OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Wednesday 17 January 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 A company produces a games computer called Alpha.

The company releases a new games computer called Beta.

When Alpha computers and Beta computers are in use, the company claims that Beta computers use on average less energy per hour than Alpha computers.

A random sample of 150 Alpha computers is taken and the amount of energy, in kJ, used in 1 hour X_α is recorded.

For this sample

$$\bar{x}_\alpha = 87.5 \quad \text{and} \quad s_\alpha = 36.2$$

A random sample of 120 Beta computers is taken and the amount of energy, in kJ, used in 1 hour X_β is recorded.

For this sample

$$\bar{x}_\beta = 75.9 \quad \text{and} \quad s_\beta = 27.4$$

Test the company's claim using the 1% level of significance.

[6 marks]

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6

Turn over for the next question

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2 The discrete random variable X has probability distribution

x	–1	2	5	7
$\mathrm{P}(X = x)$	0.1	0.2	0.6	0.1

The moment generating function of X is $\mathrm{M}_X(t)$

2 (a) Use differentiation to find the value of $\mathrm{M}'_X(0)$

[3 marks]

Answer _____

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2 (b) Use differentiation to find the value of $\mathrm{M}''_X(0)$

[2 marks]

Answer _____

2 (c) Hence find $\mathrm{Var}(X)$

[2 marks]

Answer _____

7

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3 The scores on a Mathematics test have a normal distribution.

A random sample of four students is taken and their scores are

16 22 30 44

Construct a 98% confidence interval for the population mean test score.

Give your values to one decimal place.

[4 marks]

Answer _____

4

Turn over ►

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4 Chione is studying the lifetime in days of a particular type of moth.

Chione claims that the population standard deviation for the lifetime is 5 days.

She takes a random sample of 101 moths. For this sample the standard deviation for the lifetime of the moths is 5.6 days.

4 (a) Chione's sample is used to test her claim. The χ^2 distribution is used for the hypothesis test. State a necessary assumption for the hypothesis test.

[1 mark]

4 (b) Test Chione's claim, using the 5% level of significance.

[6 marks]

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4 (c) Chione is also studying the lifetime in days of a particular type of butterfly.

She takes a random sample of 51 butterflies. For this sample the standard deviation of the lifetime of the butterflies is 5.9 days.

She claims that the population variance of the lifetime of the butterflies is greater than the population variance of the lifetime of the moths.

The lifetimes of the butterflies and the moths are independent.

Test Chione's claim, using the 5% level of significance.

[6 marks]

13

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5 The number of errors made each day by the employees of a company can be modelled by a normal distribution.

Each employee participates in a training course designed to reduce the number of errors they make.

The company takes a random sample of seven employees and records the number of errors they each make on a randomly selected day before they have participated in the training course.

The company also records the number of errors the same seven employees each make on a randomly selected day after they have participated in the training course.

The results are given in the following table.

	Number of errors	
Employee	**Before training**	**After training**
1	12	4
2	21	5
3	8	8
4	7	6
5	9	12
6	10	9
7	13	10

Using the 5% level of significance, investigate whether the employees make fewer errors in a day after they have participated in the training course.

[9 marks]

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9

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6 Ayegbeni models the mass in kilograms of adult female African bush elephants using a normal distribution with standard deviation 300 kg

He takes a random sample of n African bush elephants and uses it to construct a 95% confidence interval of width 58.8 kg for the population mean mass of adult female African bush elephants.

6 (a) Find the value of n

[3 marks]

Answer _____

6 (b) Rashida constructs a 99% confidence interval for the population mean mass of adult female African bush elephants using Ayegbeni's data.

Find the width of Rashida's confidence interval.

Give your answer to three significant figures.

[3 marks]

Answer _____

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6 (c) The sample mean of Ayegbeni's sample is 4450 kg

Using a 1% level of significance, Rashida conducts a hypothesis test with her confidence interval and the hypotheses

$$H_0 : \mu = 4500$$
$$H_1 : \mu \neq 4500$$

State with a reason whether Rashida rejects the null hypothesis.

[2 marks]

8

Turn over for the next question

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7 A hypothesis test is carried out to test for association between time of day and number of snacks eaten during the working hours of a large office.

The **expected** frequencies calculated for the test are given in the following table.

		Number of snacks eaten		
		0	**1**	**2**
Time of day	Morning	11.44	8.8	1.76
	Afternoon	14.56	11.2	2.24

7 (a) State the hypotheses for the test.

[1 mark]

7 (b) Explain why the number of degrees of freedom for the hypothesis test is equal to 1

[2 marks]

7 (c) Write down the formula used to calculate the test statistic for this test.

[1 mark]

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7 (d) The test statistic is calculated correctly to be 5.05 given to three significant figures.

The hypothesis test is carried out using the 5% level of significance.

Explain whether the null hypothesis will be rejected.

[3 marks]

7

Turn over for the next question

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8 The random variables A, B, X_1, X_2, X_3, …, X_n are independent, unbiased estimators of the mean μ of a population and each have variance σ^2

8 (a) Show that

$$R = \frac{1}{n+2}\left(A + B + \sum_{i=1}^{n} X_i\right)$$

is an unbiased estimator of μ

[3 marks]

8 (b) Show that R is a consistent estimator of μ

[3 marks]

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8 (c) The random variable

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

is also an unbiased estimator of μ

A student claims that $\bar{X}$ is a more efficient estimator than R

By calculating the relative efficiency, determine whether the student is correct.

[4 marks]

10

Turn over for the next question

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9 The random variable X has a normal distribution with population mean μ and population variance 10.24

A random sample of six observations is taken from X

The random sample is used to carry out a hypothesis test at the 2% level of significance with the hypotheses

$$H_0 : \mu = 8$$
$$H_1 : \mu \neq 8$$

9 (a) Find the critical region of the hypothesis test.

Give the values in your answer to three decimal places.

[3 marks]

Answer _____

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9 (b) Subsequently μ is found to be 11.4

Find the power of the hypothesis test.

Give your answer to two significant figures.

[2 marks]

Answer _____

5

Turn over for the next question

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10 The lifetime T billion years of a particular type of star is to be modelled by a normal distribution.

A random sample of 50 stars is taken and the data is shown in the grouped frequency table below.

t	$t \leq 9.25$	$9.25 < t \leq 9.5$	$9.5 < t \leq 9.75$	$t > 9.75$
Frequency	6	22	13	9

The summary statistics for the data are:

$$\sum t = 475 \quad \text{and} \quad \sum (t - \bar{t})^2 = 1.96$$

Investigate the goodness of fit of the normal distribution to this data using the 1% level of significance.

[11 marks]

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END OF QUESTIONS

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