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INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

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246XFM05/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$0^2 = u^2 + 2\times(-9.8)\times 0.4$	**M1**	Uses constant acceleration equation or two term energy equation
	$u = \sqrt{7.84} = \frac{14}{5} = 2.8 \left[\text{m s}^{-1}\right]$	**A1**	Correct rebound speed
		2	

Q	Answer	Marks	Comments
1(b)	$3.24 = 0.3\times 2.8 - 0.3\times -u$	**M1**	Uses impulse formula to create equation, **PI** by 8 m s^{-1}
	$u = 8$	**A1**	Correct impact speed Accept -8
	Impact Speed $\times e = 2.8$	**M1**	Uses their impact speed to form an equation for e **PI** by $e = 0.35$
	$e = \frac{2.8}{8} = \frac{7}{20} = 0.35$	**A1**	Correct value for e
		4	

	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2(a)	Resultant force in vertical direction		
	$6a = \frac{300}{2} \times 0.5 - 6 \times 9.8$	**M1**	Three term equation of motion
	$a = 2.7 \left[\text{m s}^{-2}\right]$	**A1**	Correct magnitude AWRT 2.7
	Direction is vertically upwards	**B1**	Correct direction
		3	

Q	Answer	Marks	Comments
2(b)	Maximum speed when		
	$6 \times 9.8 = \frac{300}{2} \times e$	**M1**	Equation to find extension for max speed
	$e = 0.392$	**A1**	Correct extension
	$\frac{1}{2} \times \frac{300}{2} \times 0.5^2 = \frac{1}{2} \times \frac{300}{2} \times 0.392^2 + 6 \times 9.8 \times 0.108 + \frac{1}{2} \times 6 \times v^2$	**M1 A1**	**M1**: Four term energy equation **A1**: Correct equation
	$v = 0.54 \left[\text{m s}^{-1}\right]$	**A1**	Correct speed AWRT 0.54
		5	

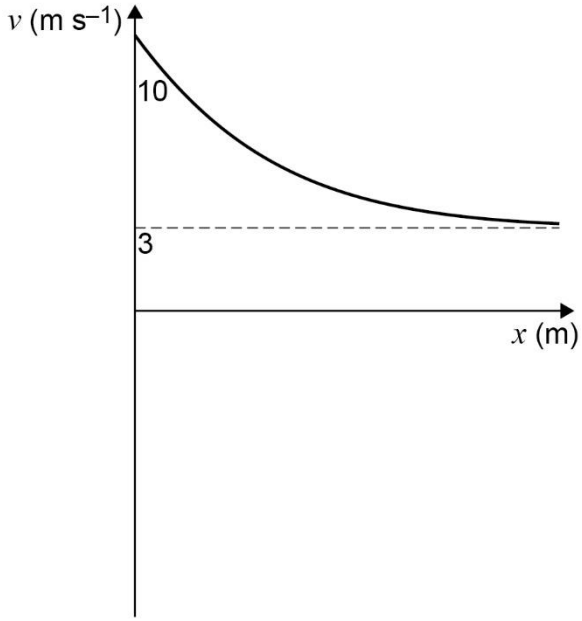
	Question 2 Total	**8**	

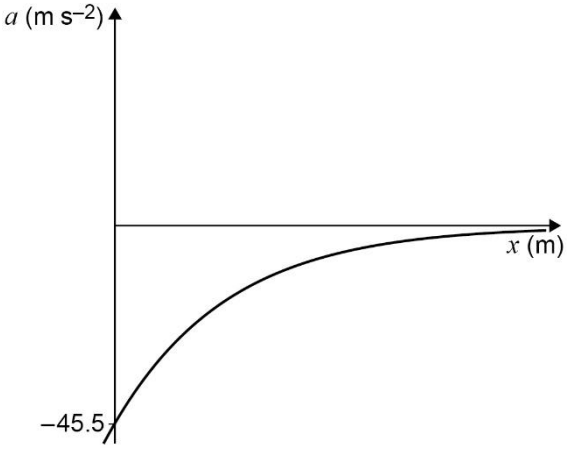
Q	Answer	Marks	Comments
3(a)	$\int_0^5 10\mathrm{e}^{-0.1x}\ \mathrm{d}x$	**M1**	Uses formula for work done by a variable force Condone missing or incorrect limits
	$= \left[-100\mathrm{e}^{-0.1x}\right]_0^5$	**A1**	Correct integration Condone missing or incorrect limits
	$= 100 - 100\mathrm{e}^{-0.5}$		
	$= 39.346...$		
	$= 39\ \left[\text{J to 2 sf}\right]$	**A1**	Correct work done **AWRT** 39
		3	

Q	Answer	Marks	Comments
3(b)(i)	Friction $= 0.2\times 2\times 9.8$		
	$= 3.92$	**B1**	Correct friction force
	$\frac{1}{2}\times 2\times v^2 = 39.3 - 5\times 3.92$	**M1**	Forms three term energy equation
	$v = 4.4\ \left[\text{m s}^{-1}\right]$	**A1**	Correct speed
		3	

Q	Answer	Marks	Comments
3(b)(ii)	$\int_0^d 10\mathrm{e}^{-0.1x}\ \mathrm{d}x = 3.92d$	**M1**	Forms equation using the formula for the work done by a variable force
	$\mathrm{f}(d) = 100 - 100\mathrm{e}^{-0.1d} - 3.92d = 0$	**A1**	Correct formula
	If $d = 22.5$ $\quad \mathrm{f}(d) = 1.26$ If $d = 23.5$ $\quad \mathrm{f}(d) = -1.66$	**M1**	Uses numerical method to justify answer If $d = 22$ $\quad \mathrm{f}(d) = 2.67$ If $d = 24$ $\quad \mathrm{f}(d) = -3.15$
	$\therefore\ d = 23$ to the nearest metre	**A1**	Correct conclusion from correct working
		4	

	Question 3 Total	10	
Q	**Answer**	**Marks**	**Comments**
4(a)	$2v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\left(v^2 - 9\right)$	**B1**	Correct differential equation
	$\displaystyle\int \frac{v}{v^2-9}\,\mathrm{d}v = \int -\frac{1}{2}\,\mathrm{d}x$	**M1**	Separates variables
	$\dfrac{1}{2}\ln\left(v^2-9\right) = -\dfrac{1}{2}x + c$	**M1 A1**	**M1**: Integrates to get a ln term **A1**: Correct integration Condone missing constant of integration
	$x = 0,\ v = 10 \implies c = \dfrac{1}{2}\ln 91$	**M1**	Finds constant of integration
	$\dfrac{1}{2}\ln\left(v^2-9\right) = -\dfrac{1}{2}x + \dfrac{1}{2}\ln 91$		
	$\dfrac{v^2-9}{91} = \mathrm{e}^{-x}$		
	$v = \sqrt{9 + 91\mathrm{e}^{-x}}$	**A1**	Correct expression for v
		6	

Q	Answer	Marks	Comments
4(b)		**B1** **B1**	Correct shape Correct values of 10 on intercept and 3 for asymptote
		2	

Q	Answer	Marks	Comments
4(c)	 $2a = -91$ $a = -45.5$	**B1** **M1 A1**	Correct shape with asymptote value of 0 **M1**: Calculates intercept, using either the differential equation or the derivative of the velocity **A1**: Correct intercept
		3	

	Question 4 Total	**11**	

Q	Answer	Marks	Comments
5	$0 = 10\sin\alpha\, t - 4.9\cos(30°)t^2$	**M1**	Equation for the time of flight
	$t = \dfrac{10\sin\alpha}{4.9\cos(30°)}$	**A1**	Correct time of flight
	$4 = 10\cos\alpha\left(\dfrac{10\sin\alpha}{4.9\cos(30°)}\right) - 4.9\sin(30°)\left(\dfrac{10\sin\alpha}{4.9\cos(30°)}\right)^2$	**M1**	Equation for motion parallel to plane using their time
	$\dfrac{4\times 4.9}{100} = \dfrac{\cos\alpha\sin\alpha}{\cos(30°)} - \dfrac{\sin(30°)\sin^2\alpha}{\cos^2(30°)}$	**A1**	Correct equation
	$0.588 = 2\sqrt{3}\cos\alpha\sin\alpha - 2\sin^2\alpha$		
	$\dfrac{0.588}{\cos^2\alpha} = 2\sqrt{3}\dfrac{\cos\alpha\sin\alpha}{\cos^2\alpha} - 2\dfrac{\sin^2\alpha}{\cos^2\alpha}$		
	$0.588\left(1+\tan^2\alpha\right) = 2\sqrt{3}\tan\alpha - 2\tan^2\alpha$	**M1**	Uses trig identity to create a quadratic in $\tan\alpha$
	$2.588\tan^2\alpha - 2\sqrt{3}\tan\alpha + 0.588 = 0$	**A1**	Correct quadratic equation
	$\tan\alpha = 0.199$ or 1.139	**M1**	Solves their quadratic to obtain two real roots
	$\alpha = 11$ or 49 [to nearest degree]	**A1**	Correct values for α

	Question 5 Total	**8**	

Q	Answer	Marks	Comments
6(a)	$I = 2\begin{bmatrix}0.4\\3\end{bmatrix} - 2\begin{bmatrix}4\\3\end{bmatrix}$	**M1**	Uses vector form of impulse formula
	$= \begin{bmatrix}-7.2\\0\end{bmatrix}$	**A1**	Correct impulse as a vector
	$I = 7.2 \quad [\text{N s}]$	**A1ft**	Correct magnitude, must be positive **ft** their impulse vector of the form $\begin{bmatrix}k\\0\end{bmatrix}$
		3	

Q	Answer	Marks	Comments
6(b)	$2\begin{bmatrix}4\\3\end{bmatrix} + 3\begin{bmatrix}-1\\5\end{bmatrix} = 2\begin{bmatrix}0.4\\3\end{bmatrix} + 3v_B$	**M1 A1**	**M1**: Uses conservation of momentum or impulse to form a vector equation for the velocity of B **A1**: Correct equation
	$v_B = \frac{1}{3}\begin{bmatrix}4.2\\15\end{bmatrix} = \begin{bmatrix}1.4\\5\end{bmatrix} \quad [\text{m s}^{-1}]$	**A1**	Correct velocity
		3	

Q	Answer	Marks	Comments
6(c)	Velocity of A does not change in the direction $\begin{bmatrix}0\\1\end{bmatrix}$, so line of centres must	**M1**	States that the velocity of A or B does not change in one direction or that the impulse is in the direction $\begin{bmatrix}0\\1\end{bmatrix}$
	be perpendicular to this vector, which is in the direction $\begin{bmatrix}1\\0\end{bmatrix}$	**A1**	Uses this to create a valid argument
		2	

Q	Answer	Marks	Comments
6(d)	$0.4-1.4=-e\left(4-\left(-1\right)\right)$ $e=0.2$	M1 A1	Uses coefficient of restitution formula Correct value
		2	

	Question 6 Total	**10**	

Q	Answer	Marks	Comments
7(a)	$e_B = 3a-(a+e_A)-a$		
	$e_B = a-e_A$	**M1**	Finds extension of second spring
	$ke_A = 2k\left(a-e_A\right)$	**M1**	Uses Hooke's Law to form an equation
	$e_A = 2a-2e_A$		
	$e_A = \frac{2a}{3}$ [m]	**A1**	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
7(b)(i)	$T_A = k\left(x+\frac{2a}{3}\right)$	**B1**	Correct tension for one spring
	$\left[T_B = 2k\left(3a-\left(x+\frac{5a}{3}\right)-a\right)\right]$		
	$T_B = 2k\left(\frac{a}{3}-x\right)$	**B1**	Correct tension in the other spring
	$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 2k\left(\frac{a}{3}-x\right)-k\left(x+\frac{2a}{3}\right)$	**M1**	Forms correct differential equation
	$= -3kx$		
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -3\frac{kx}{m}$	**A1**	Correct simplified differential equation
	SHM as the acceleration is proportional to the displacement and in the opposite direction	**B1**	Correct explanation
		5	

Q	Answer	Marks	Comments
7(b)(ii)	$\pi = 2\pi\sqrt{\dfrac{m}{3k}}$	**M1**	Uses formula for the period with their ω
	$k = \dfrac{4m}{3}$	**A1ft**	Correct expression Follow through their ω
		2	

Q	Answer	Marks	Comments
7(b)(iii)	$\omega = 2$	**B1**	Correct value for ω
	$v^2 = 2^2\left(\left(\dfrac{a}{10}\right)^2 - \left(\dfrac{a}{20}\right)^2\right)$	**M1**	Uses SHM speed formula with their ω
	$v = \dfrac{\sqrt{3}\,a}{10}$ $\left[\text{m s}^{-1}\right]$	**A1ft**	Correct speed Follow through their ω
		3	

Q	Answer	Marks	Comments
7(b)(iv)	$x = \dfrac{a}{10}\cos(2t)$	**B1**	Correct expression for displacement **PI**
	$\dfrac{a}{20} = \dfrac{a}{10}\cos(2t)$	**M1**	Forms an equation to find a time **PI**
	$t = \dfrac{\pi}{6}$	**A1**	Correct time, **PI**
	Required time $= 2\times\dfrac{\pi}{6}$		
	$= \dfrac{\pi}{3}$ [seconds]	**A1**	Correct time to move from *C* to *C* **CSO** AWRT 1.05
		4	

	Question 7 Total	**17**	

Q	Answer	Marks	Comments
8	Let the angle between *OA* and the vertical $= \alpha$ Let r = radius		
	At C $\quad \dfrac{mv^2}{r} = mg\cos 60°$	**M1**	Resolves radially at C
	$v^2 = \dfrac{gr}{2}$	**A1**	Correct velocity at C
	$mgr\left(\cos\alpha - \cos 60°\right) = \dfrac{1}{2}mv^2 = \dfrac{mgr}{4}$	**M1**	Energy equation at A
	$\cos\alpha - \dfrac{1}{2} = \dfrac{1}{4}$		
	$\cos\alpha = \dfrac{3}{4}$	**A1**	Correct value for $\cos\alpha$
	At A $\quad R = mg\cos\alpha = \dfrac{3mg}{4}$	**M1**	Reaction force at A
	At B $\quad R = \dfrac{3mg}{8}$	**A1**	Correct reaction force at B
	$\dfrac{mv^2}{r} = mg\cos\theta - R$	**M1**	Resolves radially at B
	$v^2 = gr\left(\cos\theta - \dfrac{3}{8}\right)$		
	$mgr\left(\cos\alpha - \cos\theta\right) = \dfrac{1}{2}mv^2$	**M1**	Energy equation
	$\dfrac{3}{4} - \cos\theta = \dfrac{\cos\theta}{2} - \dfrac{3}{16}$	**A1**	Correct equation
	$\dfrac{3}{2}\cos\theta = \dfrac{15}{16}$		
	$\cos\theta = \dfrac{5}{8}$	**A1**	Correct value for $\cos\theta$

	Question 8 Total	**10**	