

INTERNATIONAL A-LEVEL MATHEMATICS MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

June 2023

Version: 1.0 Final



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Key to mark scheme abbreviations

| Μ | Mark is for method |
|-------------------------|--|
| m | Mark is dependent on one or more M marks and is for method |
| Α | Mark is dependent on M or m marks and is for accuracy |
| В | Mark is independent of M or m marks and is for method and accuracy |
| E | Mark is for explanation |
| $\sqrt{\mathbf{or}}$ ft | Follow through from previous incorrect result |
| CAO | Correct answer only |
| CSO | Correct solution only |
| AWFW | Anything which falls within |
| AWRT | Anything which rounds to |
| ACF | Any correct form |
| AG | Answer given |
| SC | Special case |
| oe | Or equivalent |
| A2, 1 | 2 or 1 (or 0) accuracy marks |
| – <i>x</i> EE | Deduct <i>x</i> marks for each error |
| NMS | No method shown |
| Ы | Possibly implied |
| SCA | Substantially correct approach |
| sf | Significant figure(s) |
| dp | Decimal place(s) |

| Q | Answer | Marks | Comments |
|---------|--|-------|--|
| 1(a)(i) | $\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right] = kx^4 (3x^5 - 4)^7$ | M1 | <i>k</i> is an integer or product |
| | $\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right] = 120x^4 (3x^5 - 4)^7$ | A1 | Accept $5 \times 3 \times 8 \times x^4 (3x^5 - 4)^7$ |
| | | 2 | |

| Q | Answer | Marks | Comments |
|----------|---|-------|----------------------|
| 1(a)(ii) | $\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right] = \frac{p \times x^3(7x-5) - qx^4}{(7x-5)^2}$ | M1 | <i>p, q</i> integers |
| | $\left[\frac{dy}{dx} = \right] \frac{12x^3(7x-5) - 21x^4}{(7x-5)^2}$ | A1 | oe |
| | or $\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right] px^4 (7x-5)^{-2} + qx^3 (7x-5)^{-1}$ | (M1) | |
| | $\left[\frac{\mathrm{d}y}{\mathrm{d}x}\right] 3x^4(-1)(7)(7x-5)^{-2} + (7x-5)^{-1}12x^3$ | (A1) | |
| | $\left[\frac{dy}{dx} = \frac{63x^4 - 60x^3}{(7x - 5)^2}\right]$ | | |
| | | 2 | |

| Q | Answer | Marks | Comments |
|-----------|--|-------|--|
| 1(a)(iii) | $\frac{2x}{2} + 2y\frac{dy}{dt} = y + x\frac{dy}{dt}$ | M1 | Correct differentiation of y^2 or xy |
| | x^2 dx dx | B1 | Correct differentiation of $ln(x^2)$ |
| | $\begin{bmatrix} \frac{dy}{dx} = \end{bmatrix} \frac{y - \frac{2x}{x^2}}{2y - x}$ $\begin{bmatrix} \frac{dy}{dx} = \frac{xy - 2}{x(2y - x)} \end{bmatrix}$ | A1 | oe |
| | | 3 | |

| Q | Answer | Marks | Comments |
|---------|--|-------|---|
| 1(b)(i) | $\int \frac{x-6}{x^2-12x+5} dx = k \ln \left x^2 - 12x + 5 \right $ | M1 | $k \neq 0$ |
| | $=\frac{1}{2}\ln x^2-12x+5 $ [+c] | A1 | oe Must be in terms of <i>x</i> condone brackets as moduli signs |
| | | 2 | |

| Q | Answer | Marks | Comments |
|----------|--|-------|--|
| 1(b)(ii) | $\left[\frac{\mathrm{d}}{\mathrm{d}x}(2x^2+3x-1)\right]=4x+3$ | B1 | PI |
| | $\left[\int \frac{8x+6}{(2x^2+3x-1)^3} \mathrm{d}x = \right] k(2x^2+3x-1)^{-2}$ | М1 | $k \neq 0$ |
| | $=\frac{-1}{(2x^{2}+3x-1)^{2}} [+c]$ | A1 | oe Must be in terms of <i>x</i> |
| | | 3 | |
| | | | |

| Question 1 Total 12 |
|---------------------|
|---------------------|

| Q | Answer | Marks | Comments |
|---------|--|-------|---|
| 2(a)(i) | $f(x) = 2^{-x} - 4 + 2x$ $f(1.8) = 2^{-1.8} - 4 + 3.6 = -0.1[12825]$ $f(1.9) = 2^{-1.9} - 4 + 3.8 = 0.06[794]$ | M1 | Or reverse Both values rounded or truncated to at least 1sf |
| | Change of sign, 1.8 $< \alpha <$ 1.9 | A1 | Must have both statement and interval in words or symbols or comparing 2 sides: at 1.8, $2^{-1.8} = 0.2[871] < 0.4$; at 1.9, $2^{-1.9} = 0.26[79] > 0.2$ Accuracy as before (M1) Conclusion as before (A1) |
| | | 2 | |

| Q | Answer | Marks | Comments |
|----------|----------------------------|-------|-------------------------------|
| 2(a)(ii) | $2x = 4 - 2^{-x}$ | | |
| | $x = 2 - \frac{2^{-x}}{2}$ | | |
| | $x = 2 - 2^{-x-1}$ | | |
| | $x = 2 - 2^{-(x+1)}$ | B1 | AG Must be convincingly shown |
| | | 1 | |

| Q | Answer | Marks | Comments |
|-----------|---------------|-------|-------------------|
| 2(a)(iii) | $x_2 = 1.856$ | B1 | AWRT 1.856 |
| | $x_3 = 1.862$ | B1 | CAO |
| | | 2 | |

| Q | | Answer | Marks | Comments |
|------|----------------------------------|---|-------|--|
| 2(b) | x | у | | |
| | 1 | $2^{-1} = 0.5$ | | |
| | 1.75 | 2 ^{-1.75} = 0.2973018 | B1 | All five correct <i>x</i> values (and no extra used) PI by five correct <i>y</i> values |
| | 2.5 | $2^{-2.5} = 0.1767767$ | M1 | At least four correct y values in exact |
| | 3.25 | $2^{-3.25} = 0.1051121$ | | to three dp or better (in table or formula) |
| | 4.0 | $2^{-4} = 0.0625$ | | (PI by AWRT correct answer) |
| | $\frac{1}{3}$ × 0.75[+0.1051 | 0.5+0.0625+4(0.2973018 121)+2×0.1767767] | m1 | Correct sub into formula with $h = 0.75$ OE and at least four correct <i>y</i> values either listed, with + signs, or totalled. (PI by AWRT correct answer) |
| | 0.6314 | | A1 | CAO , must see this value exactly and no error seen |
| | | | 4 | |

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| Q | Answer | Marks | Comments |
|------|--|-------|----------|
| 3(a) | Translation | B1 | |
| | $\begin{bmatrix} -1\\ 0 \end{bmatrix}$ | B1 | |
| | Stretch | B1 | |
| | SF 0.5, parallel to <i>y</i> -axis | B1 | |
| | | | |
| | | 4 | |

| Q | Answer | Marks | Comments |
|------|---|----------------|--|
| 3(b) | $\begin{array}{c c} & & & \\ & & & \\ & & & \\ \hline -2 & -1 & 0 & 1 & 2 & \theta \\ & & & 1 & 2 & \theta \\ & & & 1 & 2 & \theta \\ & & & 1 & 2 & \theta \\ & & & 1 & 2 & \theta \end{array}$ | B1 B1 B1 | Correct shape and position Min at (–1, 0.5) may be written as coordinates or indicated on the diagram <i>y</i> -intercept as (0, 0.9) AWRT oe may be written as coordinates or indicated on the diagram |
| | | 3 | |

| Q | Answer | Marks | Comments |
|---------|---|-------|----------|
| 3(c)(i) | $\left[\frac{\mathrm{d}x}{\mathrm{d}y}\right] = \frac{1}{2}\sec y \tan y$ | B1 | oe |
| | | 1 | |

| Q | Answer | Marks | Comments |
|----------|---|-------|---|
| 3(c)(ii) | $\frac{\mathrm{d}y}{\mathrm{d}x} \left[= \frac{2}{\sec y \tan y} \right] = \frac{2\cos^2 y}{\sin y}$ | M1 | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$ |
| | $=\frac{2(1-\sin^2 y)}{\sin y}$ | A1 | ое |
| | | 2 | |

| Question 3 Tota | 10 | |
|-----------------|----|--|
|-----------------|----|--|

| Q | Answer | Marks | Comments |
|-----------|---|----------|--|
| 4(a) | $[f(x)] \ge 0$ | B1 | Do not allow $x \ge 0$ |
| | | 1 | |
| Q | Answer | Marks | Comments |
| 4(b)(i) | $fg(x) = \sqrt{1 - \frac{2}{x - 1}}$ | B1 | oe eg $\sqrt{\frac{x-3}{x-1}}$ |
| | | 1 | |
| Q | Answer | Marks | Comments |
| 4(b)(ii) | $\begin{bmatrix} 1 - \frac{2}{x - 1} \ge 0 \end{bmatrix}$ $x \ge 3$ x < 1 | B1 B1 | oe oe |
| | | 2 | |
| Q | Answer | Marks | Comments |
| 4(b)(iii) | $1 - \frac{2}{x - 1} = 9$ [x - 1 = -0.25] | M1 | Attempt to solve by eliminating square root. |
| | <i>x</i> = 0.75 | A1 | only |
| | | 2 | |
| Q | Answer | Marks | Comments |
| 4(c)(i) | $x = \sqrt{1 - \frac{2}{y - 1}}$ | М1 | Interchange x and y |
| | $x^2 = 1 - \frac{2}{y - 1}$ | М1 | Attempt to solve |
| | $h(x) = \frac{3 - x^2}{1 - x^2}$ | A1 | Allow $p = 3, q = 1$ |
| | | 3 | |
| Q | Answer | Marks | Comments |
| 4(c)(ii) | $\left[9-3x^2=11-11x^2\right]$ | | |
| | $8x^2 = 2$ | M1 | |
| | $x = [\pm]0.5$ | A1 | At least one correct |
| | | 2 | |
| | | • | • |

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| Q | Answer | Marks | Comments |
|------|---|-------|----------|
| 5(a) | $[7\cos\theta + 24\sin\theta =]$ $R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ | M1 | PI |
| | <i>R</i> = 25 | B1 | |
| | <i>α</i> = 1.29 | A1 | AWRT |
| | $[7\cos\theta + 24\sin\theta = 25\cos(\theta - 1.29)]$ | | |
| | | 3 | |

| Q | Answer | Marks | Comments |
|------|---|-------|---|
| 5(b) | $2\csc 4x + 2\cot 4x = \frac{2}{\sin 4x} + \frac{2\cos 4x}{\sin 4x}$ | | |
| | $=\frac{2+2(2\cos^2 2x-1)}{2\sin 2x\cos 2x}$ | M1 | Correct use of both double angle formulae |
| | $=\frac{4\cos^2 2x}{2\sin 2x\cos 2x}_4$ 2 cos 2x | A1 | |
| | $= \frac{\sin 2x}{\sin 2x}$ $= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$ | M1 | Correct use of both double angle formulae |
| | $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \cot x - \tan x$ | A1 | AG no errors seen |
| | | 4 | |

| Q | Answer | Marks | Comments |
|------|---|--------------|---|
| 5(c) | $5(\sec^2 Y - 1) = 7 - 4 \sec Y$ | M1 | Correct use of trig identity PI |
| | $5 \sec^{2} Y + 4 \sec Y - 12 = 0$ (5 \sec Y - 6)(\sec Y + 2)[= 0] $\sec Y = -2, \frac{6}{5}$ $\left[\cos Y = \frac{5}{6}, -0.5\right]$ | A1 | Both values correct |
| | <i>Y</i> = 0.585, -0.585, 5.697, 6.868, 2.094, 4.188, 8.377 | A1 | For any of these values, or rounded or truncated to 2 dp PI |
| | <i>y</i> = 0.59, 1.35, 1.93, 2.69 | B1 | Sight of any of these values |
| | | B1 | All 4 correct and no extras in interval (ignore answers outside interval) |
| | $5\frac{\sin^2 Y}{\cos^2 Y} = 7 - \frac{4}{\cos Y}$ $5(1 - \cos^2 Y) = 7\cos^2 Y - 4\cos Y$ $12\cos^2 Y - 4\cos Y - 5 = 0$ $(6\cos Y - 5)(2\cos Y + 1)[= 0]$ $\cos Y = \frac{5}{6}, -0.5$ | [M1] [A1] | |
| | | 5 | |
| | Question 5 Total | 12 | |

| Q | Answer | Marks | Comments |
|------|---|-------|-------------------------------|
| 6(a) | $4(0.5)^{3} + a(0.5)^{2} + b \times 0.5 + c = 0$ $4(1.5)^{3} + a(1.5)^{2} + b \times 1.5 + c = 15$ | М1 | One correct substitution |
| | $4((1.5)^{3} - (0.5)^{3}) + a((1.5)^{2} - (0.5)^{2}) + b(1.5 - 0.5) = 15$ | m1 | Attempt to eliminate <i>c</i> |
| | or a+2b+4c = -2 9a+6b+4c = 6 | | |
| | 13+2a+b=15 or $8a+4b=8$ leading to $2a+b=2$ | A1 | AG no errors seen |
| | | 3 | |

| Q | Answer | Marks | Comments |
|------|---|----------|--|
| 6(b) | $4(-0.5)^3 + a(-0.5)^2 + b \times (-0.5) + c = 9$ | M1 | Correct substitution |
| | a-2b+4c = 38 $4b = -40 \implies b = -10$ a = 6 c = 3 | m1 | Attempt to solve simultaneous equations in a , b and c |
| | $f(x) = 4x^{3} + 6x^{2} - 10x + 3$ $[f(-1.5)] = 18$ | A1 B1 | All 3 values correct |
| | | 4 | |

| | | Question 6 Total | 7 | |
|--|--|------------------|---|--|
|--|--|------------------|---|--|

| Q | Answer | Marks | Comments |
|------|--|-------|---|
| 7(a) | $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2t - 1 - 2t}{(2t - 1)^2} \text{or} (-t)(2t - 1)^{-2} + (2t - 1)^{-1}$ | M1 | oe Either derivative correct |
| | $\frac{\mathrm{d}y}{\mathrm{d}t} = 2 - \frac{1}{2\sqrt{t}}$ | A1 | oe Both correct |
| | $\frac{dy}{dx} = -\frac{(2t-1)^2 (4\sqrt{t}-1)}{2\sqrt{t}}$ | m1 | |
| | $t = 1, \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{2}, \ x = 1, \ y = 1$ | A1 | All correct PI |
| | Equation of normal: $y-1=\frac{2}{3}(x-1)$ | M1 | Correct attempt to find <i>their</i> normal |
| | 3y - 2x = 1 | A1 | Allow $p = 3, q = -2, r = 1,$ |
| | | | Allow integer multiples |
| | | 6 | |

| Q | Answer | Marks | Comments |
|---------|--|-------|-------------------------------|
| 7(b)(i) | $x + y = 2e^{2m}$ | M1 | Both correct |
| | $x - y = 2e^{-2m}$ | | |
| | $\frac{x+y}{2} = \frac{2}{x-y}$ | m1 | Attempt to eliminate <i>m</i> |
| | $x^2 = y^2 + 4$ | A1 | |
| | or | | |
| | $x^{2} = \left(e^{2m}\right)^{2} + \left(e^{-2m}\right)^{2} + 2$ | [M1] | Either equation correct |
| | $y^{2} = \left(e^{2m}\right)^{2} + \left(e^{-2m}\right)^{2} - 2$ | [m1] | Both correct |
| | $x^2 = y^2 + 4$ | [A1] | |
| | | 3 | |

| Q | Answer | Marks | Comments |
|----------|---|-------|----------|
| 7(b)(ii) | $\begin{bmatrix} x+y=2e^{kn} & x-y=2e^{-kn} \\ \frac{x+y}{2}=\frac{2}{x-y} \end{bmatrix}$ | | |
| | $x^2 = y^2 + 4$ | B1ft | ое |
| | | 1 | |

| Q | Answer | Marks | Comments |
|------|--|------------------|---|
| 7(c) | $x^{2} = a^{2} \sin^{2} \theta + b^{2} \cos^{2} \theta + 2ab \cos \theta \sin \theta$ $y^{2} = a^{2} \cos^{2} \theta + b^{2} \sin^{2} \theta - 2ab \cos \theta \sin \theta$ | M1 | Squaring one expression (condone one slip) |
| | $x^{2} + y^{2}$ $= a^{2}(\sin^{2}\theta + \cos^{2}\theta) + b^{2}(\cos^{2}\theta + \sin^{2}\theta)$ $x^{2} + y^{2} = a^{2} + b^{2}$ or $ax - by = \sin\theta(a^{2} + b^{2})$ $ay + bx = \cos\theta(a^{2} + b^{2})$ | m1 A1 [M1] | Attempt to eliminate θ oe One expression in either sin θ or cos θ (condone one slip) |
| | $\left(\frac{ax-by}{a^2+b^2}\right)^2 + \left(\frac{ay+bx}{a^2+b^2}\right)^2 = 1$ $\left[\left(ax-by\right)^2 + \left(ay+bx\right)^2 = \left(a^2+b^2\right)^2\right]$ | [m1] [A1] | Attempt to eliminate θ |
| | | 3 | |

| | | Question 7 Total | 13 | |
|--|--|------------------|----|--|
|--|--|------------------|----|--|

| Q | Answer | Marks | Comments |
|------|---|----------|----------|
| 8(a) | $\frac{dP}{dt} = kP$ $[3000 = k \times 1000000]$ $\frac{dP}{dt} = 0.003P$ | M1 A1 | oe |
| | | 2 | |

| Q | Answer | Marks | Comments |
|------|--|-------|---|
| 8(b) | $2x\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - y^2$ | | |
| | $\int \frac{\mathrm{d}y}{4-y^2} = \int \frac{\mathrm{d}x}{2x}$ | B1 | PI Correctly separate variables |
| | $\frac{1}{4-y^2} = \frac{A}{2-y} + \frac{B}{2+y}$ | M1 | Using partial fractions |
| | 1 = A(2+y) + B(2-y) A = 0.25, B = 0.25 | A1 | Both correct |
| | $\frac{1}{4}\int \frac{1}{2-y} + \frac{1}{2+y} dy = \frac{1}{2}\int \frac{dx}{x}$ | | |
| | $-\ln(2-y) + \ln(2+y) = 2\ln x + \ln A \text{ (or } + c)$ | m1 | Correct integration (at least 3 terms correct) |
| | $\ln\frac{2+y}{2-y} = \ln Ax^2$ | | $\int \frac{\mathrm{d}y}{4-y^2} = \frac{1}{4} \ln \left \frac{2+y}{2-y} \right \text{ scores } \mathbf{M1A1m1}$ |
| | $Ax^2 = \frac{2+y}{2-y}$ | m1 | Eliminating 'In' from <i>their</i> equation – must have scored M1m1 |
| | $(1,1) \Rightarrow A = 3$ $3x^{2}(2-y) = 2+y$ | | or $(1,1) \Rightarrow c = \pm \frac{1}{4} \ln 3$ |
| | $y = \frac{2(3x^2 - 1)}{(3x^2 + 1)}$ | A1 | ACF |
| | | 6 | |
| | | | |

| | | Question 8 Total | 8 | |
|--|--|------------------|---|--|
|--|--|------------------|---|--|

| Q | Answer | Marks | Comments |
|------|---|-------|------------------------------------|
| 9(a) | $\frac{\mathrm{d}y}{\mathrm{d}x} = Ax^{0.5}\mathrm{e}^{-0.5x} + Bx^{-0.5}\mathrm{e}^{-0.5x}$ | M1 | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = x^{0.5} (-0.5) \mathrm{e}^{-0.5x} + 0.5 x^{-0.5} \mathrm{e}^{-0.5x}$ | A1 | |
| | $x^{0.5}(-0.5)e^{-0.5x} + 0.5x^{-0.5}e^{-0.5x} = 0,$ $0.5x^{0.5} = 0.5x^{-0.5}$ x = 1 | m1 | Equating to 0 and attempt to solve |
| | $(1, e^{-0.5})$ | A1 | Allow $x = 1$, $y = e^{-0.5}$ |
| | | 4 | |

| Q | Answer | Marks | Comments |
|------|---|-------|---|
| 9(b) | $V = \pi \int_1^2 \left(\sqrt{x} \mathrm{e}^{-0.5x}\right)^2 \mathrm{d}x$ | B1 | PI by final answer |
| | $\int x \mathrm{e}^{-x} \mathrm{d}x = -x \mathrm{e}^{-x} - \int -\mathrm{e}^{-x} \mathrm{d}x$ | M1 | Use of parts formula |
| | $= -xe^{-x} - e^{-x}$ | A1 | |
| | $\int_{1}^{2} = [\pi] \left(-x e^{-x} - e^{-x} \right)_{1}^{2}$ | | |
| | $= [\pi] \left(\left(-2e^{-2} - e^{-2} \right) - \left(-e^{-1} - e^{-1} \right) \right)$ | m1 | Correct substitution of <i>their</i> 1, 2 into <i>their</i> expression in the form $axe^{-x}+be^{-x}$ |
| | $=\pi\left(\frac{2}{e}-\frac{3}{e^2}\right)$ | A1 | Allow $a = 2, b = -3$ |
| | | 5 | |

| Question 9 Tota | 9 | |
|-----------------|---|--|
|-----------------|---|--|

| Q | Answer | Marks | Comments |
|----|--|-------|--|
| 10 | $\frac{\mathrm{d}u}{\mathrm{d}\theta} = \cos\theta$ | B1 | oe Pl |
| | $\left[\int \frac{\cos^3\theta}{\left(1+\sin\theta\right)^{1.5}} d\theta = \right] \int \frac{1-\left(u-1\right)^2}{\left(u\right)^{1.5}} du$ | М1 | All in terms of u , condone omission of du |
| | $\int \frac{2u-u^2}{(u)^{1.5}} \mathrm{d}u$ | | |
| | $=\int 2u^{-0.5} - u^{0.5} du$ | A1 | Must see du here, or earlier |
| | $=4u^{0.5}-\frac{2}{2}u^{1.5}$ [+c] | M1 | $Au^{0.5} + Bu^{1.5}$ |
| | 3 | A1ft | Correct integration |
| | $\begin{bmatrix} \theta \end{bmatrix}_{0}^{\frac{\pi}{2}} = \begin{bmatrix} u \end{bmatrix}_{1}^{2}$ $\begin{bmatrix} \frac{\pi}{2} & \cos^{3}\theta \\ 0 & (1+\sin\theta)^{1.5} & d\theta \end{bmatrix} = \left(4u^{0.5} - \frac{2}{3}u^{1.5}\right)_{1}^{2}$ | B1 | Change of limits, maybe seen earlier (may change back to θ and not change limits) |
| | $= \left(4 \times 2^{0.5} - \frac{2}{3} \times 2^{1.5}\right) - \left(4 - \frac{2}{3}\right)$ $= \frac{8}{3} \times 2^{0.5} - \frac{10}{3}$ | М1 | Correct substitution of <i>their</i> limits into <i>their</i> expression of the form $Au^{0.5} + Bu^{1.5}$ |
| | $=\frac{2}{3}(4\sqrt{2}-5)$ | A1 | Allow $p = 4, q = -5$ |
| | | 8 | |
| | | | T |

| | Question 10 Total | 8 | |
|--|-------------------|---|--|
|--|-------------------|---|--|

| Q | 4Answer | Marks | Comments |
|-------|--|-------|---|
| 11(a) | $f(x) = \frac{A}{(3-x)} + \frac{B}{(3-x)^2} + \frac{C}{(1-3x)}$ | | |
| | $7x^{2} - 17x + 12 = A(3 - x)(1 - 3x) + B(1 - 3x) + C(3 - x)^{2}$ | B1 | Correctly eliminating fractions |
| | x = 3, 24 = -8B, B = -3 | M1 | Attempt at finding one constant |
| | $x = \frac{1}{3}, \frac{64}{9} = \frac{64}{9}C, C = 1$ x = 0, 12 = 3A + B + 9C, A = 2 | A1 | Two constants correct |
| | $f(x) = \frac{2}{(3-x)} - \frac{3}{(3-x)^2} + \frac{1}{(1-3x)}$ | A1 | Allow <i>A</i> = 2, <i>B</i> = –3, <i>C</i> = 1 |
| | | | Allow equivalent methods |
| | | 4 | |

| Q | Answer | Marks | Comments |
|-------|---|-------|----------|
| 11(b) | $(3-x)^{-1} = \frac{1}{3}(1-\frac{1}{3}x)^{-1}$ | M1 | |
| | $=\frac{1}{3}+\frac{1}{9}x+\frac{1}{27}x^{2}$ | A1 | |
| | | 2 | |

| Q | Answer | Marks | Comments |
|-------|---|----------|--|
| 11(c) | $(3-x)^{-2} = \left(\frac{1}{3}\right)^2 \left(1 + \frac{2}{3}x + \frac{1}{3}x^2\right)$ | M1 | oe either expansion correct |
| | $(1-3x)^{-1} = 1 + 3x + 9x^2$ | A1 | Both correct |
| | f(x): $2(\frac{1}{3} + \frac{1}{9}x + \frac{1}{27}x^{2}) - 3 \times \frac{1}{9}(1 + \frac{2}{3}x + \frac{1}{3}x^{2}) + 1(1 + 3x + 9x^{2})$ f(x) = $\frac{4}{3} + 3x + \frac{242}{27}x^{2}$ $\left[D = \frac{4}{3}, E = 3, F = \frac{242}{27}\right]$ | m1 A1 | Correct substitutions of <i>their</i> expansions into <i>their three term part (a)</i> |
| | | 4 | |

| Question 11 Total | 10 | |
|-------------------|----|--|
| | | |

| | Answer | Marks | Comments |
|-------|---|-------|---|
| 12(a) | $\begin{bmatrix} l_1 : \mathbf{r} = \end{bmatrix} \begin{bmatrix} 2\\-1\\3 \end{bmatrix} + \lambda \begin{bmatrix} 3\\-1\\-4 \end{bmatrix}$ | B1 | oe $\begin{bmatrix} l_1 \\ \mathbf{r} \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$ |
| | | 1 | |

| Q | Answer | Marks | Comments |
|----------|---|-------|--|
| 12(b)(i) | $2+3\lambda = -2-\mu$ -1- $\lambda = 5-2\mu$ | M1 | Both equations correct, ft from (a) |
| | $\begin{bmatrix} 7\lambda = -14 \Longrightarrow \lambda = -2, \end{bmatrix} \mu = 2$ | A1 | |
| | $[z:3-4\times -2=11=7+2k]$ k=2 | A1 | |
| | | 3 | 2 |

| Q | Answer | Marks | Comments |
|-----------|-------------|-------|----------|
| 12(b)(ii) | (-4, 1, 11) | B1 | |
| | | 1 | |

| Q | Answer | Marks | Comments |
|-------|---|----------|--|
| 12(c) | Coords of $D(-2-d, 5-2d, 7+2d)$ | B1ft | Ft <i>their 'k'</i> in (b)(i) |
| | $\overrightarrow{CD} = \begin{bmatrix} -5 - d \\ 1 - 2d \\ 3 + 2d \end{bmatrix}$ | M1 | oe Seen or used, Ft <i>their 'D'</i> |
| | $\begin{bmatrix} -5-d\\1-2d\\3+2d \end{bmatrix} \begin{bmatrix} -1\\-2\\2 \end{bmatrix} [=0]$ | m1 | Correct use of dot product with their CD and their k |
| | $9d = -9 \implies d = -1$ | A1 | |
| | Coords of D (-1, 7, 5) Dist = $\sqrt{(-1-3)^2 + (7-4)^2 + (5-4)^2}$ = $\sqrt{26}$ | M1 A1 | Must have scored m1 |
| | | 6 | |

| | | Question 12 Total | 11 | |
|--|--|-------------------|----|--|
|--|--|-------------------|----|--|