OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA04) Unit S2 Statistics

Tuesday 16 January 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 The probability density function f for a continuous random variable X is given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{7}\mathrm{e}^{-\frac{1}{7}x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1 (a) State the name of the distribution of X

[1 mark]

Answer _____

1 (b) Find the mean of X

[1 mark]

Answer _____

1 (c) Find $\mathrm{P}(2 < X < 7)$

Give your answer to four significant figures.

[2 marks]

Answer _____

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1 (d) It is given that $P(X < a) = 0.8$

Find the value of a

Give your answer to four significant figures.

[2 marks]

Answer _____

1 (e) Find $P(X > 8 \mid X > 5)$

Give your answer to four significant figures.

[2 marks]

Answer _____

8

Turn over for the next question

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2 For users of an app, the time spent per day using the app has a mean of 300 seconds and a standard deviation of 40 seconds.

A new version of the app is released.

A random sample of 200 users is taken and their time spent per day using the new version of the app is recorded.

The sample mean is 306 seconds.

Assume that the population standard deviation is unchanged.

2 (a) Test at the 2% level of significance whether the mean time spent per day using the app has increased.

[7 marks]

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2 (b) Explain why it is not necessary to assume that the time spent per day using the app is normally distributed to carry out the test in **part (a)**.

[2 marks]

9

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3 The continuous random variable X has probability density function $\mathrm{f}(x)$ defined by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{8}x^2 & 0 \leq x < 1 \\ k(x-1)+\frac{1}{8} & 1 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

3 (a) Find $\mathrm{P}(X < 1)$

Give your answer in exact form.

[2 marks]

Answer _____

3 (b) Show that $k = \frac{2}{75}$

[3 marks]

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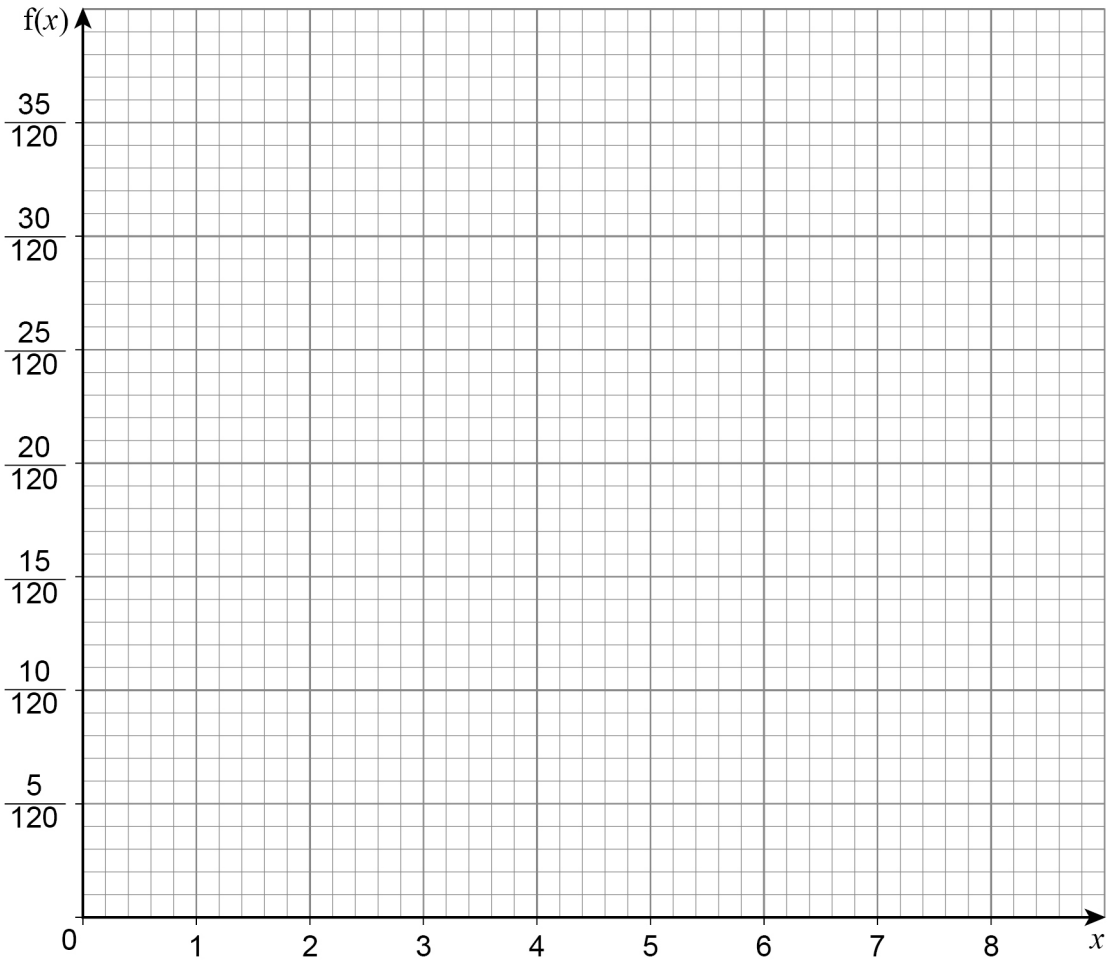
3 (c) Draw the graph of $y = f(x)$ for $0 \leq x \leq 8$

[3 marks]



Question 3 continues on the next page

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3 (d) Find $\mathrm{F}(x)$, the cumulative distribution function of X

You are given that $\mathrm{F}(x) = 1$ for $x > 6$

[4 marks]

$$\mathrm{F}(x) = \begin{cases} \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ 1 \qquad\qquad x > 6 \end{cases}$$

12

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4 (a) The random variables C, D, E and F are all binomially distributed, as shown in the table.

Distribution
$C \sim \mathrm{B}(10,\ 0.04)$
$D \sim \mathrm{B}(200,\ 0.51)$
$E \sim \mathrm{B}(400,\ 0.91)$
$F \sim \mathrm{B}(100,\ 0.05)$

State, with a reason, which one of these distributions would be most suitable to approximate by a Poisson distribution.

[2 marks]

Distribution _____

Reason _____

_____

_____

Question 4 continues on the next page

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4 (b) In any given hour the number of aeroplanes which fly over an island can be modelled as a Poisson distribution with a mean of 1.8

4 (b) (i) Find the probability that exactly 3 aeroplanes fly over the island between 9.00 am and 10.00 am on a randomly selected day.

Give your answer to three significant figures.

[2 marks]

Answer _____

4 (b) (ii) Find the probability that more than 1 and less than 4 aeroplanes fly over the island in a given 20 minute period on a randomly selected day.

Give your answer to three significant figures.

[4 marks]

Answer _____

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4 (c) A different Poisson distribution is used to model the number of helicopters flying over the island in any given hour. The mean of this distribution is 2.7

Assume that the number of helicopters and the number of aeroplanes flying over the island in any given hour are independent random variables.

The random variable G represents the total number of helicopters and aeroplanes flying over the island in any given hour.

4 (c) (i) Write down the parameter of G

[1 mark]

Answer _____

4 (c) (ii) It is given that $P(G < a) > 0.95$ where a is an integer.

By considering probabilities, find the smallest possible value of a

[3 marks]

Answer _____

12

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5 A stellar camera is used to take photographs of stars in the night sky.

The mean maximum exposure time for a stellar camera is normally distributed.

Alice has a stellar camera which has a mean maximum exposure time of 13 seconds.

The camera accidentally gets dropped.

As a result, Alice believes the camera's mean maximum exposure time has changed.

Alice records the maximum exposure times X in seconds of 10 randomly selected pictures taken.

She compiles the following summary statistics.

$$\sum x = 129.5 \quad \text{and} \quad \sum x^2 = 1677.05$$

5 (a) Test whether the mean maximum exposure time of Alice's stellar camera has changed, using the 1% level of significance.

[10 marks]

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5 (b) Benga uses the same summary statistics as Alice.

He then performs a hypothesis test correctly but reaches a different conclusion to Alice.

Suggest one possible difference in Benga's hypothesis test.

[1 mark]

11

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6 The mass V grams of flour in a bag can be modelled by a normal distribution with mean 502 grams and standard deviation 2.7 grams.

A bakery purchases 30 bags of flour.

The masses of the bags are independent and each bag is selected at random.

6 (a) Find the probability that the mean mass of the 30 bags is less than 501 grams.

Give your answer to four decimal places.

[4 marks]

Answer _____

6 (b) Find the probability that the mass of flour in each of the 30 bags is more than 496 grams.

Give your answer to three decimal places.

[4 marks]

Answer _____

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6 (c) The mass W grams of butter in a packet is modelled by $W \sim \mathrm{N}\left(251,\ 1.5^2\right)$

The mass Y grams of sugar in a bag is modelled by $Y \sim \mathrm{N}\left(503,\ 2^2\right)$

The bakery has a dessert recipe mix which uses

1 bag of flour

2 packets of butter, and

1 bag of sugar

The distributions of mass for the flour, butter and sugar are all independent of each other.

It is given that 95% of the dessert recipe mixes have a mass greater than a grams.

Find the value of a

Give your answer to four significant figures.

[5 marks]

Answer _____

13

Turn over ►

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7 The continuous random variable X has a cumulative distribution function $\mathrm{F}(x)$ defined by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2 \\ \frac{1}{2} - \frac{2}{x^2} & 2 \leq x \leq 4 \\ \frac{1}{96}\left(x^2 + 3x + 8\right) & 4 < x \leq 8 \\ 1 & x > 8 \end{cases}$$

7 (a) Show that $\mathrm{Var}(X) = 4\ln(2) + \frac{1343}{1296}$

[7 marks]

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7 (b) The continuous random variable Y has a standard deviation of 2.5

The random variables X and Y are independent.

Find $\text{Var}(2X + 3Y)$

Give your answer to three significant figures.

[2 marks]

Answer _____

9

Turn over ►

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8 A student takes a test where the answer to each question is either True or False.

The test has 20 different questions.

The student answers each of the 20 questions.

The student selects the correct answer for 15 of the 20 questions.

The student's teacher claims that the student has randomly selected their answer to each question.

Test at the 3% level of significance whether there is evidence to support the teacher's claim.

[6 marks]

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END OF QUESTIONS

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