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INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Friday 2 June 2023 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
TOTAL	

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Answer **all** questions in the spaces provided.

1 (a) For each of the following find $\frac{dy}{dx}$

1 (a) (i) $y = \left(3x^5 - 4\right)^8$

[2 marks]

Answer _____

1 (a) (ii) $y = \frac{3x^4}{7x - 5}$

[2 marks]

Answer _____

1 (a) (iii) $\ln\left(x^2\right) + y^2 = xy$

[3 marks]

Answer _____

Do not write outside the box

1 (b) (i) Find $\int \frac{x-6}{x^2-12x+5}\,\mathrm{d}x$

[2 marks]

Answer _____

1 (b) (ii) Find $\int \frac{8x+6}{\left(2x^2+3x-1\right)^3}\,\mathrm{d}x$

[3 marks]

Answer _____

12

Turn over ►

Do not write outside the box

2 (a) The curve with equation $y = 2^{-x}$ intersects the line $y = 4 - 2x$ at a point where $x = \alpha$ and $\alpha > 0$

2 (a) (i) Show that α lies between 1.8 and 1.9

[2 marks]

2 (a) (ii) Show that the equation $4 - 2x = 2^{-x}$ can be rearranged into the form $x = 2 - 2^{-(x+1)}$

[1 mark]

2 (a) (iii) Use the iterative formula

$$x_{n+1} = 2 - 2^{-(x_n + 1)}$$

with $x_1 = 1.8$ to find the values of x_2 and x_3 giving your answers to three decimal places.

[2 marks]

$x_2 =$ _____ $x_3 =$ _____

Do not write outside the box

2 (b) Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^4 2^{-x}\,\mathrm{d}x$$

giving your answer to four significant figures.

[4 marks]

Answer _____

9

Do not write outside the box

3 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sec\theta$ onto the graph of $y = 0.5\sec(\theta + 1)$

[4 marks]

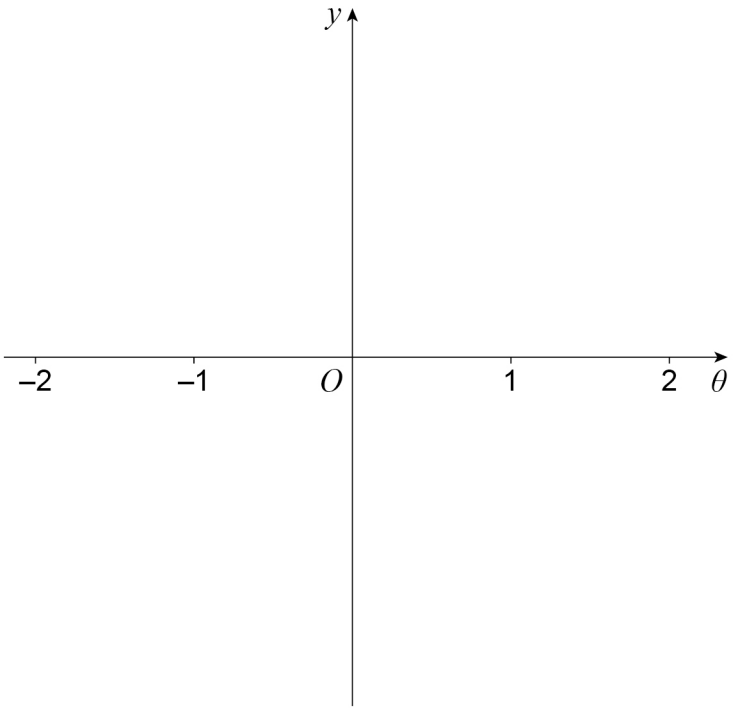
3 (b) Sketch on the axes below the curve with equation

$$y = 0.5\sec(\theta + 1) \quad \text{for} \quad -2 < \theta < 2$$

where θ is in radians.

State the coordinates of the stationary point and the coordinates of any intercept with the axes.

[3 marks]



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3 (c) It is given that $x = \dfrac{1}{2\cos y}$

3 (c) (i) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$

[1 mark]

Answer _____

3 (c) (ii) Hence find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\sin y$

[2 marks]

Answer _____

10

Do not write outside the box

4 The functions f and g are defined by

$$\mathrm{f}(x) = \sqrt{1 - 0.25x} \quad \text{for } x \leq 4$$

$$\mathrm{g}(x) = \frac{8}{x - 1} \quad \text{for all real values of } x,\ x \neq 1$$

4 (a) State the range of the function f

[1 mark]

Answer _____

4 (b) (i) Find $\mathrm{fg}(x)$

[1 mark]

Answer _____

4 (b) (ii) Find the domain of the function fg

[2 marks]

Answer _____

4 (b) (iii) Solve the equation $\mathrm{fg}(x) = 3$

[2 marks]

Answer _____

Do not write outside the box

4 (c) The inverse of the function fg is h

4 (c) (i) Find $\mathrm{h}(x)$ giving your answer in the form $\mathrm{h}(x) = \dfrac{p - x^2}{q - x^2}$ where p and q are integers.

[3 marks]

Answer _____

4 (c) (ii) Solve the equation $\mathrm{h}(x) = \dfrac{11}{3}$

[2 marks]

Answer _____

11

Turn over ►

Do not write outside the box

5 (a) Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α in radians to three significant figures.

[3 marks]

Answer _____

5 (b) Show that

$$2\operatorname{cosec} 4x + 2\cot 4x = \cot x - \tan x$$

for all x where $x \neq \frac{n\pi}{4}$

[4 marks]

Do not write outside the box

5 (c) By using a suitable trigonometrical identity, solve the equation

$$5\tan^2(2y+3) = 7 - 4\sec(2y+3)$$

Give all values of y to two decimal places in the interval $0 < y < \pi$

[5 marks]

Answer _____

12

Do not write outside the box

6 The polynomial $f(x)$ is defined by

$$f(x) = 4x^3 + ax^2 + bx + c$$

where a, b and c are constants.

It is given that $(2x - 1)$ is a factor of $f(x)$

When $f(x)$ is divided by $(2x - 3)$ the remainder is 15

6 (a) Show that $2a + b = 2$

[3 marks]

Do not write outside the box

6 (b) When $f(x)$ is divided by $(2x + 1)$ the remainder is 9

Find the remainder when $f(x)$ is divided by $(2x + 3)$

[4 marks]

Answer _____

7

Do not write outside the box

7 (a) A curve is defined by the parametric equations

$$x = \frac{t}{2t-1} \quad \text{and} \quad y = 2t - \sqrt{t}$$

Find an equation of the normal to the curve at the point where $t = 1$

Give your answer in the form $py + qx = r$ where p, q and r are integers.

[6 marks]

Answer _____

7 (b) A second curve is defined by the parametric equations

$$x = e^{2m} + e^{-2m} \quad \text{and} \quad y = e^{2m} - e^{-2m}$$

7 (b) (i) Find a Cartesian equation of this curve, giving your answer in the form $x^2 = f(y)$

[3 marks]

Do not write outside the box

Answer _____

7 (b) (ii) Hence write down a Cartesian equation of the curve defined by the parametric equations

$$x = e^{kn} + e^{-kn} \quad \text{and} \quad y = e^{kn} - e^{-kn}$$

where n is the parameter and k is a positive integer.

[1 mark]

Answer _____

7 (c) A third curve is defined by the parametric equations

$$x = a\sin\theta + b\cos\theta \quad \text{and} \quad y = a\cos\theta - b\sin\theta$$

where θ is the parameter and a and b are constants.

Find a Cartesian equation of this curve.

[3 marks]

Answer _____

13

Do not write outside the box

8 (a) The rate at which the population of a colony of ants is increasing is directly proportional to the population P of the colony.

Currently the population is $1\,000\,000$ and is increasing at a rate of 3000 per month.

Construct a differential equation for the population of the colony.

[2 marks]

Answer _____

8 (b) It is given that $y = \mathrm{f}(x)$ satisfies the differential equation

$$2x\frac{\mathrm{d}y}{\mathrm{d}x} + y^2 = 4$$

Find the solution of the differential equation for which $y = 1$ when $x = 1$

Give your answer in the form $y = \mathrm{f}(x)$

[6 marks]

Do not write outside the box

Answer _____

8

Turn over for the next question

Do not write outside the box

9 (a) Find the coordinates of the stationary point of the curve with equation

$$y = x^{0.5}e^{-0.5x}$$

Give the coordinates in an exact form.

[4 marks]

Answer _____

9 (b) The region bounded by the curve $y = x^{0.5}e^{-0.5x}$, the lines $x = 1$, $x = 2$ and the x-axis is rotated through 2π radians about the x-axis to form a solid.

Find the exact value of the volume of this solid.

Give your answer in the form $\pi\left(\frac{a}{e} + \frac{b}{e^2}\right)$ where a and b are integers.

[5 marks]

Do not write outside the box

Answer _____

9

Do not write outside the box

10 Use the substitution $u = 1 + \sin\theta$ to find the value of

$$\int_0^{\frac{\pi}{2}} \left(\frac{\cos\theta}{\sqrt{1+\sin\theta}} \right)^3 \mathrm{d}\theta$$

giving your answer in the form $\frac{2}{3}\left(p\sqrt{2}+q\right)$ where p and q are integers.

[8 marks]

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Answer _____

8

Do not write outside the box

11 (a) Express $\dfrac{7x^2-17x+12}{(3-x)^2(1-3x)}$ in the form $\dfrac{A}{(3-x)}+\dfrac{B}{(3-x)^2}+\dfrac{C}{(1-3x)}$

[4 marks]

Answer _____

11 (b) Find the binomial expansion of $(3-x)^{-1}$ up to and including the term in x^2

[2 marks]

Answer _____

Do not write outside the box

11 (c) Use your answers to **part (a)** and **part (b)** to show that

$$\frac{7x^2 - 17x + 12}{(3-x)^2(1-3x)} = D + Ex + Fx^2$$

for small values of x stating the rational values of D, E and F

[4 marks]

Answer _____

10

Turn over ►

Do not write outside the box

12 (a) Find a vector equation of the line l_1 which passes through the points $A(2, -1, 3)$ and $B(5, -2, -1)$

[1 mark]

Answer _____

12 (b) The line l_2 has equation $\mathbf{r} = \begin{bmatrix} -2 \\ 5 \\ 7 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ -2 \\ k \end{bmatrix}$

The lines l_1 and l_2 intersect.

12 (b) (i) Find the value of k

[3 marks]

Answer _____

12 (b) (ii) Find the coordinates of the point of intersection.

[1 mark]

Answer _____

Do not write outside the box

12 (c) The point C has coordinates (3, 4, 4)

The perpendicular from C to the line l_2 meets l_2 at the point D

Find the length of CD

[6 marks]

Answer _____

11

END OF QUESTIONS

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