OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Monday 8 January 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 A curve passes through the point (–2, 3) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x^2 - y + 3}$$

Use Euler's step-by-step method with a step length of 0.1 to estimate the value of y when $x = -1.8$

Give your answer to four decimal places.

[5 marks]

Answer _____

5

Turn over ►

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2 The matrix **A** is defined by $\mathbf{A} = \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{bmatrix}$

The matrix **B** represents a reflection in the line $y = \left(\tan\frac{\pi}{3}\right)x$

2 (a) Write down the matrix **B**

Give each element of the matrix in exact form.

[1 mark]

Answer _____

2 (b) (i) Find the matrix **BA**

Give each element of the matrix to four decimal places.

[2 marks]

Answer _____

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2 (b) (ii) Describe fully the transformation represented by the matrix **BA**

[3 marks]

6

Turn over for the next question

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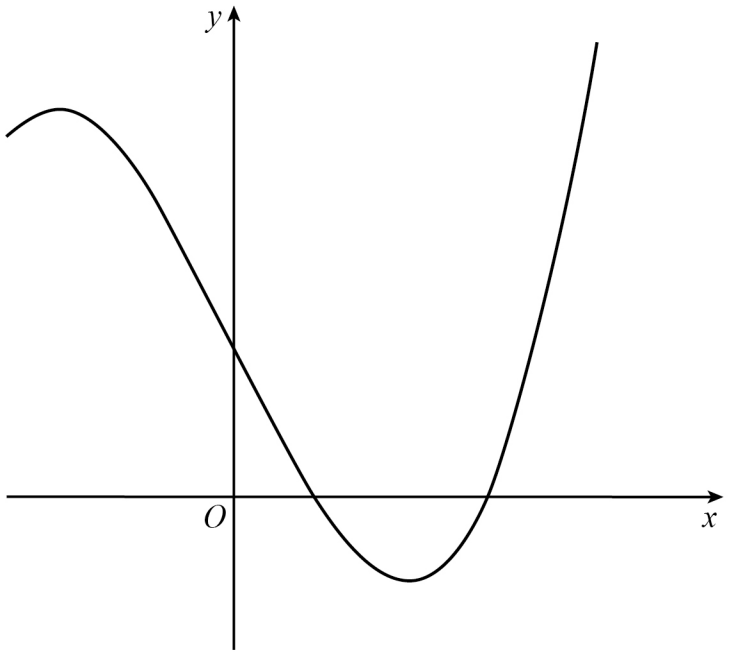
3 The graph shows part of the curve $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 1 - 2x + \frac{1}{2}x^3$



The equation $\mathrm{f}(x) = 0$ has two positive roots α and β

It is given that $\alpha < 1$ and that $\beta > 1$

3 (a) Show that $\frac{3}{2} < \beta < \frac{7}{4}$

[2 marks]

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3 (b) Starting with the interval $\frac{3}{2} < \beta < \frac{7}{4}$

use linear interpolation once to show that $\frac{209}{126} < \beta < \frac{7}{4}$

[5 marks]

7

4 The variables P and Q are believed to be related by the equation

$$Q = aP^b$$

where a and b are constants.

The table below shows some values of P and Q obtained in an experiment.

P	5	7	10	15	20	30
Q	3.0	2.6	1.9	1.5	1.3	1.0

4 (a) Let $X = \log_{10} P$ and $Y = \log_{10} Q$

Complete the table below, giving values to two decimal places.

[2 marks]

X	0.70	0.85	1.00			
Y	0.48	0.41		0.18		0.00

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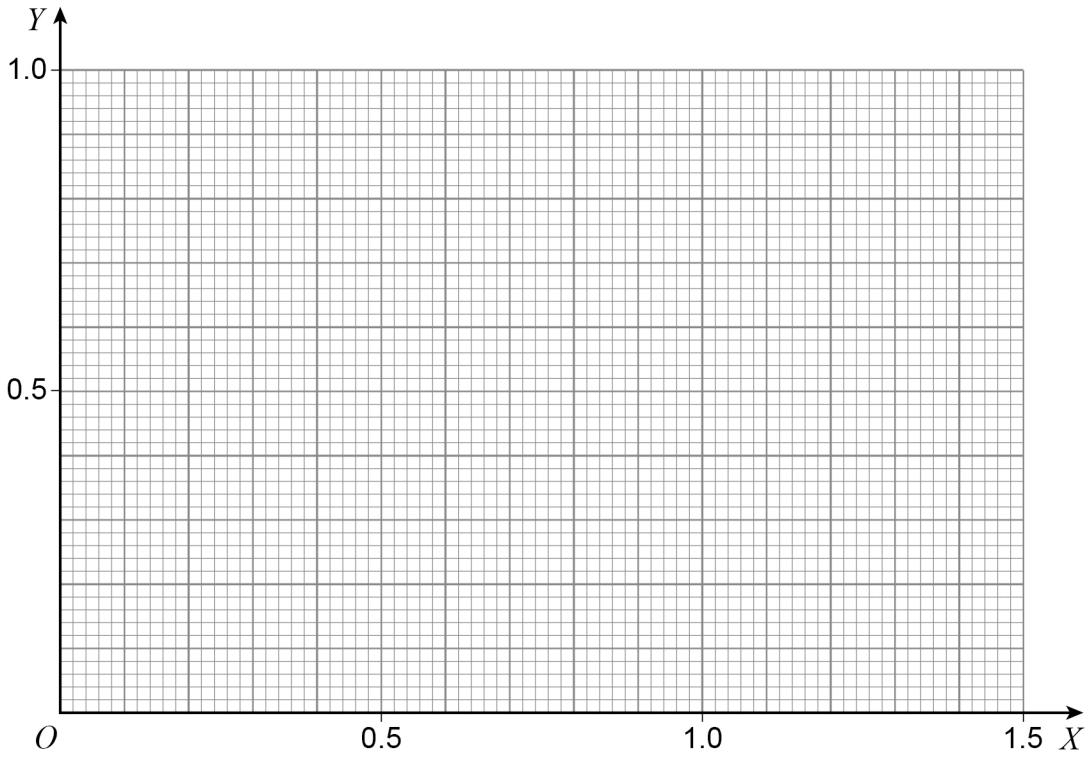
4 (b) Plot the values of X and Y on the grid below and draw a line of best fit.

[2 marks]



4 (c) Use your line of best fit to estimate the value of a and the value of b

[4 marks]

$a =$ _____ $b =$ _____

8

Turn over ►

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5 The matrix $\mathbf{M}$ is defined by $\mathbf{M} = \begin{bmatrix} 0.4 & 1.2 \\ 1.2 & -1.4 \end{bmatrix}$

5 (a) Find the equations of the invariant lines of the transformation represented by $\mathbf{M}$

[6 marks]

Answer _____

5 (b) Find the equation of the line of invariant points of the transformation represented by $\mathbf{M}$

[2 marks]

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Answer

5 (c) The transformation represented by **M** is a combination of two transformations: a reflection in the line $y = \frac{1}{2}x$ followed by a transformation represented by the matrix **N**

Find the matrix **N**

[6 marks]

Answer

14

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 A bag contains 32 blue counters, 14 red counters and 17 white counters.

A counter is randomly selected from the bag, its colour is recorded and it is **not** replaced.

A second counter is randomly selected from the bag and its colour is recorded.

6 (a) Draw a tree diagram to represent this information.

[3 marks]

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6 (b) Find the exact probability that the first counter is blue, given that the second counter is white.

[3 marks]

Answer _____

6

Turn over for the next question

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7 On a test, 7% of students gain an A grade.

A teacher selects students at random and notes whether they have gained an A grade.

The random variable N represents the number of students the teacher selects up to and including the first student who gained an A grade on the test.

7 (a) Find the probability that the fourth student the teacher selects is the first to have gained an A grade.

Give your answer to four decimal places.

[1 mark]

Answer

7 (b) Find the probability that the first three students the teacher selects did **not** gain an A grade.

[2 marks]

Answer

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7 (c) Find $G_N(t)$, the probability generating function for N

Give your answer in the form $\frac{at}{1-bt}$ where a and b are positive constants.

[3 marks]

Answer _____

7 (d) The random variable M is such that the probability generating function

$$G_M(t) = 0.07t + 0.93$$

The random variables M and N are independent.

Find $G_{M+N}(t)$, the probability generating function for $M + N$

Give your answer in the form $\frac{xt^2 + yt}{1-zt}$ where x, y and z are positive constants.

[2 marks]

Answer _____

8

Turn over ►

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8 The random variable X has a discrete uniform distribution and takes values 1, 2, 3, …, n

It is given that $\mathrm{P}(X > 26) = 2\mathrm{P}(X \leq 9)$

8 (a) Find the value of n

[2 marks]

Answer _____

Do not write outside the box

8 (b) The random variable Y is independent of X and $Y \sim \mathrm{B}(34,\ 0.73)$

Find the exact value of $\mathrm{Var}(4X - 10Y + 8)$

[4 marks]

Answer _____

6

Turn over for the next section

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 (a) Given that θ is an angle, use the cosine rule formula to show that $\cos\theta$ is dimensionless.

[2 marks]

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9 (b) A student is answering a mechanics question using the formula

$$U^2 = rg\left(1-\cos\theta\right)$$

where

U ms^{-1} is a speed

r metres is a radius

g ms^{-2} is the acceleration due to gravity

θ is an angle

Show that the formula is dimensionally consistent.

[3 marks]

5

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10 A boat is moving in the sea.

The boat has constant velocity $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ m s^{-1} relative to the sea.

The sea water has constant velocity $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ m s^{-1}

At time $t = 0$ the boat has displacement $\begin{bmatrix} -200 \\ -60 \end{bmatrix}$ metres from a lighthouse.

10 (a) Find the resultant velocity of the boat.

[1 mark]

Answer _____

10 (b) Find the minimum distance between the boat and the lighthouse.

[5 marks]

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Answer _____

6

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11 Two particles A and B are moving towards each other along a straight line on a smooth horizontal surface.

The particles collide.

Particle A has mass 4 kg and particle B has mass 6 kg

Before the collision particle A has speed 5 m s^{-1} and particle B has speed 3 m s^{-1}

After the collision particle B has speed 1.48 m s^{-1}

The coefficient of restitution between the two particles is e

11 (a) Find the value of e

Fully justify your answer.

[5 marks]

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11 (b) Find the magnitude of the impulse exerted on A by B during the collision.

[2 marks]

Answer _____

11 (c) Assume that the particles exert a constant force of 672 newtons on each other during the collision.

Find the time that the particles are in contact with each other.

[2 marks]

Answer _____

9

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