OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA04) Unit S2 Statistics

Tuesday 13 June 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 (a) The random variable W has a Poisson distribution with a mean of 15

1 (a) (i) State the value of the standard deviation of W

[1 mark]

Answer _____

1 (a) (ii) It is given that $\mathrm{P}(W \leq a) > 0.9$

Find the smallest possible value of a

[2 marks]

Answer _____

1 (a) (iii) It is given that $\mathrm{P}(W > b) < 0.01$

Find the smallest possible value of b

[2 marks]

Answer _____

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1 (b) The random variable X has a Poisson distribution with a mean of 3

Find $\mathrm{P}(3 \leq X < 5)$ giving your answer to three significant figures.

[2 marks]

Answer _____

1 (c) The random variable T is defined as $T \sim \mathrm{B}(500, 0.02)$

1 (c) (i) Explain why T can be approximated by the random variable $Y \sim \mathrm{Po}(\lambda)$

[1 mark]

1 (c) (ii) State the value of λ in **part 1(c)(i)**.

[1 mark]

Answer _____

1 (c) (iii) The random variable C is defined as

$$C = W + X + Y$$

It is given that W, X and Y are independent.

Find $\mathrm{P}(C < 3)$ giving your answer in the form $p\mathrm{e}^{-q}$ where p and q are integers.

[3 marks]

Answer _____

12

Turn over ►

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2 The owner of a bus company records the time taken for a bus to travel between two towns.

The time taken is known to be modelled by a normal distribution with mean 38 minutes and standard deviation 11 minutes.

2 (a) Find the probability that the time taken for a bus to travel between the two towns is more than 50 minutes.

Give your answer to three significant figures.

[3 marks]

Answer _____

2 (b) Parts of the road between the two towns are repaired. As a result, the owner of the bus company claims that the mean time taken to travel between the two towns is reduced.

Since the repairs, the owner records the new time taken for a bus to travel between the two towns. A random sample of 30 of the recorded times has a mean of 34 minutes.

Test the owner's claim using the 2% level of significance, assuming that the standard deviation has not changed.

[7 marks]

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10

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3 The lifetime (in thousands of hours) of a component for a photocopier is modelled as T where $\mathrm{P}(T \leq t) = 1 - \mathrm{e}^{-\frac{t}{8}}$

3 (a) State the distribution model and the parameter of T

[1 mark]

Model _____

_____

Parameter _____

_____

3 (b) Find the probability of a component lasting longer than 7 thousand hours, giving your answer to four significant figures.

[2 marks]

_____

_____

_____

_____

_____

Answer _____

3 (c) Only 5% of components have lifetimes greater than L hours.

Find the value of L giving your answer to four significant figures.

[2 marks]

_____

_____

_____

_____

_____

Answer _____

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3 (d) Given that a component still works after 7 thousand hours of use, find the probability that the component has a lifetime of less than 10 thousand hours.

Give your answer to four significant figures.

[2 marks]

Answer _____

3 (e) Explain why your answer in **part 3(d)** is not likely to be representative of a real-life component.

[1 mark]

8

Turn over for the next question

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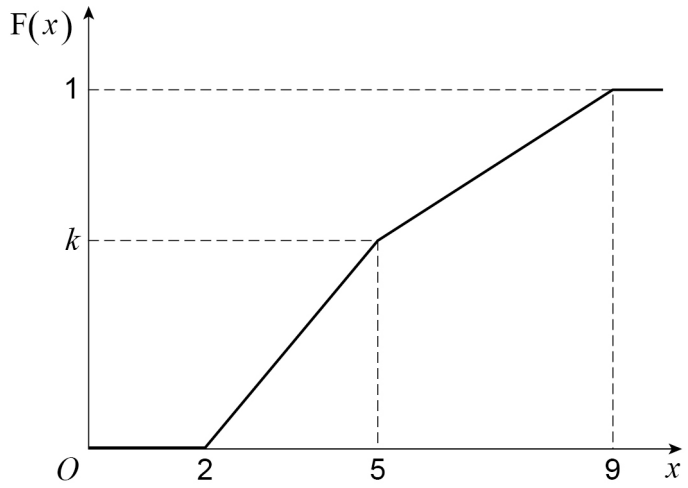
4 The continuous random variable X has cumulative distribution function $\mathrm{F}(x)$ as shown in the graph below.

The graph is made up from line segments where $\mathrm{F}(2) = 0$, $\mathrm{F}(5) = k$ and $\mathrm{F}(9) = 1$



It is given that $\mathrm{P}(X \leq 4) = 0.4$

4 (a) Find the value of k

[2 marks]

Answer _____

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4 (b) Find $\mathrm{F}(x)$

[5 marks]

$$\mathrm{F}(x) = \begin{cases} \\ \\ \end{cases}$$

4 (c) Hence find $\mathrm{P}(4 < X < 6)$

[2 marks]

Answer _____

9

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5 Sam has a business repairing mobile phones which have been damaged by water. He has a 90% success rate for repairing the damaged mobile phones.

5 (a) Sam designs a new gel to remove moisture from damaged mobile phones.

He claims that using the gel has improved his success rate.

Sam randomly selects 50 mobile phones sent to be repaired to trial the new gel.

He successfully repairs 49 of these 50 damaged mobile phones.

Test Sam's claim at the 1% level of significance.

[7 marks]

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5 (b) Explain what is meant by a Type I error in the context of the hypothesis test in **part 5(a)**.

[1 mark]

5 (c) All of the mobile phones owned by a family are damaged by a household flood. These damaged mobile phones are sent to Sam to be repaired.

Explain why a binomial distribution is not likely to be a good model for the number of successful repairs of these damaged mobile phones.

[1 mark]

9

Turn over ►

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6 Hannah is a long jump athlete. The length of a long jump, in metres, made by Hannah is modelled by the random variable X

The mean length of her competitive long jumps during 1 year is 6.03 metres.

For the next year Hannah appoints a new coach who changes her training programme.

The new coach claims that the changes made to her training programme have increased the mean lengths of Hannah's competitive long jumps.

Following the changes to her training programme, the length, in metres, of 12 randomly selected competitive long jumps are shown below.

6.08	5.90	6.21	5.98	6.23	6.07
5.87	6.00	6.14	6.30	6.15	6.15

Test the new coach's claim at the 10% level of significance, assuming that the length of Hannah's long jumps is normally distributed.

[10 marks]

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7 The continuous random variable X has probability density function $\mathrm{f}(x)$ defined by

$$\mathrm{f}(x) = \begin{cases} \dfrac{3\sqrt{x} + k}{6} & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

7 (a) Show that $k = 4$

[2 marks]

7 (b) Find the exact value of $\mathrm{E}(X)$

[3 marks]

Answer _____

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7 (c) Show that $\text{Var}(X) = \frac{127}{1575}$

[5 marks]

Question 7 continues on the next page

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7 (d) The continuous random variable Y has $\mathrm{E}(Y) = 2$ and $\mathrm{Var}(Y) = \frac{5}{7}$

It is given that X and Y are independent random variables.

7 (d) (i) Find the value of $\mathrm{E}(15X - 9Y)$

[2 marks]

Answer _____

7 (d) (ii) Find the value of $\mathrm{Var}(15X - 9Y)$

[2 marks]

Answer _____

14

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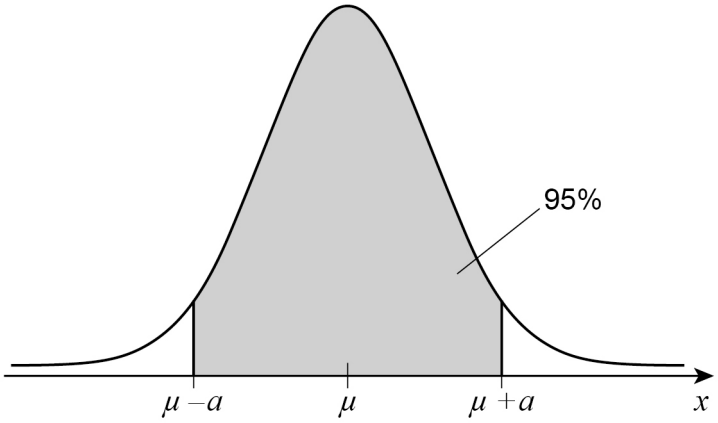
8 The random variable X is defined as $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ and graphs of its probability distribution function are shown in **Figures 1–3** below.

8 (a) (i) The shaded region between the lines $x = \mu - a$ and $x = \mu + a$ in **Figure 1** is 95% of the total area between the graph and the x-axis.

Figure 1



Find an expression for a in terms of σ

Give the coefficient of σ to four decimal places.

[1 mark]

Answer _____

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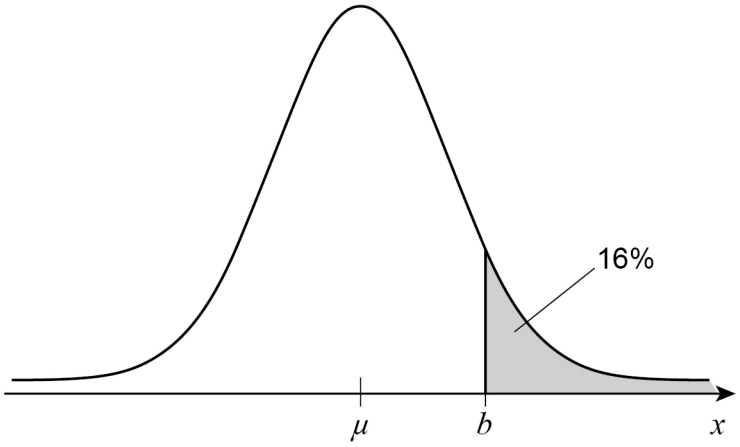
8 (a) (ii) The shaded region to the right of the line $x = b$ in **Figure 2** is 16% of the total area between the graph and the x-axis.

Figure 2



Find an expression for b in terms of μ and σ

Give the coefficient of σ to four decimal places.

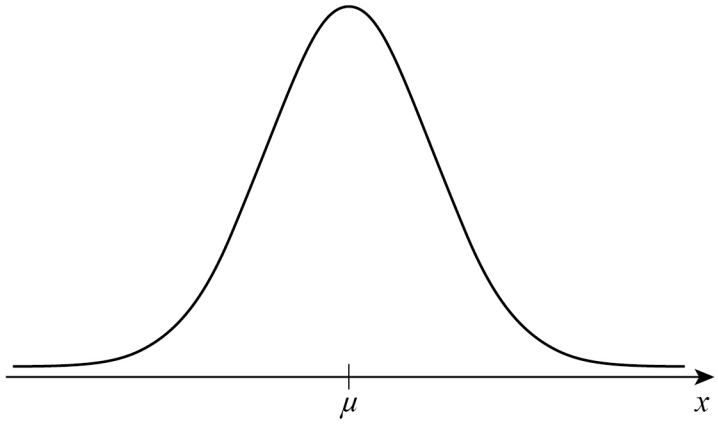
[1 mark]

Answer _____

8 (a) (iii) Sketch on **Figure 3** the total region represented by $\mathrm{P}\left(\left(X-\mu\right)^2 > c^2\right)$ where c is a positive constant by shading the appropriate area and labelling the x-axis.

[1 mark]

Figure 3



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8 (b) The random variable Y is defined as $Y \sim \text{N}(m, 0.8m)$

It is given that $\text{P}(Y < 205) = 0.975$

Find the value of m giving your answer to two decimal places.

[5 marks]

Answer _____

8

END OF QUESTIONS

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