OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Wednesday 24 May 2023 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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12	
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14	
TOTAL	

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Answer **all** questions in the spaces provided.

1 The 3×3 matrix **N** represents a reflection in the plane $y = 0$

The 3×3 matrix **M** represents an enlargement, scale factor 2 with the origin as the centre of enlargement.

Find the matrix **NM**

[2 marks]

NM =

2

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2 The cubic equation

$$z^3 - 4z^2 + 3z + c = 0$$

where c is a non-zero constant, has roots α, β and γ

2 (a) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = 10$$

[3 marks]

2 (b) Explain why

$$\beta^3 - 4\beta^2 + 3\beta + c = 0$$

[1 mark]

2 (c) Show that

$$\alpha^3 + \beta^3 + \gamma^3 = 28 - 3c$$

[2 marks]

6

Turn over ►

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3 Two 3×3 matrices $\mathbf{A}$ and $\mathbf{B}$ are such that

$$\det(\mathbf{AB}) = 10 \quad \text{and} \quad \det(\mathbf{A}^{-1}) = 5$$

A three-dimensional shape S_1 with volume 6 cm^3 is mapped onto the shape S_2 by the transformation represented by matrix **B**

Find the volume of S_2

[4 marks]

Answer ____________________

4

4 A curve has Cartesian equation

$$y = x\sqrt{x} - \frac{1}{3}\sqrt{x}$$

4 (a) Show that

$$1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \left(px^n + qx^{-n}\right)^2$$

where p, q and n are rational numbers.

[2 marks]

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4 (b) The arc of the curve from $x = 1$ to $x = 4$ is rotated through 2π radians about the x-axis.

Show that the area of the curved surface generated is $\frac{173}{3}\pi$

[4 marks]

6

Turn over ►

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5 Evaluate the improper integral

$$\int_0^{e} \left(9x^2 \ln x + \frac{4}{1+4x^2} \right) dx$$

showing the limiting process used.

[7 marks]

Answer ______________________

7

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6 By using an integrating factor, find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{8x}{x^2+2}y = 2x^3 + \frac{1}{\left(x^2+2\right)^{\frac{9}{2}}}$$

Give your answer in the form $y = f\left(x\right)$

[7 marks]

$y =$ ______

7

Turn over ►

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7 The sequence $u_1,\ u_2,\ u_3, \ldots$ is defined by

$$u_1 = 3 \qquad u_{r+1} = 3u_r + 4$$

7 (a) By writing $u_{r+1} = 3u_r + 4$ in the rearranged form $u_{r+1} - u_r = 2u_r + 4$ use the method of differences to show that

$$\sum_{r=1}^{n} u_r = \frac{1}{2}u_{n+1} - 2n - \frac{3}{2}$$

[3 marks]

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7 (b) Prove by induction that, for all integers $n \geq 1$

$$u_n = 5 \times 3^{n-1} - 2$$

[4 marks]

7 (c) Hence, write down $\sum_{r=1}^{n} u_r$ in terms of n

[1 mark]

$$\sum_{r=1}^{n} u_r = $$

8

Turn over ►

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8 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

[2 marks]

Answer ____________________

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8 (b) It is given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 6e^{-2x}$$

and when $x = 0$ it is given that both $y = 0$ and $\frac{d^2y}{dx^2} = 4$

Find the value of y when $x = 3$

[7 marks]

$y =$ ______

9

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9 The non-singular matrix **A** is defined as

$$\mathbf{A} = \begin{bmatrix} 3-k & 1-k & 3 \\ 5 & 7 & 4 \\ 3 & 5 & 3 \end{bmatrix}$$

where k is a **positive** constant.

9 (a) Find $\mathbf{A}^{-1}$ in terms of k

[6 marks]

$\mathbf{A}^{-1} =$

Do not write outside the box

9 (b) (i) Use your answer to **part (a)** to solve the equations

$$\begin{aligned}(3-k)x + (1-k)y + 3z &= 1\\ 5x + 7y + 4z &= 1\\ 3x + 5y + 3z &= 1\end{aligned}$$

Give your solution in terms of k

[3 marks]

$x =$ ______ $y =$ ______ $z =$ ______

9 (b) (ii) Hence, state the range of possible values of $x+y+z$

[1 mark]

Answer ______

10

Turn over ►

Do not write outside the box

10 (a) (i) Write down $e^{\frac{i\theta}{2}} + e^{-\frac{i\theta}{2}}$ in terms of $\cos\left(\frac{\theta}{2}\right)$

[1 mark]

$$e^{\frac{i\theta}{2}} + e^{-\frac{i\theta}{2}} = \_\_\_\_\_\_\_\_$$

10 (a) (ii) Hence, given that $e^{i\theta} \neq -1$ show that

$$\frac{1}{e^{i\theta}+1} = \frac{1}{2} - \frac{i}{2}\tan\left(\frac{\theta}{2}\right)$$

[2 marks]

10 (b) Hence, by replacing θ by $\pi - \theta$ in the equation in **part (a)(ii)**, show that for $e^{i\theta} \neq 1$

$$\frac{1}{e^{i\theta}-1} = -\frac{1}{2} - \frac{i}{2}\cot\left(\frac{\theta}{2}\right)$$

[3 marks]

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10 (c) Deduce that, for $e^{2i\theta} \neq 1$

$$\frac{1}{\cos 2\theta - 1 + i\sin 2\theta} = a + i\,b\left(\tan\left(\frac{\theta}{2}\right) - \cot\left(\frac{\theta}{2}\right)\right)$$

where a and b are rational numbers.

[2 marks]

8

Turn over ►

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11 The line L has equation $\left(\mathbf{r} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}\right) \times \begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix} = \mathbf{0}$

The plane Π has equation $\mathbf{r} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$

11 (a) Find $\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$

[1 mark]

Answer ______________________________

11 (b) Use a scalar triple product to determine whether or not $\begin{bmatrix} -4 \\ 3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$ are coplanar vectors.

[2 marks]

Answer ______________________________

11 (c) Calculate the acute angle between the line L and the plane Π giving your answer to the nearest degree.

[4 marks]

Answer ______________________

11 (d) The line L intersects the plane Π at the point P

The point Q has coordinates $(1, 0, 2)$

Calculate the length PQ

[4 marks]

Question 11 continues on the next page

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Answer

11 (e) Hence, or otherwise, find the **exact** value for the shortest distance from the point Q to the plane Π

[3 marks]

Answer

14

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12 A curve C has equation

$$y = e^{\frac{7}{25}x} \operatorname{sech} x$$

The curve has exactly one stationary point P

12 (a) Find the x-coordinate of P giving your answer in the form $\ln k$ where k is a constant.

[6 marks]

Answer

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12 (b) The line L is the tangent to the curve at P

Find the shortest distance of L from the origin.

Give your answer in the form $a(b)^c$ where a, b and c are rational numbers.

[3 marks]

Answer ______________________

12 (c) Hence, determine whether or not the line L intersects the curve $y = \tanh x$

Justify your answer.

[2 marks]

11

Turn over ►

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13 (a) Write down the Maclaurin series expansion of $\ln(1+4x)$ in ascending powers of x up to and including the term in x^3 and state the range of values of x for which this expansion is valid.

[2 marks]

$\ln(1+4x) =$ ______________ valid for ______________

13 (b) It is given that $y = \ln(\cos x - \sin x)$

13 (b) (i) Show that

$$\frac{d^2y}{dx^2} = \frac{-2}{1-\sin 2x}$$

[3 marks]

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13 (b) (ii) Hence, show that the first three non-zero terms in ascending powers of x in the Maclaurin series expansion of $\ln(\cos x - \sin x)$ are $-x - x^2 - \frac{2}{3}x^3$

[3 marks]

13 (c) **Hence**, show that

$$\lim_{x \to 0}\left[\frac{\ln\left(\left(1 - \sin 2x\right)\sqrt{1+4x}\right)}{5x^2 + 6x^3}\right]$$

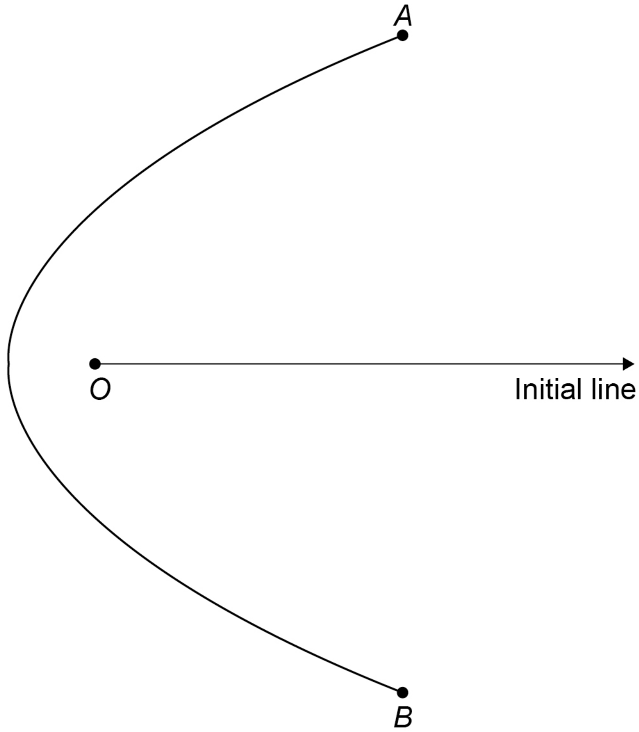
exists and state its value.

[5 marks]

Answer ______________________

13

14 The diagram shows a sketch of a curve C, the pole O and the initial line.



The end points A and B of the curve C are shown on the diagram above.

The curve C has polar equation

$$r = \frac{3}{2}\operatorname{cosec}^2\left(\frac{\theta}{2}\right) \quad \text{for} \quad \frac{\pi}{4} \leq \theta \leq \frac{7\pi}{4}$$

14 (a) The end point A has polar coordinates $\left(6 + 3\sqrt{2}\,,\ \frac{\pi}{4}\right)$

Show that the area of triangle AOB is $27 + 18\sqrt{2}$

[2 marks]

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14 (b) Find the Cartesian equation of C giving your answer in the form $y^2 = \mathrm{f}(x)$

[4 marks]

Answer ______________________

14 (c) The straight line with polar equation $\tan\theta = \sqrt{3}$ intersects the curve C at the points P and Q

14 (c) (i) Find the polar coordinates of P and Q

[3 marks]

Answer ______________________

Question 14 continues on the next page

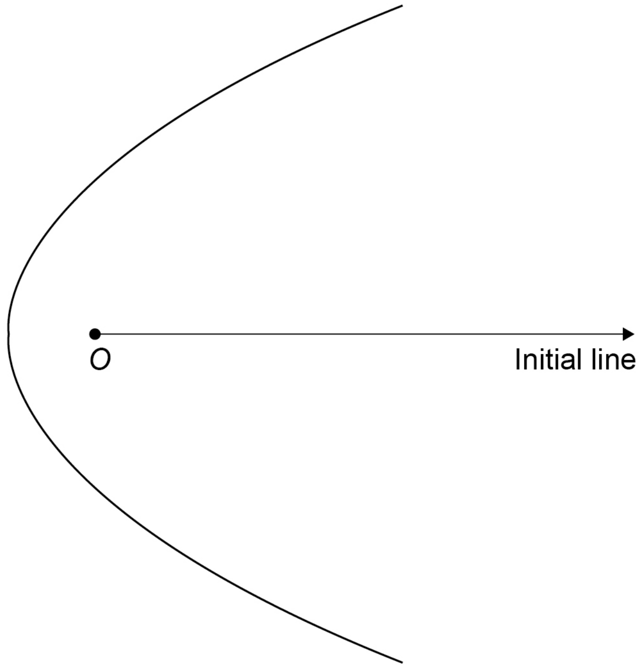
14 (c) (ii) Find $\displaystyle\int\left(1+\cot^2\left(\frac{\theta}{2}\right)\right)\operatorname{cosec}^2\left(\frac{\theta}{2}\right)\mathrm{d}\theta$

[3 marks]

Answer ______________________________

14 (c) (iii) Hence, find the area of the region bounded by the curve C and the line segment PQ giving your answer in the form $k\sqrt{n}$ where k is a rational number and n is a prime number.

[3 marks]



Answer ______________________

15

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