OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Monday 15 May 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 A curve passes through the point (2, 1) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{y + \sqrt{x}}$$

Use Euler's step-by-step method with a step length of 0.2 to estimate the value of y when $x = 2.4$

Give your answer to three decimal places.

[5 marks]

Answer _____

5

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2 (a) The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 2p & -2 & 3 \\ -1 & 3p & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

where p is a constant.

Find $\mathbf{A}^{\mathrm{T}}\mathbf{B}$ in terms of p

[2 marks]

Answer _____

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2 (b) The matrix $\mathbf{C}$ is defined by $\mathbf{C} = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$

2 (b) (i) It is given that $\mathbf{C}^2 = k\mathbf{I}$ where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix.

Find the value of k

[2 marks]

$k =$ _____

2 (b) (ii) Hence find $\mathbf{C}^{13}$

[2 marks]

Answer _____

6

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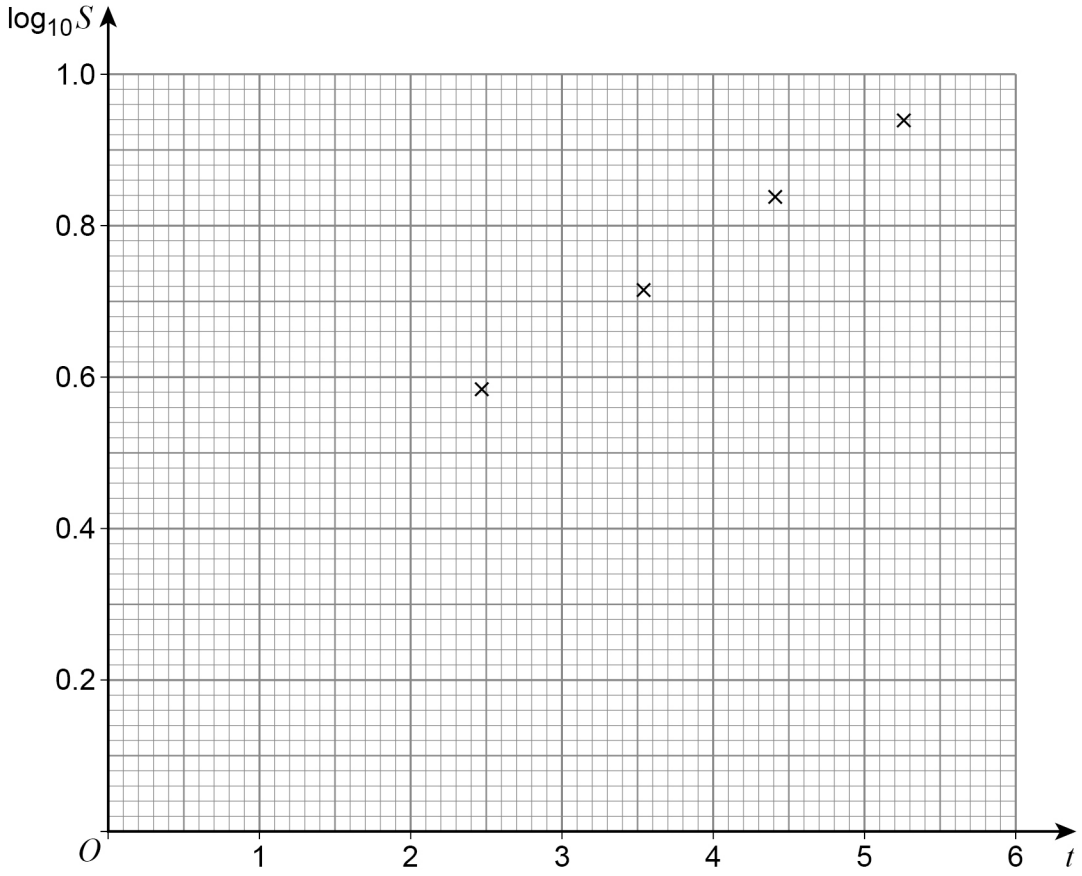
3 The variables t and S are related by the equation $S = a\,b^t$ where a and b are constants.

Some values of t and S are found in an experiment.

The values of $\log_{10} S$ are plotted against t on the grid below.



3 (a) (i) Draw a line of best fit on the grid.

[1 mark]

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3 (a) (ii) Use your line of best fit to estimate the value of a and the value of b

Give your values to two significant figures.

[4 marks]

$a =$ _____ $b =$ _____

3 (b) Use your values for a and b to estimate the value of t when $S = 11.2$

Give your answer to two significant figures.

[2 marks]

Answer _____

7

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4 The equation $3x^3 - x^2 - 5x - 3 = 0$ has a single real root α

4 (a) Show that α lies in the interval $1 < x < 2$

[2 marks]

4 (b) Starting with the interval $1 < x < 2$, use interval bisection **twice** to find an interval of width 0.25 in which α must lie.

[3 marks]

Answer _____

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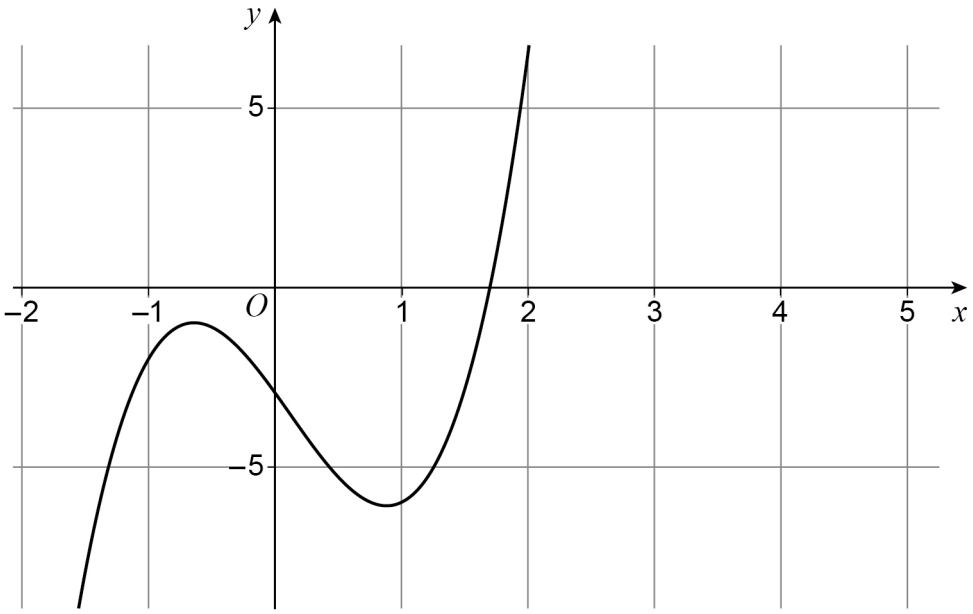
4 (c) The Newton–Raphson method is to be used once to find a second approximation to α

4 (c) (i) Explain why an initial value $x_1 = 1$ would **not** give an improved approximation to α after one iteration.

Draw an appropriate straight line on the graph of $y = 3x^3 - x^2 - 5x - 3$ below as part of your explanation.

[2 marks]



4 (c) (ii) Taking $x_1 = 1.75$ as a first approximation to α, use the Newton–Raphson method to find a second approximation x_2 to α

Give your answer to four decimal places.

[4 marks]

Answer _____

11

Turn over ►

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5 The matrix **A** is defined by $\mathbf{A} = \begin{bmatrix} 1-0.4k & -0.8k \\ 0.2k & 1+0.4k \end{bmatrix}$ where k is a non-zero constant.

5 (a) Show that **A** is non-singular for all values of k

[2 marks]

5 (b) It is given that $k = -1$

5 (b) (i) The image of the point $P(1, 2)$ under the transformation represented by **A** is the point P'

Find the coordinates of P'

[2 marks]

Answer _____

5 (b) (ii) Find the equations of the invariant lines of the transformation represented by **A**

[5 marks]

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Answer _____

5 (b) (iii) The transformation represented by **A** is a shear.

Using your answers to **part (a)** and **part (b)(ii)**, state **two** reasons that confirm this statement.

[2 marks]

Reason 1 _____

Reason 2 _____

11

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 The random variable W has probability generating function

$$G_W(t) = 0.1 + pt + (0.9 - p)t^3$$

where p is a constant.

6 (a) Find $G'_W(t)$ in terms of p

[1 mark]

Answer _____

6 (b) The mean of W is 2.5

Use your answer to **part (a)** to find $P(W \leq 1)$

[4 marks]

Answer _____

5

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7 A company makes batteries.

Each battery is made using one of three different machines: *A*, *B* or *C*

Machine *A* is used to make 45% of the batteries.

Machine *B* is used to make 32% of the batteries.

A randomly selected battery is checked to see if it is damaged.

The probability that the battery is damaged given that it is made by machine *A* is 0.015

The probability that the battery is damaged given that it is made by machine *B* is 0.018

The probability that the battery is damaged given that it is made by machine *C* is 0.03

7 (a) Using the probabilities given above, draw a tree diagram to represent this information in the space provided below.

[2 marks]

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7 (b) Show that the probability that the battery is made by machine *C* given that it is **not** damaged is 0.228, correct to three significant figures.

[3 marks]

Answer _____

5

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8 The random variable X has a discrete uniform distribution which takes the values 1, 2, 3, …, n and $\mathrm{E}(X) = 15$

8 (a) Find $\mathrm{Var}(X)$

[2 marks]

Answer _____

8 (b) The random variable Y has a geometric distribution with $\mathrm{P}(Y = 2) = 0.1824$ and $\mathrm{E}(Y) > 2$

8 (b) (i) Show that $\mathrm{Var}(Y) = \frac{475}{36}$

[5 marks]

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8 (b) (ii) The value of $\text{Var}(X - 6Y)$ is 551

Explain whether X and Y are independent.

[3 marks]

10

Turn over for the next section

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 (a) Use the definition of impulse to find its dimensions.

[2 marks]

Answer _____

9 (b) Show that the coefficient of restitution e is dimensionless.

[1 mark]

9 (c) A ball of mass m kg is moving with speed u m s^{-1} when it collides with a wall that is perpendicular to its path.

The coefficient of restitution between the ball and the wall is e

The magnitude of the impulse on the ball during its contact with the wall is given by the formula $I = mu(1+e)$

Show that the formula $I = mu(1+e)$ is dimensionally consistent.

[3 marks]

6

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10 Two spheres P and Q of equal radius are moving in the same direction on a straight line on a smooth horizontal table.

Sphere P has mass 0.1 kg and is moving with speed 12 m s^{-1} towards Q

Sphere Q has mass 0.4 kg and is moving with speed 8 m s^{-1}

The spheres collide.

The spheres are in contact for 0.08 seconds during the collision.

The force that Q exerts on P while they are in contact is modelled as a constant force with magnitude 6 newtons.

10 (a) Find the magnitude of the impulse that Q exerts on P during the collision.

[1 mark]

Answer _____

10 (b) Show that the speed of P after the collision is 7.2 m s^{-1}

[2 marks]

10 (c) Find the coefficient of restitution between the spheres.

[3 marks]

Answer _____

6

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11 Two drones A and B are flying with constant velocities at the same constant height above horizontal ground.

Drone A is initially at the point with position vector $\begin{bmatrix} 80 \\ 100 \end{bmatrix}$ metres and moves with a velocity of $\begin{bmatrix} 5 \\ p \end{bmatrix}$ m s^{-1} where p is a constant.

Drone B is initially at the point with position vector $\begin{bmatrix} 200 \\ 40 \end{bmatrix}$ metres and moves with a velocity of $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ m s^{-1}

11 (a) Find the value of p for which the drones collide.

[4 marks]

Answer _____

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11 (b) In the case where $p = 4$ find the shortest distance between the drones.

Give your answer to the nearest metre.

[4 marks]

Answer _____

8

END OF QUESTIONS

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