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I declare this is my own work.

# INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Monday 12 June 2023

07:00 GMT

Time allowed: 1 hour 30 minutes

## Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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<b>TOTAL</b>	



J U N 2 3 F M 0 4 0 1

IB/G/Jun23/E10

**FM04**

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ANSWER IN THE SPACES PROVIDED**



Answer **all** questions in the spaces provided.

- 1** The masses of apples from an orchard are assumed to be normally distributed with mean  $\mu$  grams and standard deviation  $\sigma$  grams.

A random sample of 10 apples is taken and their masses, in grams,  $M_i \{i = 1, 2, \dots, 10\}$  measured.

You may assume the  $M_i$  variables are independent.

- 1 (a)** Complete the table below of sampling distributions for the statistics shown.

**[3 marks]**

Statistic	$M_1$	$M_{10} - M_1$	$\sum_{i=1}^{10} M_i$	$\frac{1}{10} \sum_{i=1}^{10} M_i$
Sampling Distribution	$N(\mu, \sigma^2)$			

- 1 (b)** It is given that  $X_i = \frac{M_i - \mu}{\sigma}$

- 1 (b) (i)** Write down the distribution of  $X_i$

**[1 mark]**

Answer \_\_\_\_\_

- 1 (b) (ii)** Give a reason why  $X_i$  is not a statistic.

**[1 mark]**

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Turn over ►



[illegible]

[illegible]

8

**3** The random variable  $X$  has a Poisson distribution  $\text{Po}(\lambda)$

A test is conducted at the 5% level of significance with the hypotheses

$$H_0 : \lambda = 1.8$$

$$H_1 : \lambda > 1.8$$

**3 (a)** Verify that the critical region is  $X \geq 5$

**[2 marks]**

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**3 (b)** The actual value of  $\lambda$  is 3.4

Find the probability that a Type II error is made, giving your answer to three significant figures.

**[2 marks]**

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Answer \_\_\_\_\_

**3 (c)** Find the power of the test, giving your answer to three significant figures.

**[1 mark]**

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Answer \_\_\_\_\_

5

**Turn over for the next question**

**Turn over ►**



- 4** Two random variables  $X$  and  $Y$  are independent with

$$X \sim N(\mu_X, 5^2) \quad \text{and} \quad Y \sim N(\mu_Y, 7^2)$$

A random sample of size 50 is taken from  $X$  and a random sample of size 14 is taken from  $Y$

The sample means  $\bar{X}$  and  $\bar{Y}$  are calculated for both samples and the difference of the means,  $\bar{X} - \bar{Y}$  is calculated.

- 4 (a)** Write down  $E(\bar{X} - \bar{Y})$  in terms of  $\mu_X$  and  $\mu_Y$

**[1 mark]**

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Answer \_\_\_\_\_

- 4 (b)** Show that  $\text{Var}(\bar{X} - \bar{Y}) = 4$

**[2 marks]**

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- 4 (c)** Explain why  $\bar{X} - \bar{Y}$  is normally distributed.

**[1 mark]**

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**[6 marks]**

[illegible]

10

A trial is conducted which takes a random sample of plants of the original variety and a random sample of the new variety. You may assume that the samples are from normal populations.

The table below shows the summary statistics for the samples.

Variety	Number of plants	Sample mean yield (kilograms)	Sample variance $s^2$ (kilograms <sup>2</sup> )
Original	9	2.6	0.09
New	5	3.1	0.12

Investigate at the 1% level of significance whether the yield of tomato fruit is greater for the new variety compared with the original variety.

**[9 marks]**

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[illegible]

9

**6**

A random sample of size  $n$  is taken from a population.

Another random sample of size  $n$  is taken from a second independent population.

The first population has mean  $\mu$  and variance  $\sigma^2$

The sample mean from this population is  $\bar{X}$

The second population has mean  $a\mu$  and variance  $b\sigma^2$  where  $a$  and  $b$  are positive real numbers. The sample mean from this population is  $\bar{Y}$

**6 (a)**

The statistic  $S = 3\bar{X} - 4\bar{Y}$  is an unbiased estimator of  $\mu$

Find the value of  $a$

**[3 marks]**


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Answer \_\_\_\_\_

**6 (b)**

Show that the statistic  $S$  is also a consistent estimator of  $\mu$

**[3 marks]**


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6 (c) The statistic  $T = \frac{3}{2}\bar{X} - \bar{Y}$  is an unbiased estimator of  $\mu$

6 (c) (i) Show that the efficiency of  $T$  relative to  $S$  is  $\frac{36+64b}{9+4b}$

[2 marks]

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6 (c) (ii) Prove that the efficiency of  $T$  relative to  $S$  is greater than 4 and less than 16

[3 marks]

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6 (c) (iii) State with a reason which of  $S$  or  $T$  is the more efficient estimator of  $\mu$

[1 mark]

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A random sample of 11 cars of the model are investigated and the CO<sub>2</sub> emission value,  $X$  grams per km, is measured.

$$\sum x = 1034 \quad \text{and} \quad \sum x^2 = 97\,846$$

Give your values to one decimal place.

[illegible]

Answer



- 7 (b)** State an assumption you have made about the distribution of CO<sub>2</sub> emission values for your confidence interval calculation from **part (a)** to be valid.

[1 mark]

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- 7 (c)** All car models with a mean CO<sub>2</sub> emission value of 100 grams per km or below qualify for a partial government refund.

- 7 (c) (i)** State with a reason if your confidence interval provides evidence that the car model being investigated should qualify for the refund.

[2 marks]

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- 7 (c) (ii)** The car model is now investigated further by increasing the sample size to 65 cars.

The new 95% confidence interval is (97.3, 101.1)

Explain how this new confidence interval affects your answer to **part (c)(i)**.

[1 mark]

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- 8** A researcher measures the distance between mutations on a DNA strand. They believe that the distances follow an exponential distribution with  $\lambda = \frac{1}{1500}$  micrometres<sup>-1</sup> and use this to determine expected distribution frequencies.
- The measured and expected distribution frequencies from 1394 mutations are shown in the table below.

Interval (micrometres)	< 500	500 to 1000	1000 to 1500	1500 to 2000	2000 to 2500	2500 to 3000	> 3000
Measured frequency ( $M$ )	407	309	195	125	109	70	179
Expected frequency ( $E$ )	395	283	203	145	104	75	189
$(M - E)^2$	144	676	64	400	25	25	100

- 8 (a)** Use integration to verify that the expected frequency for the 1500 to 2000 micrometres interval is 145 to the closest integer.

[2 marks]

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- 8 (b)** Determine the  $\chi^2$  test statistic using the values from the table for  $(M - E)^2$

Give your answer correct to three significant figures.

[2 marks]

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Answer \_\_\_\_\_





**[5 marks]**

[illegible]

9

**Turn over for the next question**

**Turn over ►**



- 9** A discrete random variable  $X$  has a Poisson distribution with population parameter  $\lambda$ . Its moment generating function is given by

$$M_X(t) = e^{\lambda(e^t - 1)}$$

- 9 (a) (i)** Use  $M_X(t)$  to show that  $E(X) = \lambda$

**[2 marks]**

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- 9 (a) (ii)** Use  $M_X(t)$  to determine  $\text{Var}(X)$

**[3 marks]**

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Answer \_\_\_\_\_



- 9 (b)** A second discrete random variable  $Y$  has a Poisson distribution with population parameter  $\mu$

The random variables  $X$  and  $Y$  are independent.

- 9 (b) (i)** Find an expression for the moment generating function  $M_Z(t)$  of the random variable  $Z = X + Y$

**[2 marks]**

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Answer \_\_\_\_\_

- 9 (b) (ii)** Using your answer for **part (b)(i)** verify that  $Z \sim \text{Po}(\nu)$  writing down  $\nu$  in terms of  $\lambda$  and  $\mu$

**[1 mark]**

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Answer \_\_\_\_\_

**Question 9 continues on the next page**

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**9 (c)**

- $\lim_{n \rightarrow \infty} \left(1 + \frac{\alpha}{n}\right)^n = e^\alpha$  where  $\alpha$  is a constant
- If a random variable has the moment generating function of a Poisson random variable with parameter  $\lambda$  then it also has a Poisson probability distribution with parameter  $\lambda$

A discrete random variable  $W$  has a binomial distribution with parameters  $n$  and  $p$

Its moment generating function is  $M_W(t) = (1 + p(e^t - 1))^n$

Prove that as  $n \rightarrow \infty$  the probability distribution of  $W$  tends to the Poisson distribution with parameter  $\lambda = np$  given that both  $\lambda$  and  $t$  are constant during the limiting process.

**[3 marks]**

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**9 (d)**

**[1 mark]**

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**END OF QUESTIONS**

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