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FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$2\begin{bmatrix}4\\7\end{bmatrix} + 6\begin{bmatrix}6\\-4\end{bmatrix} = 2\begin{bmatrix}7\\1\end{bmatrix} + 6\mathbf{v}_B$	**M1**	Applies conservation of momentum in a vector form.
		A1	Correct vector equation
	$6\mathbf{v}_B = \begin{bmatrix}30\\-12\end{bmatrix}$	**m1**	Solves their vector equation for velocity of B
	$\mathbf{v}_B = \begin{bmatrix}5\\-2\end{bmatrix}\ [\text{m s}^{-1}]$	**A1**	Correct velocity
		4	

Q	Answer	Marks	Comments
1(b)	$\mathbf{I} = 2\begin{bmatrix}7\\1\end{bmatrix} - 2\begin{bmatrix}4\\7\end{bmatrix}$	**M1**	Uses impulse equation with vectors
	$= \begin{bmatrix}6\\-12\end{bmatrix}$	**A1**	Correct impulse
	$I = \sqrt{6^2 + 12^2}$	**M1**	Finds magnitude of their impulse
	$= 6\sqrt{5}\ [\text{N s}]$	**A1**	Correct magnitude of the impulse
		4	

	Question 1 Total	8	

Q	Answer	Marks	Comments
2(a)	$4 = \dfrac{2\pi}{\omega}$ $\omega = \dfrac{\pi}{2}$	**M1**	Correct value of ω
	$v_{\max} = \dfrac{3}{4} \times \dfrac{\pi}{2} = \dfrac{3\pi}{8} \quad \left[\text{m s}^{-1}\right]$	**A1**	Correct speed **oe** in terms of π (eg 0.375π)
		2	

Q	Answer	Marks	Comments
2(b)	$v^2 = \omega^2\left(a^2 - x^2\right)$ $v^2 = \left(\dfrac{\pi}{2}\right)^2\left(0.75^2 - 0.45^2\right)$	**M1**	Uses SHM speed formula or other SHM method with their amplitude and consistent displacement.
	$v = \dfrac{3\pi}{10} \quad \left[\text{m s}^{-1}\right]$	**A1ft**	Correct speed for their amplitude. **oe** in terms of π (eg 0.3π)
		2	

Q	Answer	Marks	Comments
2(c)	$x = \dfrac{3}{4}\cos\left(\dfrac{\pi t}{2}\right)$	**M1**	Finds expression for displacement in terms of time
	$0.45 = \dfrac{3}{4}\cos\left(\dfrac{\pi t}{2}\right)$	**A1ft**	Correct equation for time, for their amplitude.
	$t = \dfrac{2}{\pi}\cos^{-1}\left(\dfrac{3}{5}\right)$		
	$t = 0.5903 \ \left[\text{seconds}\right]$	**A1**	Correct time
		3	

	Question 2 Total	**7**	

Q	Answer	Marks	Comments
3(a)	$8\cos 60° = v\cos 30°$ $4 = \frac{\sqrt{3}}{2}v$ $v = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$	**M1 A1** **A1**	**M1**: Equation for motion parallel to wall with at least one side correct **A1**: Fully correct Correct value for v **ACF**
		3	

Q	Answer	Marks	Comments
3(b)	$e \times 8\sin 60° = v\sin 30°$ $e \times 4\sqrt{3} = \frac{4\sqrt{3}}{3}$ $e = \frac{1}{3}$	**M1 A1ft** **A1**	**M1**: Equation for motion perpendicular to wall with at least one side correct **A1ft**: Fully correct **ft** their v Correct value for e **ACF**
		3	

Q	Answer	Marks	Comments
3(c)	$I = 0.2 \times \frac{4\sqrt{3}}{3} - 0.2 \times \left(-4\sqrt{3}\right)$ $= \frac{16\sqrt{3}}{15}$ [N s]	**M1 A1** **A1**	**M1**: Impulse equation with correct values and any signs **A1**: Fully correct Correct impulse **ACF**
		3	

	Question 3 Total	**9**	

Q	Answer	Marks	Comments
4	At highest point the tension in the string is zero when U is the minimum possible value		
	$m \times 9.8 = \frac{mv^2}{0.8}$	**M1**	Equation for motion at highest point
	$v^2 = 7.84$		
	$v = 2.8$	**A1**	Correct speed at highest point ($v^2 = rg$)
	Conservation of energy $\frac{1}{2} \times mU^2 = \frac{1}{2} \times m \times 7.84 + m \times 9.8 \times 1.6$	**M1 A1**	**M1**: Three term equation for conservation of energy **A1**: Fully correct
	$U^2 = 39.2$		($U^2 = 5rg$)
	$U = \frac{14\sqrt{5}}{5} = 6.3$ [to 2sf]	**A1**	Correct value for U **AWRT** 6.3

	Question 4 Total	**5**	

Q	Answer	Marks	Comments
5	$m\dfrac{\mathrm{d}v}{\mathrm{d}t} = mg\sin 30° - \mu \times mg\cos 30° - mkv$ $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{g}{2} - \mu g\dfrac{\sqrt{3}}{2} - kv$	**M1**	Four term differential equation from Newton's second law
	$\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{1}{2}\left(g - \mu g\sqrt{3} - 2kv\right)$	**A1**	Correct differential equation
	$\displaystyle\int \frac{1}{\left(2kv - g + \mu g\sqrt{3}\right)}\,\mathrm{d}v = \int -\frac{1}{2}\,\mathrm{d}t$	**M1**	Uses separation of variables or integrating factor
	$\dfrac{1}{2k}\ln\left(2kv - g + \mu g\sqrt{3}\right) = -\dfrac{1}{2}t + c$	**m1** **A1**	Integrates Correct integration
	$v = 0, t = 0 \Rightarrow c = \dfrac{1}{2k}\ln\left(\mu g\sqrt{3} - g\right)$	**A1**	Finds correct constant of integration
	$\ln\left(2kv - g + \mu g\sqrt{3}\right) = -kt + \ln\left(\mu g\sqrt{3} - g\right)$ $\dfrac{2kv - g + \mu g\sqrt{3}}{\mu g\sqrt{3} - g} = \mathrm{e}^{-kt}$	**M1**	Solves a logarithmic equation for v
	$v = \dfrac{g}{2k}\left(1 - \mu\sqrt{3}\right)\left(1 - \mathrm{e}^{-kt}\right)$	**A1**	Correct expression for v **ACF** not involving trigonometric functions

	Question 5 Total	**8**	

Q	Answer	Marks	Comments
6(a)	Let θ be the angle between the string and the vertical at time t		
	$m\times 1.4\ \ddot{\theta} = -m\times 9.8\sin\theta$	**M1**	Forms differential equation
		A1	Correct differential equation
	$\sin\theta \approx \theta$	**B1**	Uses small angle approximation
	$\ddot{\theta} \approx -\frac{9.8}{1.4}\theta = -7\theta$ $\therefore$ SHM	**A1**	Simplifies to SHM form Accept $\ddot{\theta} \approx -\frac{g}{l}\theta$
	As acceleration proportional to the displacement and in the opposite sense	**E1**	Concludes that motion is SHM from correct working
		5	

Q	Answer	Marks	Comments
6(b)	Period $= \frac{2\pi}{\sqrt{7}} = \frac{2\sqrt{7}}{7}\pi \quad [= 2.3748...]$	**B1**	Correct period stated during working
	Total angle for 1.8 metres $= \frac{1.8}{1.4} = \frac{9}{7} \quad [= 1.286 \text{ radians}]$	**M1**	Finds total angle for the motion
	Angle per complete period $= 4 \times \frac{\pi}{20} = \frac{\pi}{5} \quad [= 0.6283... \text{ radians}]$ $\left(\frac{9}{7}\right) \div \left(\frac{\pi}{5}\right) = \frac{45}{7\pi} \quad [= 2.04627...]$	**A1**	Obtains exact value or **AWRT** 2.047
	[So between 2 and 3 periods required.] $\frac{9}{7} - \frac{2\pi}{5} \quad [= 0.029077... \text{ radians}]$	**A1**	Correct angle for remaining motion since last complete period or correct distance (0.04071…)
	$\theta = \frac{\pi}{20} - \left(\frac{9}{7} - \frac{2\pi}{5}\right) = \frac{9\pi}{20} - \frac{9}{7} \quad [= 0.12800...]$	**M1** **A1**	Finds θ Correct θ
	$\frac{9\pi}{20} - \frac{9}{7} = \frac{\pi}{20}\cos\left(\sqrt{7}t\right)$ $t = 0.23368...$ seconds	**M1**	Finds time for motion since last period
	Total Time $= 2 \times \frac{2\pi}{\sqrt{7}} + 0.23368$ $= 4.98332...$ $= 5.0$ seconds	**A1**	Correct total time **AWRT** 5.0
		8	

	Question 6 Total	**13**	

Q	Answer	Marks	Comments
7(a)	$e = \sqrt{5^2 + 3^2} - 5$ $= \sqrt{34} - 5$	**M1** **A1**	Finds extension of strings Correct extension
	$\text{EPE} = \dfrac{800 \times \left(\sqrt{34} - 5\right)^2}{2 \times 5}$ $= 55.238...$	**M1**	Uses EPE formula with their extension
	Total EPE $= 110$ [J]	**A1**	Correct total EPE
		4	

Q	Answer	Marks	Comments
7(b)	Strings slack when height above C is 1 metre	**B1**	Finds height when strings at natural length **PI**
	Change in GPE $= 7 \times 9.8 \times 1$ $= 68.6$	**B1**	Correct change in GPE
	$110.47... - 68.6 = \dfrac{1}{2} \times 7 \times v^2$	**M1 A1**	**M1**: Three term energy equation **A1**: Fully correct
	$v^2 = 11.971$		
	$v = 3.4599...$ $= 3.5$ [m s^{-1} to 2 sf]	**A1**	**AWRT** 3.5 Allow **AWRT** 3.4 if 110 J used
		5	

Q	Answer	Marks	Comments
7(c)	$110.5 = 7 \times 9.8h$	**M1**	Two term energy equation
	$h = 1.6$ [m] (to 2 sf)	**A1**	Correct height
		2	

	Question 7 Total	**11**	

Q	Answer	Marks	Comments
8(a)	$0 = 20\sin 35°t - \frac{1}{2}g\cos 30°t^2$	M1	Equation for motion perpendicular to the plane
	$t = \frac{40\sin 35°}{g\cos 30°} \quad [= 2.703...]$	A1	Correct equation
		A1	Correct time
	$\dot{x} = 20\cos 35° - g\sin 30° \times \frac{40\sin 35°}{g\cos 30°}$	M1	Finds both components of velocity
	$= 3.1368...$	A1	Correct parallel component
	$\dot{y} = 20\sin 35° - g\cos 30° \times \frac{40\sin 35°}{g\cos 30°}$		
	$= -20\sin 35°$		
	$= -11.4715...$	A1	Correct perpendicular component
	$v = \sqrt{11.4715...^2 + 3.1368...^2}$		
	$= 11.89...$		
	$= 12 \left[\text{m s}^{-1}\right]$ (to 2 sf)	A1	Correct speed
		7	

Q	Answer	Marks	Comments
8(b)	Require rebound angle to be greater than 60°	B1	Identifies 60°
	$\tan\theta = \frac{11.4715... \times e}{3.1368...}$	M1	Forms expression for tan of the rebound angle
	$\frac{11.4715... \times e}{3.1368...} > \sqrt{3}$	A1	Correct inequality
	$0.47 < e < 1$	A1	Correct range Allow lower limit between 0.46 and 0.48 Accept $0.47 \leq e \leq 1$
		4	

	Question 8 Total	**11**	

Q	Answer	Marks	Comments
9(a)	$12\times 8\cos 60^\circ = 12v_A + 3v_B$ $16 = 4v_A + 1v_B$	**M1**	Equation for conservation of momentum
	$v_B - v_A = -e\left(0 - 8\cos 60^\circ\right)$	**M1**	Equation for coefficient of restitution
	$v_B - v_A = 4e$	**A1**	Two correct equations
	$v_A = \dfrac{16-4e}{5}$	**A1**	Correct component of *A* along line of centres
	$V^2 = \left(8\sin 60^\circ\right)^2 + \left(\dfrac{16-4e}{5}\right)^2$	**M1**	Finds expression for speed of *A* after the collision
	$V = \sqrt{48 + \left(\dfrac{16-4e}{5}\right)^2}$	**A1**	Correct expression **ACF**
		6	

Q	Answer	Marks	Comments
9(b)	$V_{\min} = \sqrt{\left(8\sin 60^\circ\right)^2 + \left(\dfrac{16-4}{5}\right)^2} = \dfrac{8\sqrt{21}}{5}$	**M1**	Uses $e = 1$
	Maximum change in speed = $8 - \dfrac{8\sqrt{21}}{5} = \dfrac{40-8\sqrt{21}}{5}$	**A1**	Correct maximum change in speed **ACF**
		2	

	Question 9 Total	**8**	