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FURTHER MATHEMATICS

FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

January 2024

Version: 1.0 Final

241XFM02/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$hf(-2,3) = 0.1\times\dfrac{1}{(-2)^2-3+3}$	M1	correct substitution into RHS of this expression
	$= 0.025$	A1	PI
	$y_2 = 3+0.025 = 3.025$	M1	3 + their value of $hf(-2,3)$
	$y_3 = 3.025+0.1\times\dfrac{1}{(-1.9)^2-3.025+3}$ $[= 3.052894003]$	M1	Correct substitution using their x_2 and y_2 into second term here
	3.0529	A1	Correct answer given to 4 dp

	Question 1 Total	5	

Q	Answer	Marks	Comments
2(a)	$\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	B1	
		1	

Q	Answer	Marks	Comments
2(b)(i)	$\mathbf{BA} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{bmatrix}$	M1	Two correct elements of their **B** multiplied by **A**
	$= \begin{bmatrix} 0.1196 & -0.9928 \\ 0.9928 & 0.1196 \end{bmatrix}$	A1	
		2	

Q	Answer	Marks	Comments
2(b)(ii)	Rotation about origin	B1	**oe** Must be the only transformation mentioned
	$\theta = \cos^{-1}(0.119615)$	M1	**oe**
	$= 1.451$, anticlockwise	A1ft	**AWRT** 1.5 or 83°, sense must be clear. Allow +1.5 **ft** their **BA**, provided it is a valid rotation matrix
		3	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$f\left(\frac{3}{2}\right) = -\frac{5}{16}[= -0.3125]$ **and** $f\left(\frac{7}{4}\right) = \frac{23}{128}[= 0.1796875]$	**M1**	Correct evaluation of a suitable interval
	sign change **&** continuous function, so the root β lies in the interval $\frac{3}{2} < x < \frac{7}{4}$	**A1**	Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	

Q	Answer	Marks	Comments
3(b)	$\frac{x_1 - 3/2}{7/4 - x_1} = \frac{5/16}{23/128}$	**M1**	Obtains correct equation in terms of x_1 **oe** **ft** their values from **(a)**
	$\frac{23}{128}\left(x_1 - \frac{3}{2}\right) = \frac{5}{16}\left(\frac{7}{4} - x_1\right)$ $\frac{23}{128}x_1 - \frac{69}{256} = \frac{35}{64} - \frac{5}{16}x_1$ $\frac{63}{128}x_1 = \frac{209}{256}$	**M1**	Obtains correct linear equation in terms of x_1 **oe** **ft** their values from **(a)**
	$x_1 = \frac{209}{126}$	**A1**	Must be convincingly shown
	$[f(x_1)] = -0.03556$	**M1**	For evaluating their $f(x_1)$
	$f(x_1) < 0$ so $\frac{209}{126} < \beta < \frac{7}{4}$	**A1**	**CSO**
		5	

	Question 3 Total	**7**	

<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td>4(a)</td><td><table>
<tr><td>…</td><td>1.00</td><td>1.18</td><td>1.30</td><td>1.48</td></tr>
<tr><td>…</td><td>0.28</td><td>0.18</td><td>0.11</td><td>0.00</td></tr>
</table></td><td>M1 A1</td><td>M1: At least three correct entries; Condone 1.3 in place of 1.30

A1: All entries correct</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
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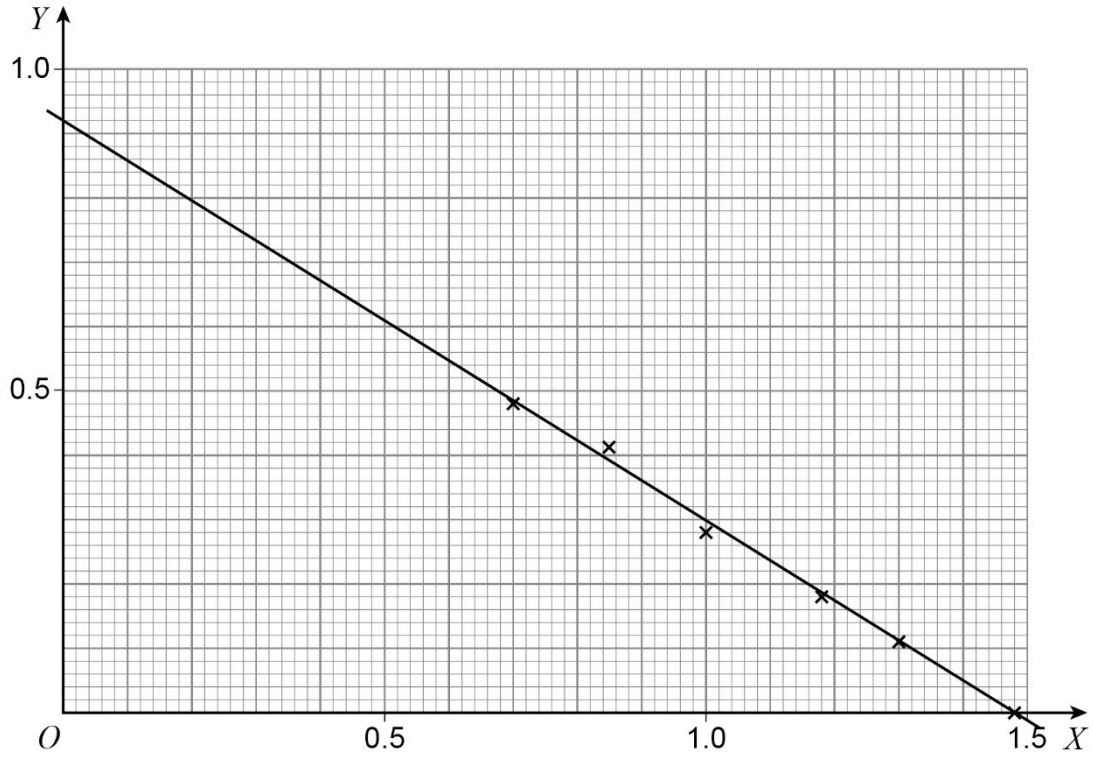
<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td>4(b)</td><td></td><td>B1ft

B1ft</td><td>Points plotted ft their entries within half a square tolerance

Line of best fit seen</td></tr>
<tr><td colspan="4"> 
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Q	Answer	Marks	Comments
4(c)	$\log_{10}Q = \log_{10}a + b\log_{10}P$	**M1**	Takes logs of both sides to reduce to linear form **PI**
	$y-$intercept $= 0.92$ $\left[= \log_{10}a\right]$ gradient $= -0.63$ $\left[= b\right]$	**M1**	Sight of correct y–intercept or gradient value for their line of best fit
	$a = 10^{0.92}$ or $b = -0.63$	**M1**	Sets $a = 10^{\text{their } y\text{-intercept}}$ or $b =$ their gradient **PI**
	$a = 8.3$ $b = -0.63$	**A1**	**AWFW** [7.8, 9.0] for a **AWFW** [–0.68, –0.58] for b
		4	

	Question 4 Total	**8**	

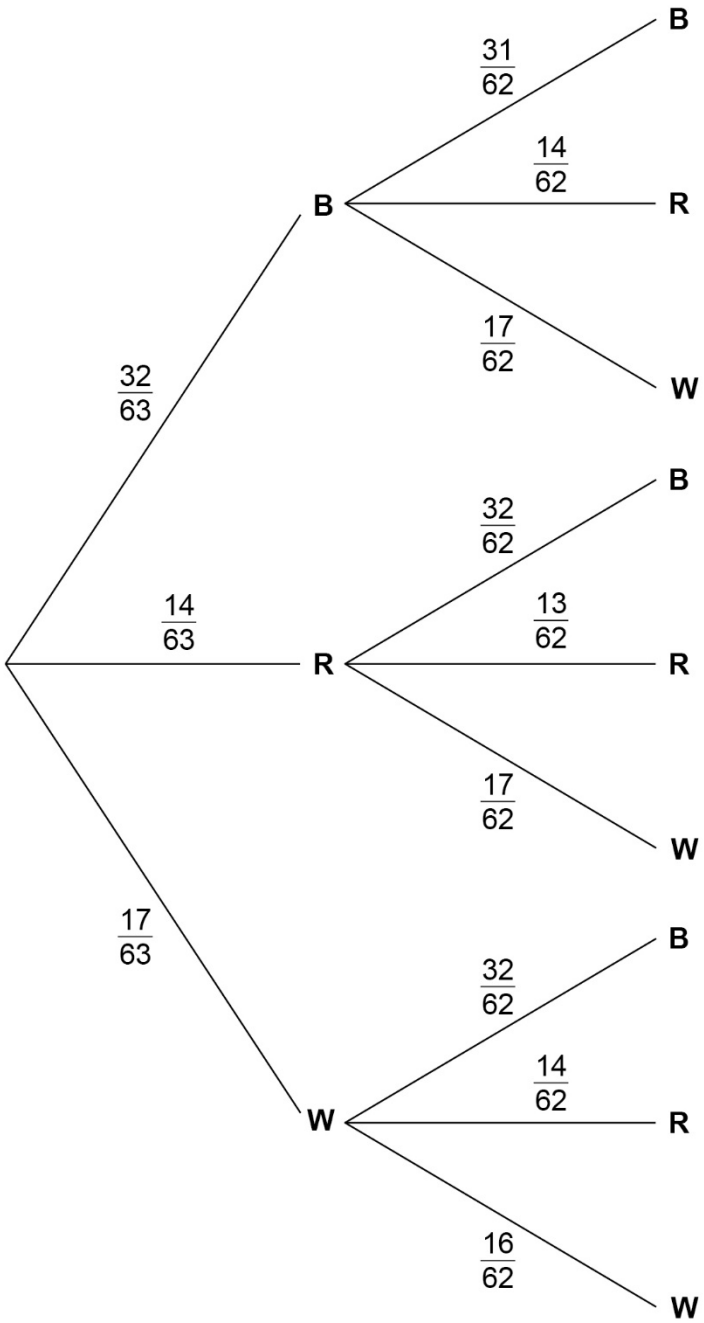
Q	Answer	Marks	Comments
5(a)	$[x' =]0.4x+1.2(mx+c)$ $[y' =]1.2x-1.4(mx+c)$	**M1**	Finds correct expressions for x' and y'
	$1.2x-1.4(mx+c)$ $=m(0.4x+1.2(mx+c))+c$	**M1**	Sets their $y'=m(\text{their } x')+c$
	$1.2m^2+1.8m-1.2=0$ $-1.4c-1.2mc-c=0$	**m1**	Attempt to find m or c by comparing coefficients
	$m=\frac{1}{2}$, $m=-2$	**A1**	Correct values of m
	lines are $y=-2x+c$ [where c is real]	**A1**	No restrictions on c
	$y=\frac{1}{2}x$	**B1**	
		6	

Q	Answer	Marks	Comments
5(b)	$0.4x+1.2y=x$ and $1.2x-1.4y=y$	**M1**	Or uses **5(a)** and demonstrates that $\begin{bmatrix}0.4 & 1.2\\ 1.2 & -1.4\end{bmatrix}\begin{bmatrix}x\\ 0.5x\end{bmatrix}=\begin{bmatrix}x\\ 0.5x\end{bmatrix}$ **PI** by correct solution
	$\Rightarrow y=\frac{1}{2}x$	**A1**	
		2	

Q	Answer	Marks	Comments
5(c)	$\tan\theta = \frac{1}{2}[\Rightarrow \theta = 0.4636]$ $\Rightarrow \cos 2\theta = 0.6$ and $\sin 2\theta = 0.8$	M1	Uses $\tan\theta = \frac{1}{2}$ to obtain values of $\cos 2\theta$ and $\sin 2\theta$ **PI** by correct matrix for reflection in $y = \frac{1}{2}x$
	$\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$	A1	Obtains matrix for reflection in $y = \frac{1}{2}x$
	so $\mathbf{M} = \mathbf{N}\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$	M1	Sets up correct equation for **N** **PI** by a correct expression for **N** later
	$\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}^{-1} = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$	B1ft	Clearly states that the reflection is self-inversing or find the correct inverse of their matrix for reflection in $y = \frac{1}{2}x$ **PI**
	$\mathbf{N} = \begin{bmatrix} 0.4 & 1.2 \\ 1.2 & -1.4 \end{bmatrix}\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$	M1	**Post**-multiplies **M** by this inverse **PI**
	$\mathbf{N} = \begin{bmatrix} 1.2 & -0.4 \\ -0.4 & 1.8 \end{bmatrix}$	A1	**oe**
		6	

	Question 5 Total	14	

Q	Answer	Marks	Comments
6(a)		**M1**	Correct structure
		A1ft	Probabilities on one set of branches correct or correct follow through from previous branches **oe**
		A1	Fully correct **oe**



		3	

Q	Answer	Marks	Comments
6(b)	$\dfrac{\frac{32}{63}\times\frac{17}{62}}{\frac{32}{63}\times\frac{17}{62}+\frac{14}{63}\times\frac{17}{62}+\frac{17}{63}\times\frac{16}{62}}$	M1	Correct numerator for their probabilities Sight of $\frac{272}{1953}$
		M1	Correct denominator for their probabilities **oe** eg $\frac{272}{1953}+\frac{17}{279}+\frac{136}{1953}$ or $\frac{17}{63}$
	$=\frac{16}{31}$	A1	**oe**
		3	

	Question 6 Total	6	

Q	Answer	Marks	Comments
7(a)	$(1-0.07)^3 \times 0.07$ $= 0.0563$	 **B1**	 **AWRT** 0.0563
		1	

Q	Answer	Marks	Comments
7(b)	$(1-0.07)^3$ $= 0.804357$	**M1** **A1**	Attempts correct calculation for either $P(N > 3)$ or $P(N \geq 3)$ **AWRT** 0.8044
		2	

Q	Answer	Marks	Comments
7(c)	$G_N(t) = \sum_{n=1}^{\infty} t^n \times 0.07 \times 0.93^{n-1}$ $= 0.07t \times (1 + 0.93t + 0.93^2t^2 + \ldots)$ $= \frac{0.07t}{1-0.93t}$	**M1** **M1** **A1**	Correct expression for $G_N(t)$ If written term by term, minimum that must be seen is $G_N(t) = 0.07t + 0.07(1-0.07)t^2 + 0.07(1-0.07)^2t^3 + \ldots$ Allow use of p for 0.07 **PI** Identify an infinite geometric series by either identifying kt $(k \neq 0)$ as a factor or substituting their a and r into $\frac{a}{1-r}$ **PI** **oe**
		3	

Q	Answer	Marks	Comments
7(d)	$G_{M+N}(t) = \frac{0.07t}{1-0.93t} \times (0.07t + 0.93)$ $G_{M+N}(t) = \frac{0.0049t^2 + 0.0651t}{1-0.93t}$	**M1** **A1**	Applies $G_{M+N}(t) = G_M(t) \times G_N(t)$ with their $G_N(t)$ **oe**
		2	

	Question 7 Total	**8**	

Q	Answer	Marks	Comments
8(a)	$\frac{n-26}{n} = 2 \times \frac{9}{n}$	M1	Forms correct equation
	$n = 44$	A1	If **M0** awarded, **SC1** for final answer of $n = 43$
		2	

Q	Answer	Marks	Comments
8(b)	$\text{Var}(X) = \frac{44^2 - 1}{12} = 161.25$	B1ft	**oe** **ft** their n **PI**
	$\text{Var}(Y) = 34 \times 0.73(1-0.73) = 6.7014$	B1	**PI**
	$\text{Var}(4X - 10Y + 8)$ $= 4^2\text{Var}(X) + 10^2\text{Var}(Y)$ $= 4^2 \times 161.25 + 10^2 \times 6.7014$	M1	Applies correct formula for $\text{Var}(4X - 10Y + 8)$ to their $\text{Var}(X)$ and $\text{Var}(Y)$
	$\text{Var}(4X - 10Y + 8) = 3250.14$	A1	**oe**
		4	

	Question 8 Total	6	

Q	Answer	Marks	Comments
9(a)	$\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$ $\left[\cos\theta\right] = \dfrac{\left[a^2+b^2-c^2\right]}{\left[2ab\right]} = \dfrac{L^2}{L^2} = L^0 = 1$	M1	Applies dimensional analysis to the cosine formula and finds either $\left[a^2+b^2-c^2\right]=L^2$ or $\left[2ab\right]=L^2$ Condone $L\times L$
	Therefore $\cos\theta$ is dimensionless	A1	Finds $\left[a^2+b^2-c^2\right]=L^2$ and $\left[2ab\right]=L^2$ and concludes that $\cos\theta$ is dimensionless
		2	

Q	Answer	Marks	Comments
9(b)	$\left[U^2\right] = L^2T^{-2}$	M1	Finds dimensions of LHS Condone use of units
	$\left[rg(1-\cos\theta)\right] = \left[r\right]\left[g\right]\left[(1-\cos\theta)\right]$ $= L\times LT^{-2}\times 1$ $= L^2T^{-2}$	M1	Finds dimensions of RHS Condone use of units
	Therefore dimensionally consistent	A1	Obtains correct dimensions of both sides and concludes dimensionally consistent
		3	

	Question 9 Total	5	

Q	Answer	Marks	Comments
10(a)	Resultant velocity $= \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $= \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\left[\text{m s}^{-1}\right]$	**B1**	Correct resultant velocity Do not ignore subsequent working
		1	

Q	Answer	Marks	Comments
10(b)	$\mathbf{r} = \begin{bmatrix} 3t - 200 \\ t - 60 \end{bmatrix}$	**M1**	Finds position of boat relative to lighthouse **ft** their resultant velocity
	$s^2 = (3t-200)^2 + (t-60)^2$ $= 10t^2 - 1320t + 43600$	**M1**	Finds distance or distance^2 between the boat and the lighthouse **ft** their position of boat relative to lighthouse
	$\frac{\mathrm{d}s^2}{\mathrm{d}t} = 20t - 1320 = 0$	**M1**	Uses a method to find the minimum distance (calculus or completing the square) for their quadratic **PI** by correct final answer
	$t = \frac{1320}{200} = 66$	**A1**	Correct time or correct completion of the square $10\left(t-66\right)^2 + 40$ **PI** by correct final answer
	$s^2 = (3\times 66 - 200)^2 + (66-60)^2$ $= 40$ $s = 2\sqrt{10}$ [metres]	**A1**	Correct minimum distance **AWRT** 6.3
		5	

	Question 10 Total	**6**	

Q	Answer	Marks	Comments
11(a)	There is a net momentum to the right, so at least B must move to the right after the collision $\therefore\ v_B = 1.48$	**B1**	Any equivalent correct statement
	$4v_A + 6\times 1.48 = 4\times 5 - 6\times 3$ $v_A = \dfrac{2-8.88}{4} = -1.72$	**M1 A1**	**M1**: An equation for conservation of momentum using one of $v_B = -1.48$ or $v_B = 1.48$ **A1**: $v_A = -1.72$ or $v_A = 2.72$ **oe**
	$1.48-(-1.72) = -e(-3-5)$	**m1**	Correct equation for coefficient of restitution **ft** their v_A and using one of $v_B = -1.48$ or $v_B = 1.48$
	$e = \dfrac{3.2}{8} = 0.4$	**A1**	Correct coefficient of restitution
		5	

Q	Answer	Marks	Comments
11(b)	$I = 4\times(-1.72) - 4\times 5$	**M1**	Uses formula for impulse Condone sign errors
	$= 26.88\ [\text{N s}]$	**A1**	Correct magnitude. Must be positive **AWRT** 27
		2	

Q	Answer	Marks	Comments
11(c)	$26.88 = 672t$	**M1**	Uses $I = Ft$
	$t = \dfrac{26.88}{672} = 0.04\ [\text{seconds}]$	**A1ft**	Correct time for their impulse
		2	

	Question 11 Total	**9**	