(ar^3+br^2\right)=\frac{a}{4}n^2(n+1)^2 + \frac{b}{6}n(n+1)(2n+1)$	M1	Correct expression in a, b and n May be unsimplified **PI** or forms a correct equation in a and/or b independent of n
	Comparing coefficients of n^4: $\frac{a}{4}=\frac{15}{12}$ Comparing coefficients of n: $\frac{b}{6}=\frac{4}{12}$	m1	Forms a correct equation in a (or b) only **PI** or forms a second correct equation in a and b independent of n
	$a = 5$	A1	
	$b = 2$	A1	

	Question 3 Total	**4**	

Q	Answer	Marks	Comments
4	$(5-\mathrm{i})+\alpha=-4$	**M1**	Writes a correct unsimplified expression for α Or forms a correct equation in α only (other than the given quadratic) ft their w if calculated first
	$\alpha=-9+\mathrm{i}$	**A1**	
	$(5-\mathrm{i})(-9+\mathrm{i})=w$	**M1**	Writes a correct unsimplified expression for w Forms a correct equation in w only eg $(5-\mathrm{i})^2+4(5-\mathrm{i})+w=0$ ft their α if calculated first
	$-45-\mathrm{i}^2+5\mathrm{i}+9\mathrm{i}=w$		
	$w=-44+14\mathrm{i}$	**A1**	

	Question 4 Total	**4**	

Q	Answer	Marks	Comments
5	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$	B1	Writes, or uses, a correct chain rule connecting V, h and t **PI**
	$\frac{dV}{dh} = 12h^2$	M1	Differentiates V with respect to h **oe** Accept mh^2 for any non-zero m or $mV^{-\frac{2}{3}}$ for $\frac{dh}{dV}$ **PI**
	When $h = 2.5$, $\frac{dV}{dh} = 12 \times 2.5^2$	m1	Substitutes $h = 2.5$ into their $\frac{dV}{dh}$ or substitutes $V = 62.5$ into their $\frac{dh}{dV}$
	$\frac{dh}{dt} = \frac{1}{12 \times 2.5^2} \times (-16)$	m1	Full correct substitution for $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ Condone 16 in place of –16
	rate of increase $= -\frac{16}{75}$ rate of decrease $= 0.21$ $\left[\text{cm s}^{-1} \text{ to 2 sf}\right]$	A1	Accept more significant figures or $\frac{16}{75}$ Condone a rate of increase instead

	Question 5 Total	**5**	

Q	Answer	Marks	Comments
6(a)(i)	$\frac{1}{r} - \frac{1}{r+1} = \frac{r+1-r}{r(r+1)}$ $= \frac{1}{r(r+1)}$	B1	Must include at least one intermediate line of working leading to **AG**
		1	

Q	Answer	Marks	Comments
6(a)(ii)	$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \sum_{r=1}^{n} \left(\frac{1}{r} - \frac{1}{r+1}\right)$	B1	Writes as a sum of $\frac{1}{r} - \frac{1}{r+1}$ Condone omission of brackets **PI** by correct use of the method of differences
	$= \frac{1}{1} - \cancel{\frac{1}{2}}$ $+ \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}$	M1	Writes at least two pairs of fractions of the form $\frac{1}{r} - \frac{1}{r+1}$
	$+ \ldots$ $+ \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}$ $+ \cancel{\frac{1}{n}} - \frac{1}{n+1}$	M1	Writes at least three pairs of fractions of the form $\frac{1}{r} - \frac{1}{r+1}$ including the correct first pair, the correct last pair, and at least one other pair
	$= \frac{1}{1} - \frac{1}{n+1}$ $= \frac{n+1-1}{n+1}$ $= \frac{n}{n+1}$	A1	**ISW**
		4	

Q	Answer	Marks	Comments
6(b)	$\sum_{r=1}^{\infty} \frac{1}{r(r+1)} = 1$	B1ft	**ft** Their **part (a)(ii)**
		1	

Q	Answer	Marks	Comments
6(c)	$\sum_{r=1001}^{2000} \frac{1}{r(r+1)} = \sum_{r=1}^{2000} \frac{1}{r(r+1)} - \sum_{r=1}^{1000} \frac{1}{r(r+1)}$	M1	Correctly splits the required sum into two sums of the form $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ **PI**
	$= \frac{2000}{2001} - \frac{1000}{1001}$		
	$= \frac{1000}{2003001}$	A1	
		2	

	Question 6 Total	**8**	

Q	Answer	Marks	Comments
7(a)	$y = 1$	**B1**	
		1	

Q	Answer	Marks	Comments
7(b)	$a^2 - 4\times 1\times 3 < 0$	**M1**	Considers the discriminant of the denominator Inequality not required for this mark
	$a^2 < 12$	**A1**	
		2	

Q	Answer	Marks	Comments
7(c)	$k = \dfrac{x^2}{x^2 + ax + 3}$		
	$kx^2 + kax + 3k = x^2$	**M1**	Forms a quadratic equation in x in terms of k
	$(k-1)x^2 + kax + 3k = 0$		
	No intersection points, so $(ka)^2 - 4(k-1)3k < 0$	**M1**	Correctly applies the discriminant to their quadratic in x Inequality not required for this mark
	$k^2a^2 - 12k^2 + 12k < 0$	**M1**	Sets their discriminant < 0
	$k^2(12 - a^2) - 12k > 0$	**A1**	**AG** Must be convincingly shown
		4	

Q	Answer	Marks	Comments
7(d)	Stationary points occur when $y^2(12-5)-12y = 0$ $y(7y-12) = 0$	**M1**	Forms an equation in y Accept k instead of y
	$y = 0$ or $y = \dfrac{12}{7}$	**A1**	At least one correct y-coordinate Condone k instead of y
		A1	Both y-coordinates correct and no incorrect y-coordinates
		3	

	Question 7 Total	**10**	

Q	Answer	Marks	Comments
8(a)	$(\alpha+\beta)^4 = \alpha^4 + 4\alpha^3\beta + 6\alpha^2\beta^2 + 4\alpha\beta^3 + \beta^4$	**M1 A1**	**M1**: At least three correct terms May be unsimplified **A1**: All correct
		2	

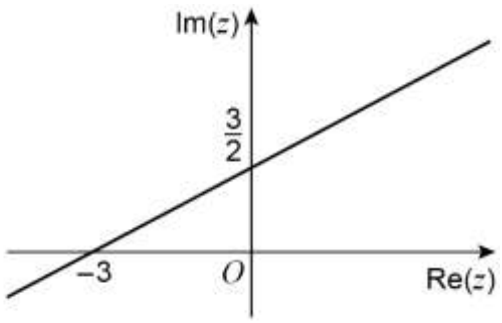
Q	Answer	Marks	Comments
8(b)	$(\alpha+\beta)^4 - 4\alpha^3\beta - 6\alpha^2\beta^2 - 4\alpha\beta^3 = \alpha^4 + \beta^4$		
	$\alpha^4+\beta^4 = (\alpha+\beta)^4 - 4\alpha\beta(\alpha^2+\beta^2) - 6(\alpha\beta)^2$	**M1**	Rearranges to write $\alpha^4+\beta^4$ in terms of $\alpha+\beta$ and $\alpha\beta$ and $\alpha^2+\beta^2$ **PI**
	$= (\alpha+\beta)^4 - 4\alpha\beta\left((\alpha+\beta)^2 - 2\alpha\beta\right) - 6(\alpha\beta)^2$	**M1**	Replaces $\alpha^2+\beta^2$ with $(\alpha+\beta)^2 - 2\alpha\beta$ **PI**
	$\alpha^4+\beta^4 = (\alpha+\beta)^4 - 4\alpha\beta(\alpha+\beta)^2 + 2(\alpha\beta)^2$	**A1**	**ACF** eg $\left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$
		3	

Q	Answer	Marks	Comments
8(c)(i)	$\alpha+\beta = \frac{1}{2}$	**B1**	
	$\alpha\beta = 3$	**B1**	
		2	

Q	Answer	Marks	Comments
8(c)(ii)	New sum $= \dfrac{\alpha^2}{\beta^2}+\dfrac{\beta^2}{\alpha^2} = \dfrac{\alpha^4+\beta^4}{\beta^2\alpha^2}$ $= \dfrac{(\alpha+\beta)^4-4\alpha\beta(\alpha+\beta)^2+2(\alpha\beta)^2}{(\alpha\beta)^2}$	**M1**	Writes the new sum in terms of $\alpha+\beta$ and $\alpha\beta$ **ft** Their $\alpha^4+\beta^4$ **PI** Accept $\dfrac{(\alpha+\beta)^2-2\alpha\beta-12(\alpha+\beta)+72}{\alpha\beta-6(\alpha+\beta)+36}$ for roots $\dfrac{\alpha-6}{\beta-6}$ and $\dfrac{\beta-6}{\alpha-6}$
	$= \dfrac{\left(\frac{1}{2}\right)^4-4\times3\times\left(\frac{1}{2}\right)^2+2\times3^2}{3^2} = \dfrac{241}{144}$	**A1**	**PI**
	New product $= \dfrac{\alpha^2}{\beta^2}\times\dfrac{\beta^2}{\alpha^2} = \dfrac{\alpha^2\beta^2}{\beta^2\alpha^2} = 1$	**B1**	**PI**
	[New equation is] $x^2-\dfrac{241}{144}x+1\ [=0]$	**M1**	**ft** Their new sum and new product
	$144x^2-241x+144=0$	**A1**	Accept any integer multiple
		5	

	Question 8 Total	**12**	

Q	Answer	Marks	Comments
9(a)	$\frac{1}{2}(2+0\mathrm{i}+0+4\mathrm{i}) = 1+2\mathrm{i}$ $c = 2$	B1	
		1	

Q	Answer	Marks	Comments
9(b)	gradient of L is $-1 \div \left(-\frac{4}{2}\right) = \frac{1}{2}$	B1	Correct calculation for the gradient of L **PI** by axis intercepts
		B1 B1 B1	Straight line through the 1st, 2nd and 3rd quadrants Correct imaginary axis intercept Condone $\frac{3}{2}\mathrm{i}$ Correct real axis intercept
		4	

Q	Answer	Marks	Comments
9(c)(i)	$a = -(-3+0\mathrm{i}) = 3$	B1ft	**ft** Their real intercept
		1	

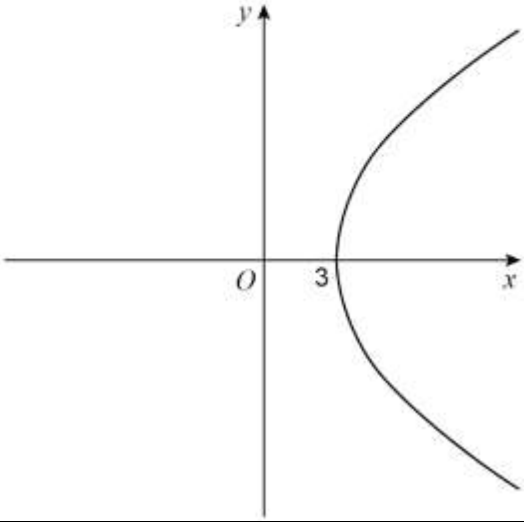
Q	Answer	Marks	Comments
9(c)(ii)	$b = \frac{1.5}{3} = \frac{1}{2}$	B1ft	**ft** Their axis intercepts or their gradient if stated
		1	

	Question 9 Total	7	

Q	Answer	Marks	Comments
10(a)	$(x-0)^2+(y-9)^2$ or $[\pm](y+3)$	**B1**	Writes a correct distance from P to $(0,9)$ or to $y=-3$ Seen or used
	$\sqrt{(x-0)^2+(y-9)^2} = \pm(y+3)$	**M1**	Forms an equation in x and y using their distances **PI**
	$(x-0)^2+(y-9)^2=(y+3)^2$	**M1**	Removes the square root correctly
	$x^2+y^2-18y+81=y^2+6y+9$		
	$x^2=24y-72$	**A1**	If **B0M0M0** then award **SC2** for $x^2=24y+b$ or $x^2=ay-72$ for non-zero a and b
		4	

Q	Answer	Marks	Comments
10(b)(i)	$y^2=24x-72$	**B1ft**	**oe** **ft** Their $x^2=ay+b$ for non-zero a and b
		1	

Q	Answer	Marks	Comments
10(b)(ii)		**B1**	Correct shape, symmetrical about the x-axis

		**B1ft**	Correct x-axis intercept **ft** Their $-\frac{b}{a}$
		2	

Q	Answer	Marks	Comments
10(b)(iii)	The line meets C_2 when $(mx)^2 = 24x - 72$	**M1**	Forms a quadratic equation in x (or y) in terms of m
	$m^2x^2 - 24x + 72 = 0$	**A1**	Correct 3-term quadratic equation in x in terms of m equal to zero [= 0 can be implied]
	Two intersection points, so $(-24)^2 - 4m^2 \times 72 > 0$	**M1**	Correctly substitutes their quadratic coefficients into $b^2 - 4ac > 0$
	$576 > 288m^2$		
	$m^2 < 2$	**A1**	**PI** by correct final inequalities
	$-\sqrt{2} < m < 0$, $0 < m < \sqrt{2}$	**A1**	
		5	

Q	Answer	Marks	Comments
10(b)(iv)	$y = mx$ is a tangent when $m^2 = 2$	**M1**	Replaces the inequality with equals **PI** By one correct tangent (allow **ft**)
	$y = x\sqrt{2}$ and $y = -x\sqrt{2}$	**A1ft**	**ft** Their $m^2 = 2$
		2	

	Question 10 Total	**14**	