OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Wednesday 3 January 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) (i) It is given that

$$w = (a + 3\mathrm{i})(2 - \mathrm{i})$$

where a is a real constant.

Express w in the form $u + \mathrm{i}v$ where u and v are real.

Give your answer in terms of a

[2 marks]

Answer _____

1 (a) (ii) Hence, or otherwise, express the complex number

$$\frac{a + 3\mathrm{i}}{2 + \mathrm{i}}$$

in the form $x + \mathrm{i}y$ where x and y are real.

Give your answer in its simplest form in terms of a

[2 marks]

Answer _____

Do not write outside the box

1 (b) The complex number z satisfies the equation

$$3z^* + \mathrm{i}z = 23 + 13\mathrm{i}$$

Find z

[5 marks]

$z =$ _____

9

Do not write outside the box

2 (a) Expand $(1+h)^5$

[1 mark]

Answer _____

2 (b) A curve has equation $y = x^5$

2 (b) (i) A line passes through two points on the curve, one where $x = 1$ and the other where $x = 1+h$ with $h > 0$

Find the gradient of this line in the form $a + bh + ch^2 + dh^3 + h^4$ where a, b, c and d are constants.

[3 marks]

Answer _____

Do not write outside the box

2 (b) (ii) Use your answer to **part (b)(i)** to find the gradient of the curve at the point where $x = 1$, showing the limiting process used.

[2 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

3 (a) Find the general solution of the equation

$$\cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$$

Give your answer in terms of π

[3 marks]

Answer _____

3 (b) Find the number of solutions of the equation

$$\cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$$

in the range $-m\pi < x \leq m\pi$ where m is a positive integer.

Give your answer in terms of m

[2 marks]

Answer _____

5

Do not write outside the box

4 By considering the derivative of $y = x^{\frac{1}{4}}$ when $x = 81$, find an estimate for $\sqrt[4]{75}$

Give your answer to three decimal places.

[6 marks]

Answer _____

6

Do not write outside the box

5 (a) Show that

$$\sum_{r=1}^{n}\left(6r^2-4r+1\right)=n^2\left(an+b\right)$$

where a and b are integers.

[4 marks]

Do not write outside the box

5 (b) Hence show that

$$\sum_{r=p+1}^{2p}\left(6r^2-4r+1\right)=p^2\left(cp+d\right)$$

where c and d are integers.

[3 marks]

7

Turn over for the next question

Do not write outside the box

6 The complex numbers α and β are the roots of the quadratic equation

$$z^2 + bz + c = 0$$

where b and c are real constants.

6 (a) (i) Write down b and c in terms of α and β

[2 marks]

$b =$ _____ $c =$ _____

6 (a) (ii) It is given that $\alpha = x + \mathrm{i}y$ where x and y are real and non-zero.

Write down β in terms of x and y

[1 mark]

Answer _____

6 (b) In the case when $b = 6$, the roots α and β are represented by the points P and Q on an Argand diagram.

The number 8 is represented by the point R on the same Argand diagram.

The area of triangle PQR is $11\sqrt{3}$

6 (b) (i) Find α and β

[4 marks]

Do not write outside the box

Answer _____ and _____

6 (b) (ii) Hence find the value of c

[2 marks]

Answer _____

6 (b) (iii) Express α and β in the form $r(\cos\theta + i\sin\theta)$ where $r \geq 0$ and $-\pi < \theta \leq \pi$

[4 marks]

Answer _____ and _____

13

Do not write outside the box

7 A curve C has equation

$$y = \frac{2}{x^2 - 3}$$

7 (a) Find the equations of the three asymptotes of C

[3 marks]

Asymptote 1 _____

Asymptote 2 _____

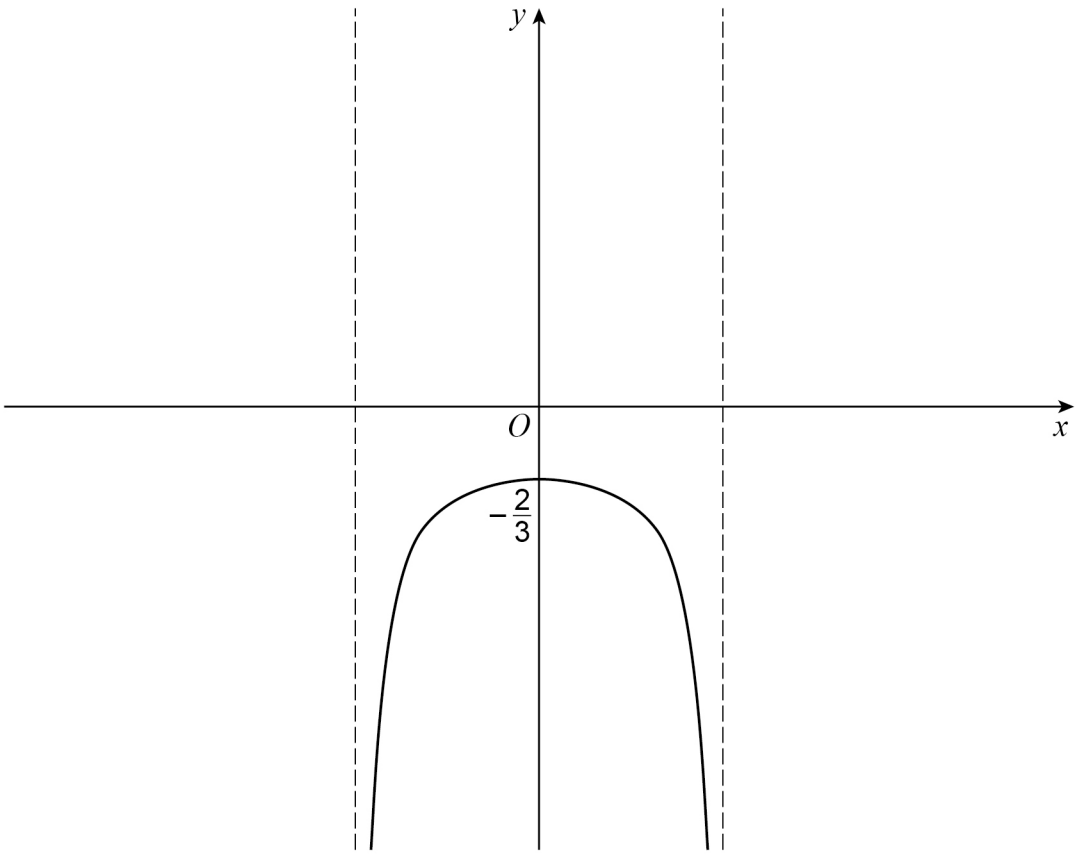
Asymptote 3 _____

7 (b) One section of the graph of C and its vertical asymptotes are shown below.

The y-intercept shown below is the only stationary point of C

On the axes below, complete the sketch of the graph of C

[2 marks]



Do not write outside the box

7 (c) The point $(2, 2)$ is a point of intersection of C with the line $y = x$

Find the coordinates of the other point where the line $y = x$ meets C

[3 marks]

Answer _____

7 (d) Hence sketch the line $y = x$ on the axes in **part (b)**.

[1 mark]

7 (e) Hence, or otherwise, solve the inequality $\frac{2}{x^2 - 3} \geq x$

[3 marks]

Answer _____

12

Do not write outside the box

8 The integral I_n is defined by

$$I_n = \int_0^4 x^n \, \mathrm{d}x$$

where n is a constant.

8 (a) Explain why I_n is an improper integral when $n < 0$

[1 mark]

8 (b) Find the exact value of I_n when $n = -\frac{3}{4}$, showing the limiting process.

[3 marks]

Answer _____

Do not write outside the box

8 (c) Write down a value of n for which I_n does **not** have a finite value.

[1 mark]

$n =$ _____

5

Turn over for the next question

Do not write outside the box

9 The locus of a point P is such that the distance from P to the point $(4, 0)$ is twice the distance from P to the line with equation $x = 1$

The locus of P is the curve C_1

9 (a) Show that the equation of C_1 is

$$\frac{x^2}{m} - \frac{y^2}{n} = 1$$

where m and n are constants.

[4 marks]

9 (b) Write down the equations of the asymptotes of C_1

Give your answers in the form $y = \mathrm{f}(x)$

[2 marks]

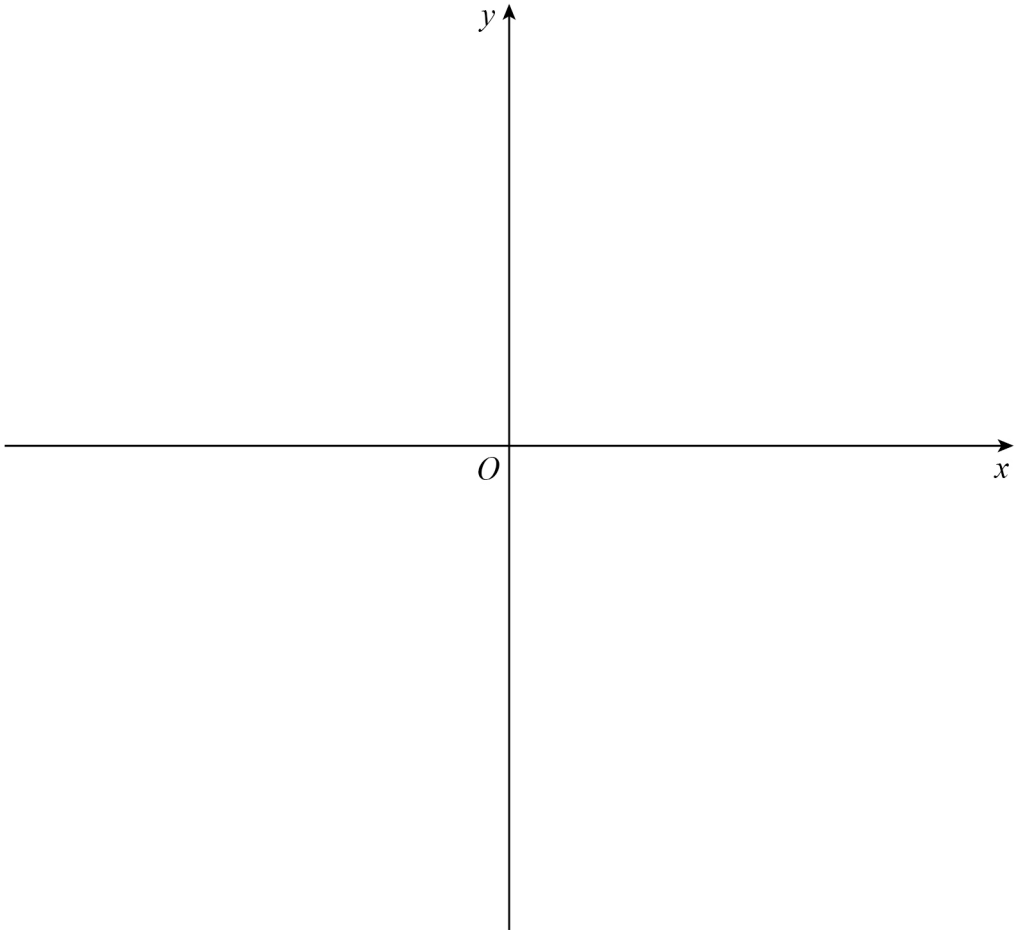
Answer _____

Do not write outside the box

9 (c) Sketch C_1 on the axes below.

Include the asymptotes and label the axis intercepts.

[2 marks]



9 (d) The locus of a point Q is such that the distance from Q to the point $(4, c)$ is twice the distance from Q to the line with equation $x = 1$

The locus of Q is the curve C_2

Write down the equations of the asymptotes of C_2

Give your answer in terms of c

[2 marks]

Answer _____

10

Turn over ►

Do not write outside the box

10 The quadratic equation $2x^2 + x + m = 0$ has roots α and β

The quadratic equation $3x^2 + nx + m = 0$ has roots $\alpha^2\beta$ and $\beta^2\alpha$

The constants m and n are both positive.

10 (a) Find the exact value of m

[3 marks]

Answer _____

Do not write outside the box

10 (b) Find the exact value of n

[4 marks]

Answer _____

7

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED