OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Monday 8 May 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
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7	
8	
9	
10	
11	
TOTAL	

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Answer **all** questions in the spaces provided.

1 By considering the derivative of $y = x^{\frac{1}{2}}$ when $x = 25$, find an estimate for $\sqrt{25.4}$

[6 marks]

Answer ______________________

6

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2 The integral I is defined by

$$I = \int_4^{\infty} x^{-3}\,\mathrm{d}x$$

2 (a) Explain why I is an improper integral.

[1 mark]

2 (b) Evaluate I showing the limiting process.

[3 marks]

Answer

4

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3 (a) Show that

$$(x+1)^3-(x-1)^3=6x^2+2$$

[1 mark]

3 (b) Use the method of differences to show that

$$\sum_{r=15}^{n}\left(6r^2+2\right)=n^3+(n+1)^3-k$$

where k is a constant.

[4 marks]

5

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4 (a) Find the general solution of the equation

$$\sin\left(\frac{x}{3}+\frac{\pi}{6}\right)=-\frac{\sqrt{2}}{2}$$

Give your answer in terms of π

[4 marks]

Answer ____________________

4 (b) Find the sum of the four smallest positive solutions of the equation

$$\sin\left(\frac{x}{3}+\frac{\pi}{6}\right)=-\frac{\sqrt{2}}{2}$$

Give your answer in terms of π

[3 marks]

Answer ____________________

7

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5 The equation

$$z^2 - az + (b + \mathrm{i}) = 0$$

where a and b are real constants, has two complex roots.

One of the roots of the equation is $2 + \mathrm{i}$

Find the other root of the equation.

[5 marks]

Answer ______________________

5

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6 (a) A curve has equation $y = px^2 - 3x$ where p is a constant.

A line passes through two points on the curve, one where $x = 7$ and the other where $x = 7 + h$

Find the gradient of this line in terms of p and h

Give your answer in its simplest form.

[3 marks]

Answer ______________________________

6 (b) The curve has a stationary point at the point where $x = 7$

Use your answer to **part (a)** to find the value of p

[2 marks]

Answer ______________________________

5

Do not write outside the box

7 The quadratic equation

$$3x^2 - 2x + 9 = 0$$

has roots α and β

7 (a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$

[2 marks]

$\alpha + \beta =$ ____________ $\alpha\beta =$ ____________

7 (b) Hence show that $\alpha^2 + \beta^2 = -\dfrac{50}{9}$

[2 marks]

Do not write outside the box

7 (c) Find a quadratic equation, with integer coefficients, which has roots α^4 and β^4

[4 marks]

Answer

8

8 The function f is defined by

$$\mathrm{f}(x) = \frac{x^2}{(x-1)(x+2)}$$

8 (a) Write down the equations of the asymptotes of the graph of $y = \mathrm{f}(x)$

[2 marks]

Answer ______________________________

8 (b) It is given that the line $y = k$, where k is a constant, intersects the graph of $y = \mathrm{f}(x)$

Find the set of possible values of k

[3 marks]

Answer ______________________________

8 (c) Hence find the coordinates of the stationary points of the graph of $y = \mathrm{f}(x)$

[3 marks]

Answer ______________________________

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8 (d) Show that the graph of $y = \mathrm{f}(x)$ intersects its horizontal asymptote at one point.

Find the coordinates of this point.

[2 marks]

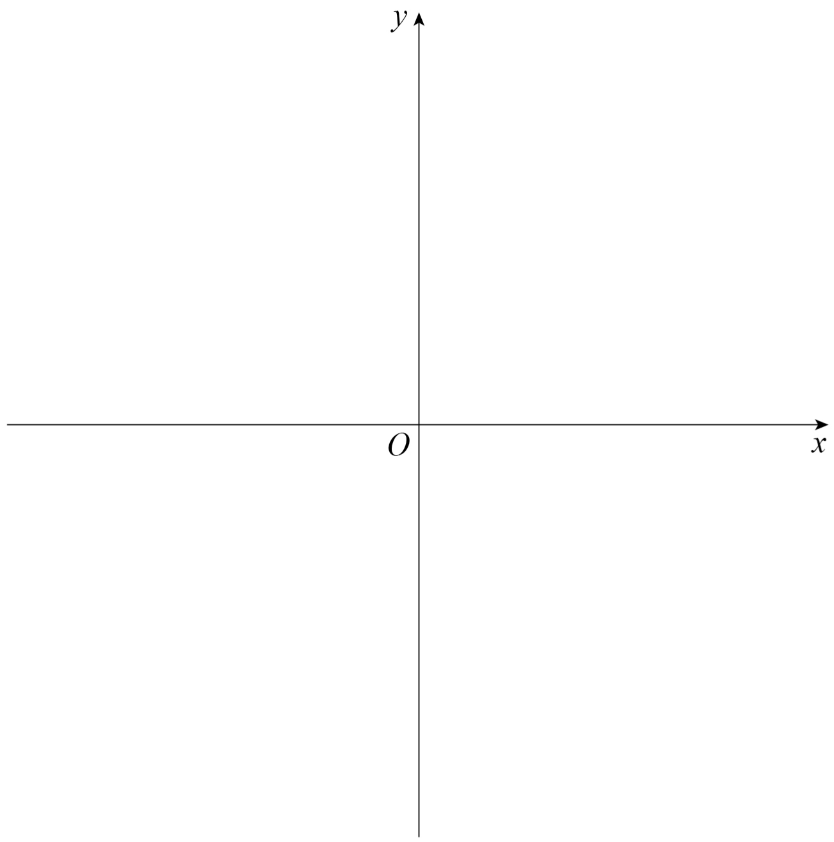
Answer ______________________

8 (e) Sketch the graph of $y = \mathrm{f}(x)$ on the axes below.

Show the coordinates of the stationary points

Show the coordinates of the point of intersection of the graph with its horizontal asymptote.

[3 marks]



13

9 (a) Show that

$$\sum_{r=1}^{n}\left(r^3+r^2\right)=\frac{1}{12}n(n+a)(n+b)(cn+a)$$

where a, b and c are integers.

[4 marks]

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9 (b) Find all the possible values of n in the range $1 \le n \le 50$ such that $\sum_{r=1}^{n}\left(r^3+r^2\right)$ is divisible by 37

[3 marks]

Answer ______

7

Turn over for the next question

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10 The locus of a point P is such that the distance from P to the point $(1, 0)$ is equal to **half** the distance from P to the line $x = 4$

The locus of P is the curve E

10 (a) Show that the equation of E is

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

[3 marks]

10 (b) The rectangular hyperbola H has equation

$$xy = \sqrt{3}$$

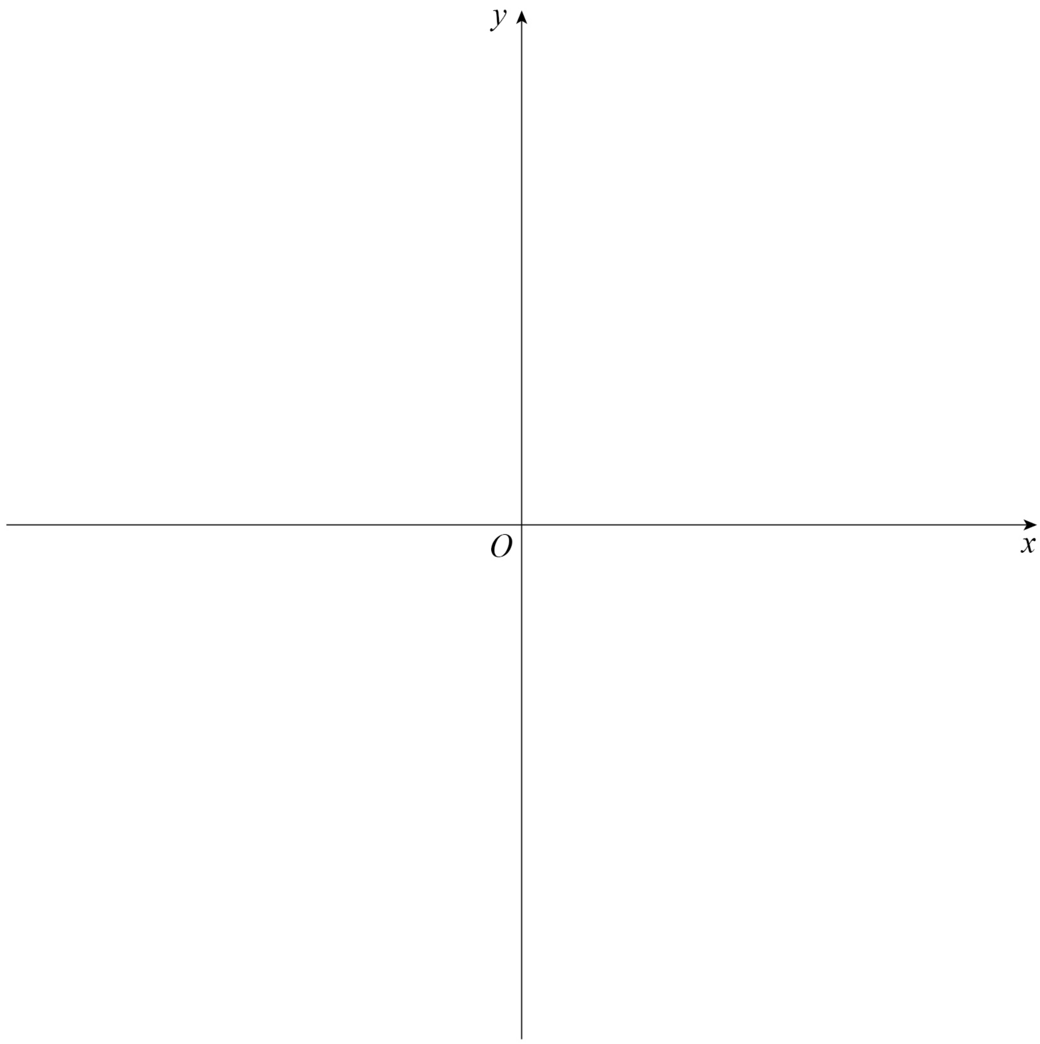
Find the coordinates of the two points of intersection of H and E

[4 marks]

Answer ______________________

10 (c) Sketch H and E on the axes below, showing all significant features.

[4 marks]



11

11 The circle C is the locus of points on an Argand diagram such that

$$|z-3|=2$$

The point Q is the centre of C

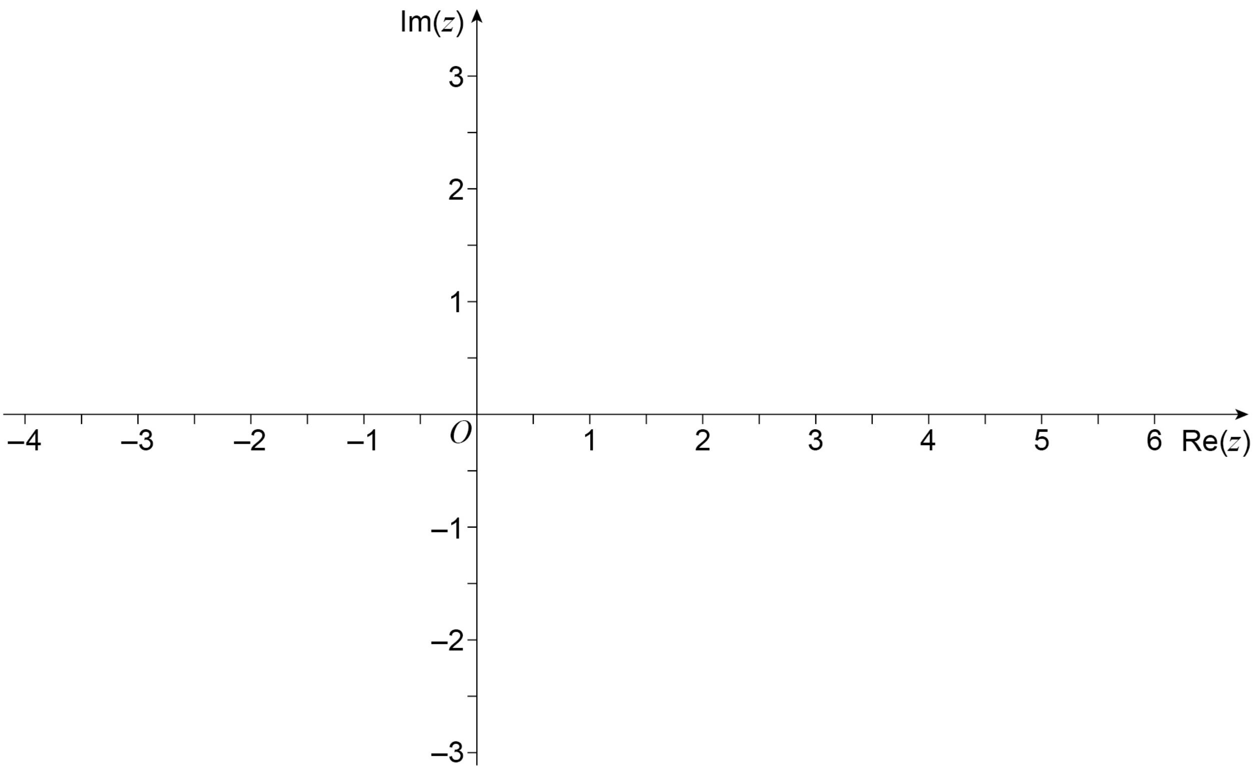
The line L is the locus of points on an Argand diagram such that

$$|z-3|=|z+3-3\text{i}|$$

The point P is the point on L which is closest to C

11 (a) On the Argand diagram, draw C and L, and mark the points P and Q

[4 marks]



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11 (b) PT and PS are tangents to C, touching the circle at T and S

Find the exact area of the quadrilateral $PTQS$

[5 marks]

Answer ______________________

9

END OF QUESTIONS

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ANSWER IN THE SPACES PROVIDED

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