OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

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I declare this is my own work.

INTERNATIONAL AS
MATHEMATICS

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Tuesday 9 January 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

For Examiner's Use	
Question	Mark
1	
2	
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6	
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8	
9	
10	
11	
12	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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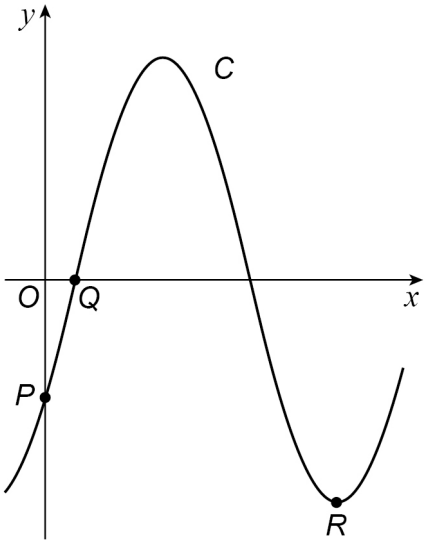
Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 The diagram below shows part of the curve C with equation $y = 4\sin(x - 30^\circ)$

Points P, Q and R lie on C



1 (a) The curve C has the following properties:

it intersects the y-axis at P

it intersects the x-axis at Q

it has a minimum at R

State the coordinates of P, the coordinates of Q and the coordinates of R

[3 marks]

P _____ Q _____ R _____

Do not write outside the box

1 (b) The curve D has equation

$$y = 4\cos(x - 15°)$$

Describe the transformation which maps C onto D

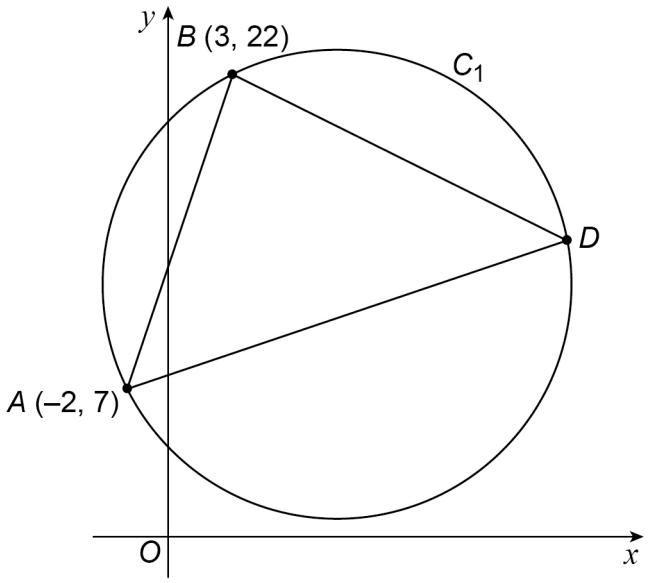
[2 marks]

5

Turn over for the next question

2 The circle C_1 and the triangle ABD are shown below.

The points $A(-2, 7)$, $B(3, 22)$ and D lie on the circle C_1



2 (a) The chord BD has gradient $-\frac{1}{2}$

Use the property of angles in a semicircle and the gradients of the chords AB and BD to show that the chord AD is **not** a diameter of C_1

[2 marks]

2 (b) (i) The point $E(18, 17)$ lies on C_1

The tangent to C_1 at A is parallel to the tangent to C_1 at E

Show that the coordinates of the centre of C_1 are $(8, 12)$

[1 mark]

Do not write outside the box

2 (b) (ii) Find the equation of C_1

Give your answer in the form $(x-p)^2+(y-q)^2=k$ where p, q and k are integers.

[3 marks]

Answer _____

2 (c) Find the coordinates of the point where the normal to C_1 at B intersects the x-axis.

[3 marks]

Answer _____

2 (d) The circle C_2 has the same radius as C_1

The translation $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ maps C_2 onto C_1

Find the equation of C_2

Give your answer in the form $(x-c)^2+(y-d)^2=n$ where c, d and n are integers.

[2 marks]

Answer _____

11

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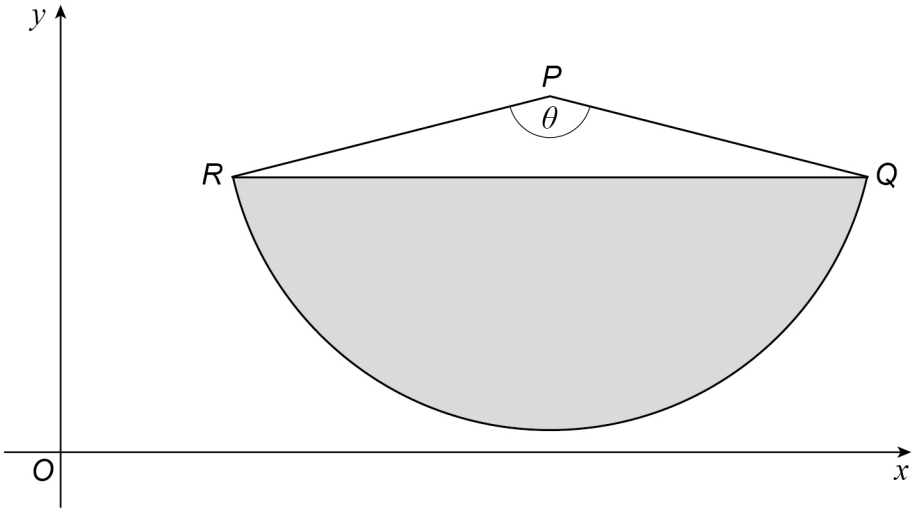
3 The diagram shows the sector PQR of a circle C with centre P and arc QR. The line segment QR is also shown, and the segment bounded by this line and the arc is shaded.



The **obtuse** angle $QPR = \theta$ radians.

3 (a) The equation of C is

$$x^2 - 24x + y^2 - 20y + 163 = 0$$

The triangle PQR has area 20.25 square units.

Show that $\theta = \frac{5\pi}{6}$

[5 marks]

Do not write outside the box

3 (b) Find the perimeter of the shaded region bounded by the arc QR and the line segment QR

Give your answer to three significant figures.

[4 marks]

Answer _____

9

Do not write outside the box

4 (a) Solve the equation

$$8 \times 6^{t} = 27 \times 6^{1-2t}$$

Give your answer in the form $p + \log_6 q$ where p and q are rational numbers.

[3 marks]

$t =$ _____

4 (b) A curve C has equation

$$y = 2\log_{10} x + \log_{10}(x+5)$$

where $x > 0$

4 (b) (i) Show that the equation of C can be written as

$$y = \log_{10}\left(x^3 + 5x^2\right)$$

[2 marks]

Do not write outside the box

4 (b) (ii) The points A and B lie on the curve C

The x-coordinate of A is 4 and the x-coordinate of B is 6

Using the result in **part (b)(i)** find the gradient of the line AB

Give your answer in the form $\log_{10}\left(k\sqrt{n}\right)$ where k and n are constants and n is prime.

[3 marks]

Answer _____

8

Turn over for the next question

Do not write outside the box

5 (a) It is given that

$$7\sin^2 x - \sin x \cos x = 6$$

Rearrange the equation to show that

$$(\tan x - 3)(\tan x + 2) = 0$$

[3 marks]

Do not write outside the box

5 (b) Use the result in **part (a)** to solve the equation

$$7\sin^2(\theta - 35°) - \sin(\theta - 35°)\cos(\theta - 35°) = 6$$

in the interval $-90° < \theta < 90°$

Show clearly each step of your working.

Give your answers to three significant figures.

[4 marks]

Answer _____

7

Turn over for the next section

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 The discrete random variables X_1, X_2, X_3 and X_4 are independent.

The mean and variance of each of the variables is given in the table.

Variable	Mean	Variance
X_1	2	1
X_2	13	6
X_3	6	4
X_4	9	3

6 (a) Find $\mathrm{E}(X_1 + X_3)$

[1 mark]

Answer _____

6 (b) Find $\mathrm{E}\left(\sum_{i=1}^{4} X_i\right)$

[1 mark]

Answer _____

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6 (c) Find $\text{Var}\left(X_2 - X_4\right)$

[1 mark]

Answer _____

6 (d) The discrete random variable X_5 is independent of X_1, X_2, X_3 and X_4

It is given that $\text{Var}\left(\sum_{i=1}^{5} X_i\right) = 39$

Find the standard deviation of X_5

[2 marks]

Answer _____

5

Turn over for the next question

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7 A game of darts has two possible outcomes: win or lose.

The probability of winning the game is 0.286

The random variable X is defined as equal to 0 if the game is lost and 1 if the game is won.

7 (a) (i) State the name of the distribution of X

[1 mark]

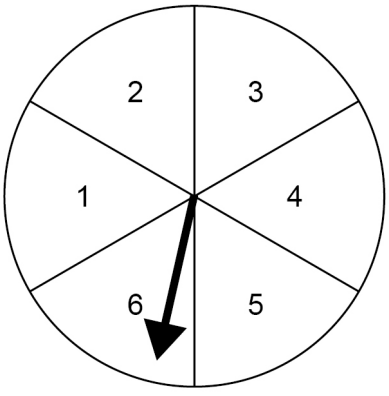
Answer _____

7 (a) (ii) State the value of $\mathrm{E}(X)$

[1 mark]

Answer _____

7 (b) An unbiased spinner is numbered 1, 2, 3, 4, 5 and 6



The arrow on the spinner is spun 5 times.

The random variable Y represents the number of times the arrow lands on a 6

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7 (b) (i) Find $\text{Var}(Y)$

[2 marks]

Answer _____

7 (b) (ii) Find $\text{P}(Y = 2)$

[2 marks]

Answer _____

7 (c) The game of darts is played and then the arrow is spun 5 times.

The random variable X is independent of Y

Find the probability that the game of darts is won and exactly 2 spins of the arrow land on a 6

[2 marks]

Answer _____

8

Turn over ►

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8 A company has 200 employees.

The company makes products P, Q and R

78 employees work on product P

101 employees work on product Q

91 employees work on product R

26 employees work on products P and Q

44 employees work on products Q and R

An employee is chosen at random.

8 (a) Find the probability that the employee works on product R given that they work on product Q

[2 marks]

Answer _____

Do not write outside the box

8 (b) Employees work on **at least one** and **at most two** of the products.

A represents the event that the employee works on product *P*

B represents the event that the employee works on product *R*

Find $\mathrm{P}(A \cap B)$ and determine whether A and B are mutually exclusive.

[5 marks]

7

Turn over for the next section

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 A particle of mass 0.6 kg is moving with speed $3u$ m s^{-1} on a smooth horizontal surface towards a fixed vertical wall.

The particle collides directly with the wall and rebounds with speed u m s^{-1}

The impulse exerted on the particle by the wall during the collision has magnitude 3 N s

Find the value of u

[2 marks]

Answer _____

2

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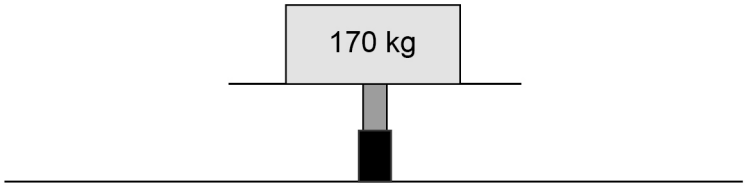
10 **The acceleration due to gravity, g, should be taken as 9.8 m s^{-2}**

A horizontal platform can move vertically upwards and vertically downwards.

A box of mass 170 kg is placed on the platform as shown in the diagram.



The reaction force between the box and the platform has magnitude R newtons.

10 (a) Find the magnitude of the acceleration of the box when $R = 1800$

[2 marks]

Answer _____

10 (b) A student claims that if $R < 1600$ then the platform must be moving downwards.

Explain why the student's claim is incorrect.

[2 marks]

4

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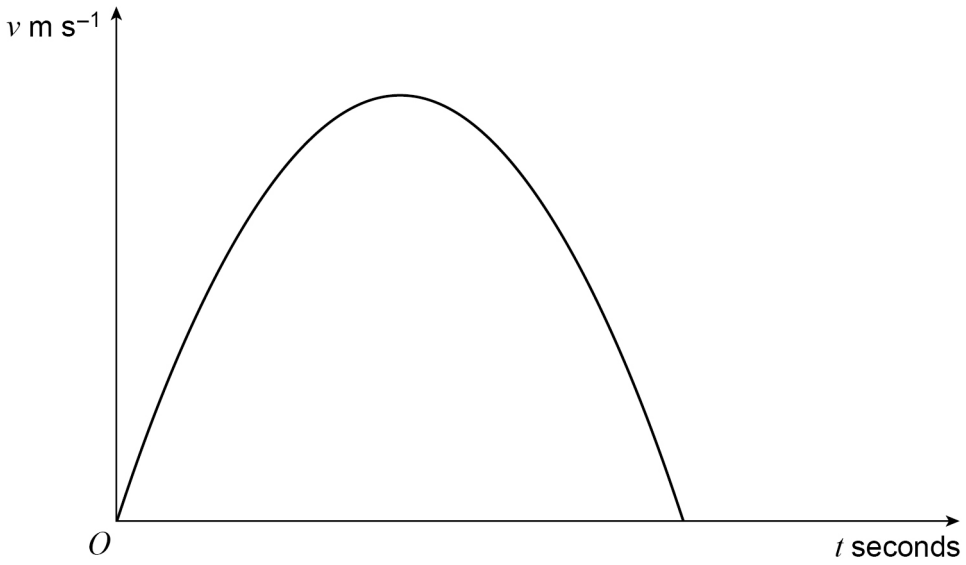
11 A car starts from rest and travels in a straight line along a horizontal road.

The car comes to rest k seconds later.

The velocity v m s^{-1} of the car at time t seconds is given by

$$v = 3t - 0.1t^2 \quad \text{for} \quad 0 \leq t \leq k$$

The velocity-time graph for the motion of the car is shown below.



11 (a) (i) Show that $k = 30$

[2 marks]

11 (a) (ii) Find the maximum speed of the car and the time at which it occurs.

[2 marks]

Maximum speed _____ Time _____

Do not write outside the box

11 (b) Find the average speed of the car during its motion.

[4 marks]

Answer _____

11 (c) Find the magnitude of the car's acceleration when $t = 0$

[2 marks]

Answer _____

10

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12 A block of mass 5 kg slides on a rough horizontal surface.

The only horizontal force acting on the block is friction due to its contact with the surface.

The coefficient of friction between the block and the surface is μ

The initial speed of the block is 6 m s^{-1}

The block moves x metres before coming to rest.

Show that $x = \frac{k}{\mu g}$ where k is a constant and g is the acceleration due to gravity.

[4 marks]

4

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