

INTERNATIONAL AS FURTHER MATHEMATICS FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

January 2024

Version: 1.0 Final



2 4 1 X F M 0 1 / M S

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)(i)	$(a+3i)(2-i) = 2a - ai + 6i + 3$ $= 2a + 3 + i(6-a)$	M1 A1	Expands the brackets and replaces i^2 with -1 oe with real and imaginary parts collected and i written as a factor of the imaginary part Accept $u = 2a + 3$ and $v = 6 - a$ Condone $2a + 3 - i(a - 6)$ ISW
		2	

Q	Answer	Marks	Comments
1(a)(ii)	$\frac{a+3i}{2+i} = \frac{(a+3i)(2-i)}{(2+i)(2-i)}$ $= \frac{2a+3 + (6-a)i}{5}$ $= \frac{2a+3}{5} + i\frac{6-a}{5}$	M1 A1ft	multiplies both the numerator and the denominator by $2-i$ or writes the fraction equivalent to $x+iy$, multiplies by the denominator and equates real and imaginary parts ft their part (a)(i) oe with real and imaginary parts collected and i written as a factor of the imaginary part Accept $x = \frac{2a+3}{5}$ and $y = \frac{6-a}{5}$ Condone $\frac{2a+3}{5} - i\frac{a-6}{5}$ ISW
		2	

Q	Answer	Marks	Comments
1(b)	Let $z = c + id$ where $c, d \in \mathbb{R}$ $3(c - id) + i(c + id) = 23 + 13i$ $3c - 3id + ci - d = 23 + 13i$ $3c - d = 23$ and $c - 3d = 13$ $c = 7$ or $d = -2$ $z = 7 - 2i$	M1 B1 M1 PI A1 A1	Uses a suitable method Uses the relationship between z and z^* Compares real and imaginary parts to form simultaneous equations in c and d where $z = c + id$ PI Finds the value of c or d PI oe
		5	

	Question 1 Total	9	
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Q	Answer	Marks	Comments
2(a)	$(1+h)^5 = 1+5h+10h^2+10h^3+5h^4+h^5$	B1	oe expanded and simplified polynomial
		1	

Q	Answer	Marks	Comments
2(b)(i)	$(1+h)^5 - 1^5 = 5h+10h^2+10h^3+5h^4+h^5$ gradient of line $= \frac{(1+h)^5 - 1^5}{(1+h)-1}$ $= \frac{5h+10h^2+10h^3+5h^4+h^5}{h}$ $= 5+10h+10h^2+5h^3+h^4$	M1 M1 A1	Subtracts 1 from their part (a) polynomial Obtains an expression for the gradient of the line Accept any correct form ft their part (a) oe polynomial Accept $a=5$, $b=10$, $c=10$, $d=5$
		3	

Q	Answer	Marks	Comments
2(b)(ii)	gradient of curve $= \lim_{h \rightarrow 0} (5+10h+10h^2+5h^3+h^4)$ $= 5$	M1 A1ft	Considers their part (b)(i) as $h \rightarrow 0$ Obtains the correct limit of their part (b)(i) as $h \rightarrow 0$ ft their $a+bh+ch^2+dh^3+h^4$ SC1 for 5 following $h=0$
		2	

	Question 2 Total	6	
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Q	Answer	Marks	Comments
3(a)	$\left[x + \frac{\pi}{4} = \right] \frac{\pi}{6}$ $x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{6}$ where $n \in \mathbb{Z}$ $x = 2n\pi \pm \frac{\pi}{6} - \frac{\pi}{4}$	B1 M1 A1	Finds one solution to $x + \frac{\pi}{4}$ Accept any correct angle in radians PI oe Removes the trig function to form a general equation Condone n not defined oe Finds a general solution May be written in two parts eg $x = n\pi - \frac{\pi}{12}$ for even n and $x = n\pi + \frac{7\pi}{12}$ for odd n
		3	

Q	Answer	Marks	Comments
3(b)	Two solutions for every 2π radians of the interval Number of solutions $= 2m$	M1 A1	PI
		2	

	Question 3 Total	5	
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Q	Answer	Marks	Comments
4	$\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$ $\frac{dy}{dx} = \frac{1}{4} \times 81^{-\frac{3}{4}}$ $= \frac{1}{108}$ $\delta y \approx \frac{dy}{dx} \times \delta x$ $\delta y \approx \frac{1}{108} \times -6$ $\delta y \approx -\frac{1}{18}$ $y \approx 81^{\frac{1}{4}} - \frac{1}{18}$ $y \approx 2.944$	M1 A1 M1 A1ft M1 A1	Correctly differentiates $x^{\frac{1}{4}}$ Substitutes $x = 81$ into a correct derivative PI May be unsimplified Sight or use of $\delta y \approx \frac{dy}{dx} \times \delta x$ PI Condone use of = sign Multiplies their $\frac{dy}{dx}$ (of the form $ax^{-\frac{3}{4}}$) by -6 (or 6) May be algebraic Full method for the required estimate from a derivative of the form $ax^{-\frac{3}{4}}$ PI CAO Do not condone $\frac{53}{18}$

	Question 4 Total	6	
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Q	Answer	Marks	Comments
5(a)	$\sum_{r=1}^n (6r^2 - 4r + 1) = 6 \sum_{r=1}^n r^2 - 4 \sum_{r=1}^n r + \sum_{r=1}^n 1$ $= 6 \times \frac{1}{6} n(n+1)(2n+1) - 4 \times \frac{1}{2} n(n+1) + n$ $= n[(n+1)(2n+1) - 2(n+1) + 1]$ $= n(2n^2 + 3n + 1 - 2n - 2 + 1)$ $= n(2n^2 + n)$ $= n^2(2n+1)$	M1 M1 M1 A1	Writes in terms of standard formulae Condone one error PI Replaces $\sum r^2$ with $\frac{1}{6}n(n+1)(2n+1)$ and $\sum r$ with $\frac{1}{2}n(n+1)$ Replaces $\sum 1$ with n Correct expression following M1M1M1
		4	

Q	Answer	Marks	Comments
5(b)	$\sum_{r=p+1}^{2p} (6r^2 - 4r + 1)$ $= \sum_{r=1}^{2p} (6r^2 - 4r + 1) - \sum_{r=1}^p (6r^2 - 4r + 1)$ $= (2p)^2(4p+1) - p^2(2p+1)$ $= p^2(16p+4-2p-1)$ $= p^2(14p+3)$	M1 m1 A1	Correctly splits the required expression into the difference of two sums of the form $\sum_{r=1}^n$ PI Writes the required expression in terms of p ft their part (a) in terms of n
		3	

	Question 5 Total	7	
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Q	Answer	Marks	Comments
6(a)(i)	$b = -(\alpha + \beta)$	B1	
	$c = \alpha\beta$	B1	
		2	

Q	Answer	Marks	Comments
6(a)(ii)	$[\beta =] x - iy$	B1	
		1	

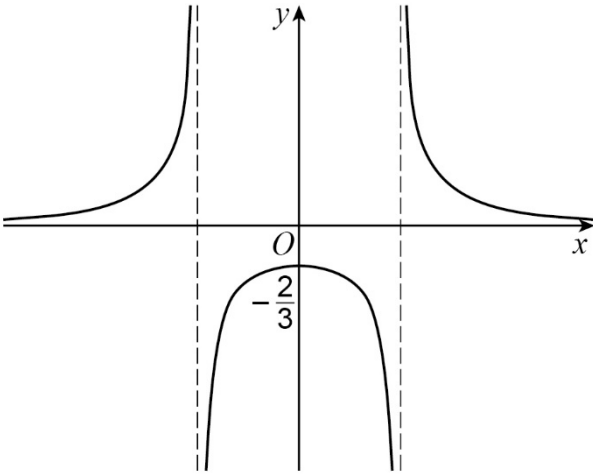
Q	Answer	Marks	Comments
6(b)(i)	Let α and β be $x \pm iy$ $x + iy + x - iy = -6$ $2x = -6$ $x = -3$	M1	Forms a correct equation in x
	Area = $\frac{1}{2} \times 2y \times (8 - x) = 11\sqrt{3}$	M1	Forms a correct equation in y
	$y = \sqrt{3}$	A1	ft their x
	$[\alpha =] -3 + i\sqrt{3}$ and $[\beta =] -3 - i\sqrt{3}$	A1	
		4	

Q	Answer	Marks	Comments
6(b)(ii)	$\frac{c}{1} = (-3 + i\sqrt{3})(-3 - i\sqrt{3})$	M1	Forms an equation in c
	$[c =] 12$	A1ft	ft a conjugate pair
		2	

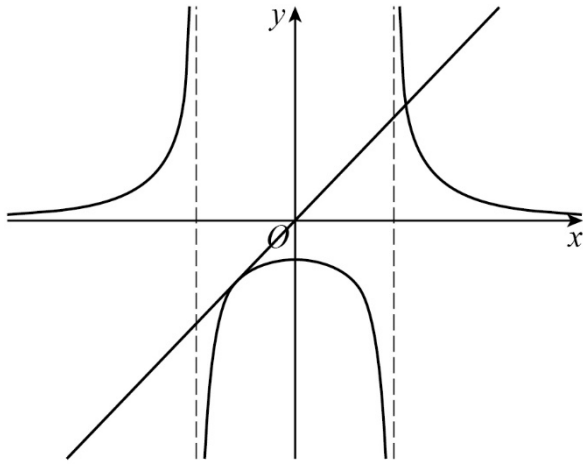
Q	Answer	Marks	Comments
6(b)(iii)	$[r =] \sqrt{(-3)^2 + (\sqrt{3})^2} \quad [= \sqrt{12}]$	M1	Correct calculation of the modulus of at least one of their roots
	$[\theta_\alpha =] \tan^{-1}\left(\frac{\sqrt{3}}{-3}\right) + \pi \quad \left[= \frac{5\pi}{6}\right]$	M1	Full method for the argument of one of their roots Accept a correct equivalent radian angle outside the interval $-\pi < \theta \leq \pi$
	$2\sqrt{3}\left(\cos\left(\frac{5\pi}{6}\right) + i \sin\left(\frac{5\pi}{6}\right)\right)$	A1	Accept $\sqrt{12}$ for $2\sqrt{3}$
	$2\sqrt{3}\left(\cos\left(-\frac{5\pi}{6}\right) + i \sin\left(-\frac{5\pi}{6}\right)\right)$	A1	
		4	

	Question 6 Total	13	
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Q	Answer	Marks	Comments
7(a)	$x = \sqrt{3}$	B1	Award a maximum of B2 if any errors are included, eg an extra asymptote
	$x = -\sqrt{3}$	B1	
	$y = 0$	B1	
		3	

Q	Answer	Marks	Comments
7(b)		M1	Any two of: $y \rightarrow 0^+$ as $x \rightarrow \infty$ or $y \rightarrow 0^+$ as $x \rightarrow -\infty$ or $y \rightarrow \infty$ as $x \rightarrow \sqrt{3}^+$ or $y \rightarrow \infty$ as $x \rightarrow -\sqrt{3}^-$
		A1	Can be drawn or written Accept implied asymptotic behaviour Accept feathering Both sections drawn correctly Do not accept feathering – must be one continuous curve for each of the two sections Must clearly show correct asymptotic behaviour for all four of $x \rightarrow \infty$, $x \rightarrow -\infty$, $x \rightarrow \sqrt{3}^+$, $x \rightarrow -\sqrt{3}^-$
		2	

Q	Answer	Marks	Comments
7(c)	$x = \frac{2}{x^2 - 3}$ $x^3 - 3x - 2 = 0$ $(x - 2)(x^2 + 2x + 1) = 0$ The other intersection point(s) occur when $x^2 + 2x + 1 = 0$ $x = -1 \text{ is a repeated root}$ $(-1, -1)$	M1 M1 A1	Forms a cubic equation PI Correct solution (other than $x = 2$) of their cubic equation PI Correct intersection point Accept $x = -1$ and $y = -1$ unambiguously written as the required coordinates Ignore $(2, 2)$ included
		3	

Q	Answer	Marks	Comments
7(d)		B1	Draws a tangent through the origin, intersecting the curve in the 1st and 3rd quadrants Coordinates of intersection not needed for this mark
		1	

Q	Answer	Marks	Comments
7(e)	$x < -\sqrt{3}$	B1	Condone $\sqrt{3} < x < 2$ for $\sqrt{3} < x \leq 2$ if only one (or neither) of the previous B1 s has been awarded
	$x = -1$	B1	
	$\sqrt{3} < x \leq 2$	B1	Award a maximum of B2 if any errors are included, eg an extra region
		3	

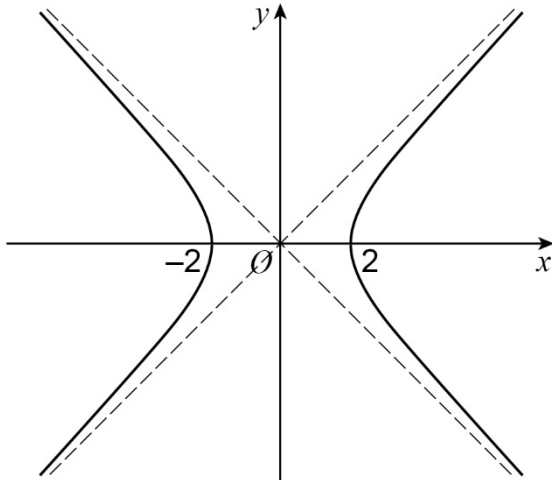
	Question 7 Total	12	
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Q	Answer	Marks	Comments
8(a)	The integrand is not defined for $x = 0$ [when $n < 0$]	E1	oe Condone 'infinite' for 'not defined' Accept 0^n is not defined (or infinite)
		1	

Q	Answer	Marks	Comments
8(b)	$I_{\frac{3}{4}} = \int_0^4 x^{-\frac{3}{4}} dx$ $= \lim_{a \rightarrow 0^+} \left(\int_a^4 x^{-\frac{3}{4}} dx \right)$ $= \lim_{a \rightarrow 0^+} \left[4x^{\frac{1}{4}} \right]_a^4$ $= \lim_{a \rightarrow 0^+} \left(4 \times 4^{\frac{1}{4}} - 4 \times a^{\frac{1}{4}} \right)$ $= 4 \times 4^{\frac{1}{4}} - 0$ $= 4\sqrt{2}$	M1 m1 A1	Replaces lower limit with a variable Correct integration and limiting process shown oe eg $4\sqrt[4]{4}$ NMS scores 1 mark out of 3 SC2 for correct integration and correct answer without use of limiting process Condone $a \rightarrow 0$ throughout
		3	

Q	Answer	Marks	Comments
8(c)	Accept any value of n where $n \leq -1$ eg $n = -1$ or $n = -\frac{5}{2}$ etc	B1	Writes down any number in the range $n \leq -1$ Accept a range of numbers within the range $n \leq -1$ Condone $-\infty$ or ∞
		1	

	Question 8 Total	5	
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Q	Answer	Marks	Comments
9(c)		B1 Correct shape Must include the asymptotes Accept asymptotes drawn as continuous lines B1ft Correct axis intercepts with no incorrect axis intercepts ft their m from part (a)	
		2	

Q	Answer	Marks	Comments
9(d)	$y = \pm x\sqrt{3} + c$	M1 Accept any vertical translation of one of the asymptotes A1ft Accept any correct form ft their part (b) of the form $y = \pm kx$	
		2	

	Question 9 Total	10	
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Q	Answer	Marks	Comments
10(a)	$\alpha\beta = \frac{m}{2}$ or $\alpha^2\beta\beta^2\alpha = \frac{m}{3}$ $\left(\frac{m}{2}\right)^3 = \frac{m}{3}$ $3m^3 = 8m$ $m^2 = \frac{8}{3}$ or $m = 0$ but $m > 0$, so $m = \frac{2}{3}\sqrt{6}$	B1 M1 A1	Uses the product of roots rule on at least one equation Forms an equation in m only oe eg $\sqrt{\frac{8}{3}}$
		3	

Q	Answer	Marks	Comments
10(b)	$\alpha + \beta = -\frac{1}{2}$ or $\alpha^2\beta + \beta^2\alpha = -\frac{n}{3}$ $\alpha\beta(\alpha + \beta) = -\frac{n}{3}$ $\frac{m}{2} \times -\frac{1}{2} = -\frac{n}{3}$ $3m = 4n$ $3 \times \frac{2}{3}\sqrt{6} = 4n$ $n = \frac{1}{2}\sqrt{6}$	B1 M1 m1 A1	Uses the sum of roots rule on at least one equation May be seen in part (a) Forms a correct equation in m and n May be seen in part (a) PI Forms an equation in n only oe eg $\sqrt{\frac{3}{2}}$
		4	

	Question 10 Total	7	
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