OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA05) Unit M2 Mechanics

Tuesday 20 June 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Five particles *R*, *S*, *T*, *U* and *V* are placed at different positions in the x-y plane.

The table below shows the mass and coordinates of each particle, where m and k are constants.

Particle	Mass (kg)	Coordinates
R	1	$(4, 2)$
S	2	$(1, 4)$
T	4	$(3, 1)$
U	8	$(5, 3)$
V	m	$(6, k)$

The coordinates of the centre of mass of the system of five particles are $(4.4, 3.9)$

Find the value of m and the value of k

[5 marks]

$m =$ _____ $k =$ _____

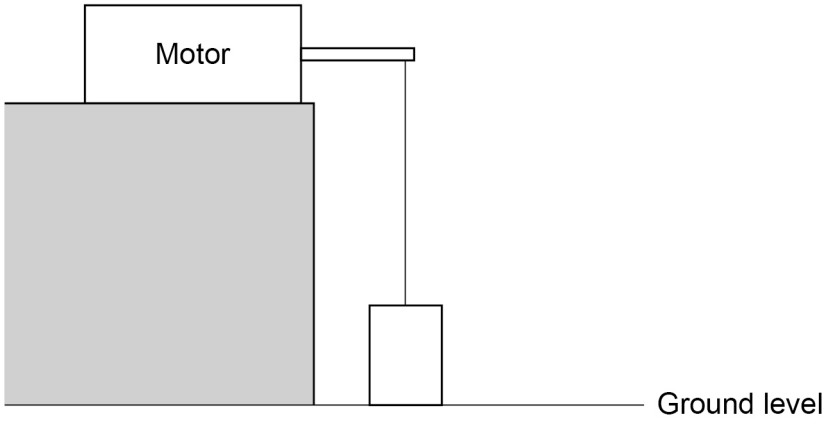
5

Do not write outside the box

2 One end of a light inextensible string is attached to a block of mass 14 kg

The other end of the string is connected to a motor.

The motor is positioned on a horizontal surface above the block so that the string is vertical and taut, as shown in the diagram.



The motor is used to raise the block vertically upwards from ground level at a constant speed of 0.8 m s^{-1}

Assume that the block experiences no air resistance during its motion.

2 (a) Find the rate at which the tension in the string does work on the block.

State the units of your answer.

[3 marks]

Answer _____ Units _____

2 (b) The string breaks when the bottom of the block is 1.6 metres above ground level.

Using an energy method, find the speed of the block immediately before it collides with the ground.

[3 marks]

Answer _____

6

Do not write outside the box

3 An electric car of mass 1800 kg moves along a straight horizontal road.

When the car moves with speed v m s^{-1}, the car's electric motor provides a driving force of magnitude $5000v^2e^{-0.32v}$ newtons.

The car also experiences a resistance force of magnitude $0.26v^2$ newtons.

3 (a) Write down an expression for the resultant force acting on the car when it moves with speed v m s^{-1}

[1 mark]

Answer _____

3 (b) Find the acceleration of the car when its speed is 8.3 m s^{-1}

[3 marks]

Answer _____

Do not write outside the box

3 (c) Find the maximum speed of the car.

[2 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

4 A body moves so that at time t seconds its position vector **r** metres relative to a fixed origin O is

$$\mathbf{r} = 5\cos(3t)\,\mathbf{i} + 37\sin(3t)\,\mathbf{j}$$

where the unit vectors **i** and **j** are directed east and north respectively.

4 (a) The body starts its motion when $t = 0$

Find the first **two** values of t for which the distance between the body and O is 19 metres.

[5 marks]

$t =$ _____ and _____

Do not write outside the box

4 (b) Find the acceleration of the body when $t = \frac{\pi}{4}$

Give your answer in an exact form.

[5 marks]

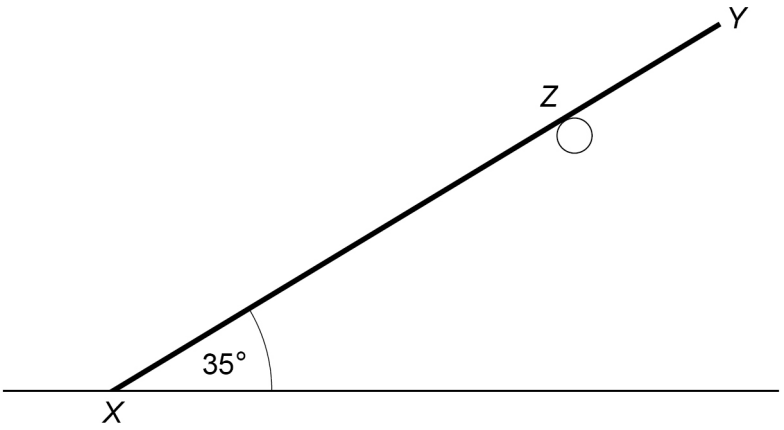
Answer _____

10

Do not write outside the box

5 A uniform rod XY has one end on rough horizontal ground and is supported by a smooth horizontal circular peg at the point Z

The rod makes an angle of $35°$ to the horizontal, as shown in the diagram below.



The rod has length 8 metres and mass 15 kg

The distance XZ is 6 metres.

The coefficient of friction between the rod and the rough horizontal ground is μ

The rod is in equilibrium.

5 (a) State what can be deduced from the rod being described as uniform.

[1 mark]

5 (b) Draw a diagram in the space below to show all of the forces acting on the rod.

Write down the names of the forces on your diagram.

[2 marks]

Do not write outside the box

5 (c) Find the minimum value of μ

Give your answer to three significant figures.

[8 marks]

Answer _____

11

Do not write outside the box

6 The three variable forces $\mathbf{F}_1$ newtons, $\mathbf{F}_2$ newtons and $\mathbf{F}_3$ newtons act on a particle of mass 0.2 kg

At time t seconds the three forces are

$$\mathbf{F}_1 = \begin{bmatrix} t^2 + 4t + 1 \\ 2t^2 - 2t \end{bmatrix} \quad \mathbf{F}_2 = \begin{bmatrix} 2t^2 - 1 \\ -t^2 - 2t + 1 \end{bmatrix} \quad \mathbf{F}_3 = \begin{bmatrix} t^2 - 12t + 3 \\ t^2 - t + 1 \end{bmatrix}$$

6 (a) Find in terms of t the resultant force acting on the particle.

[2 marks]

Answer _____

6 (b) Find the magnitude of the acceleration of the particle when $t = 2$

[3 marks]

Answer _____

Do not write outside the box

6 (c) Use your answer to **part (a)** to find the value of t when the particle is instantaneously in equilibrium.

[4 marks]

Answer _____

9

7 A particle of mass 5 kg slides along a track.

The points *A*, *B* and *C* lie on the track.

The particle starts from rest at *A* which is 10 metres vertically above ground level.

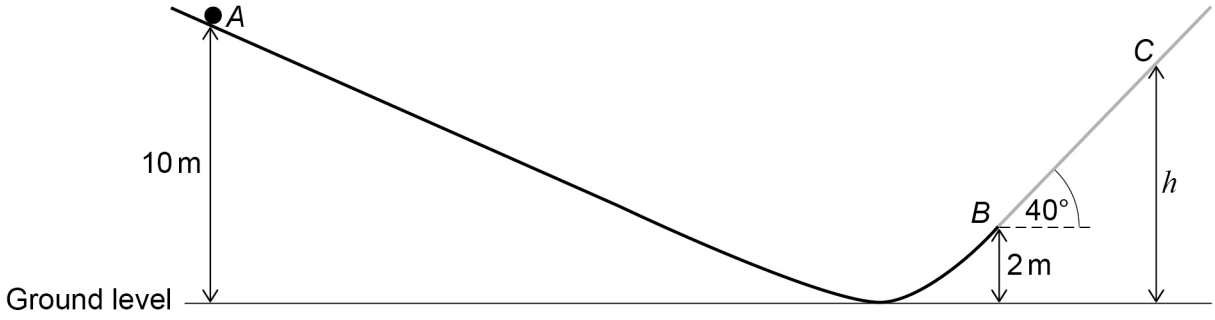
Between *A* and *B* the track is smooth and curved.

Point *B* is 2 metres vertically above ground level.

Between *B* and *C* the track is rough and straight, and makes an angle of 40° above the horizontal.

Point *C* is h metres vertically above ground level.

The track and the initial position of the particle are shown in the diagram below.



The coefficient of friction between the particle and the rough track between *B* and *C* is 0.3

At the instant the particle reaches *C* it comes to rest.

7 (a) Find the value of h

Give your answer to three significant figures.

[6 marks]

Do not write outside the box

Answer _____

7 (b) Determine whether the particle slides back down the rough track from C towards B

[3 marks]

9

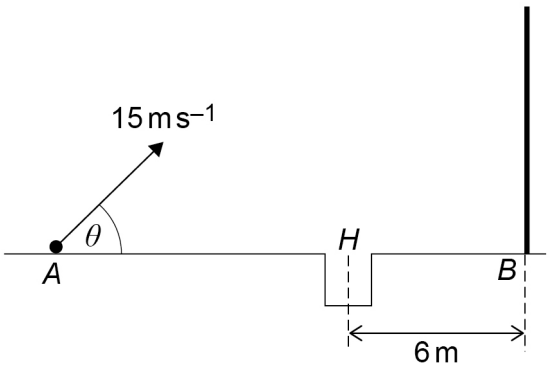
Do not write outside the box

8 At a fairground the objective of a game is for a ball of mass 0.4 kg to be kicked from horizontal ground towards a fixed vertical wall, rebound from the wall and land in a circular hole with centre H

The centre H of the hole is 6 metres from the bottom of the wall at B

H is on the horizontal line AB

A player kicks the ball from position A on the horizontal ground with a speed 15 m s^{-1} at an angle θ above the horizontal. The motion of the ball is in the vertical plane perpendicular to the wall, as shown in the diagram below.



The ball collides with the wall at a position 1.9 metres vertically above B

When the ball collides with the wall, the ball is moving horizontally.

The ball should be modelled as a particle.

8 (a) Show that θ is 24° correct to the nearest degree.

[4 marks]

Do not write outside the box

8 (b) The ball loses half of its kinetic energy during its collision with the wall.

Immediately after colliding with the wall, the ball is moving horizontally away from the wall.

8 (b) (i) Find the speed of the ball immediately after its collision with the wall.

Give your answer to three significant figures.

[3 marks]

Answer _____

8 (b) (ii) The diameter of the hole with centre H is 50 cm

Deduce whether the ball lands in the hole.

[4 marks]

11

Do not write outside the box

9 A small sphere X of mass 6 kg is attached to one end of a light inextensible string.

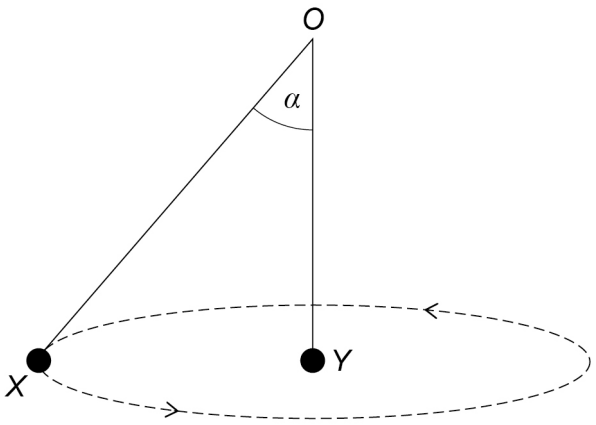
The string passes through a smooth ring which is fixed at O

Another small sphere Y of mass 8 kg is attached to the other end of the string.

X is set into motion around a horizontal circle with constant angular speed 7 rad s^{-1}

Y hangs in equilibrium vertically below O at the centre of the horizontal circle.

The angle XOY is α as shown in the diagram.



9 (a) Show that the magnitude of the tension in the string is 78.4 newtons.

[1 mark]

9 (b) (i) Show that $\cos\alpha = 0.75$

[2 marks]

Do not write outside the box

9 (b) (ii) Find the magnitude of the force exerted on the smooth ring due to its contact with the string.

Give your answer to three significant figures.

[3 marks]

Answer _____

9 (c) Find the distance OX

[3 marks]

Answer _____

Question 9 continues on the next page

Do not write outside the box

9 (d) The ground below X and Y is horizontal.

The string suddenly breaks while X is in motion around the circle.

9 (d) (i) Identify one similarity between the subsequent motions of X and Y

[1 mark]

9 (d) (ii) Identify one difference between the subsequent motions of X and Y

[1 mark]

9 (d) (iii) Explain whether X and Y reach the ground simultaneously.

[2 marks]

13

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required.** **Write the question numbers in the left-hand margin.**

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED