OXFORD

INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
MATHEMATICS
MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

June 2023

Version: 1.0 Final

236XMA05/MS

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$(1+2+4+8+m)\times 4.4$ $=1\times 4+2\times 1+4\times 3+8\times 5+m\times 6$	M1 A1	**M1**: Forming centre of mass equation using x-coordinates or y-coordinates Allow one error **A1**: Both sides of x-coordinate equation correct
	$66+4.4m=58+6m$ $m=5$	A1	Correct value for m
	$(1+2+4+8+m)\times 3.9$ $=1\times 2+2\times 4+4\times 1+8\times 3+m\times k$ $78=38+5k$	A1ft	Both sides of y-coordinate equation correct, **ft** their m if substituted
	$k=8$	A1ft	Correct value for k from their m $k=\dfrac{20.5}{\text{their } m}+3.9$
		5	

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2(a)	$P = Fv$ $P = (14 \times 9.8) \times 0.8$	M1	Use of $P = Fv$ Condone 9.81 m s^{-2} for g, but not 10 m s^{-2}
	$P = 110$	A1	Unrounded answer is 109.76
	Units = W or J s^{-1}	B1	**oe**
		3	

Q	Answer	Marks	Comments
2(b)	Initial Energy (KE & GPE) $= \frac{1}{2} \times 14 \times 0.8^2 + 14 \times 9.8 \times 1.6$ $= 224$ [J]	M1	Adds potential energy and kinetic energy An initial energy of 219.52 J has not included the initial KE
	Final Energy (KE only) $224 = \frac{1}{2} \times 14 \times v^2$	M1	Setting their 224 J equal to final KE **PI** by correct answer of 5.7
	$v = 5.7$ [m s^{-1}]	A1	**CAO** Allow 5.6568... [m s^{-1}] or $4\sqrt{2}$ [m s^{-1}] as the correct final answer. If initial KE not included, answer should be 5.6 [m s^{-1}] – award **SC2**
		3	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	[Resultant Force =] $5000v^2e^{-0.32v} - 0.26v^2$	B1	oe
		1	

Q	Answer	Marks	Comments
3(b)	$5000 \times 8.3^2 e^{-0.32 \times 8.3} - 0.26 \times 8.3^2$	M1	Substituting $v = 8.3$ into their resultant force expression from **part (a)**
	$= 24172.32125$ [N]	A1	Correct magnitude of resultant force, **AWRT** 24000 [N] **PI** by correct answer
	$F = ma \quad \Rightarrow \quad a = \dfrac{F}{m}$ $a = \dfrac{24172.32125}{1800}$ $a = 13 \left[\text{m s}^{-2}\right]$	A1ft	Unrounded answer is 13.429… [m s^{-2}]
		3	

Q	Answer	Marks	Comments
3(c)	At maximum speed, resultant force on car is zero. $5000v^2e^{-0.32v} - 0.26v^2 = 0$		
	$e^{-0.32v} = \dfrac{13}{250\,000} \quad \left[= 5.2 \times 10^{-5}\right]$	M1	This line or better **PI** by correct final answer
	$v = \dfrac{\ln\left(\dfrac{250\,000}{13}\right)}{0.32} \quad \left[= \dfrac{\ln(19230.76923)}{0.32}\right]$		Note $\ln\left(\dfrac{250\,000}{13}\right) = 9.864...$
	$v = 31 \left[\text{m s}^{-1}\right]$	A1	Unrounded answer is 30.8258… [m s^{-1}]
		2	

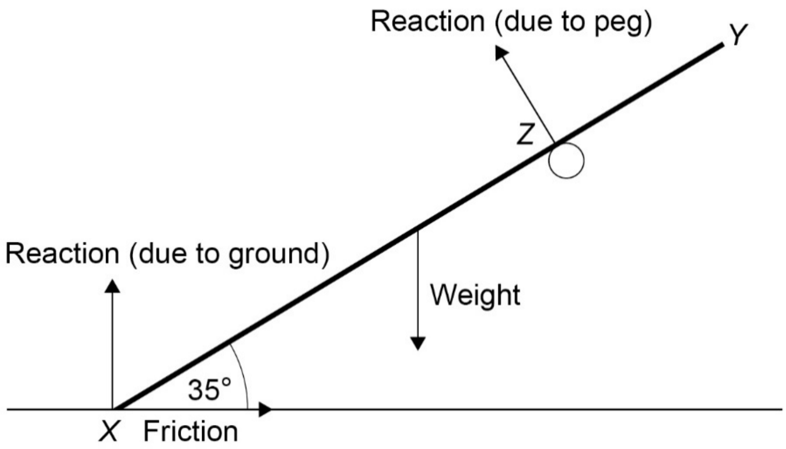
	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4(a)	$\sqrt{25\cos^2(3t) + 1369\sin^2(3t)} = 19$	M1	Forming a distance equation
	$\sqrt{25 + 1344\sin^2(3t)} = 19$	M1	Use of $\cos^2(3t)+\sin^2(3t)=1$
	$1344\sin^2(3t) = 336$ or $1344\cos^2(3t) = 1008$		
	$\sin^2(3t) = \frac{1}{4}$ or $\cos^2(3t) = \frac{3}{4}$		
	$\sin(3t) = [\pm]\frac{1}{2}$ or $\cos(3t) = [\pm]\frac{\sqrt{3}}{2}$	m1	**PI** by at least one correct value of t or sight of 10 and 50
	$t = 0.17$	A1	$t = \frac{\pi}{18}$ One correct value of t
	$t = 0.87$	A1	$t = \frac{5\pi}{18}$ The second correct value of t and no others, and no errors throughout solution
		5	

Q	Answer	Marks	Comments
4(b)	$\mathbf{r} = 5\cos(3t)\,\mathbf{i} + 37\sin(3t)\,\mathbf{j}$		
	$\mathbf{v} = -15\sin(3t)\,\mathbf{i} + 111\cos(3t)\,\mathbf{j}$	M1 A1	**M1**: At least one component correct or $\mathbf{v} = \mp 15\sin(3t)\,\mathbf{i} \pm 111\cos(3t)\,\mathbf{j}$ **A1**: Both components correct Both marks **PI** by correct acceleration vector
	$\mathbf{a} = -45\cos(3t)\,\mathbf{i} - 333\sin(3t)\,\mathbf{j}$	m1	At least one component correct
	$\mathbf{a} = -45\cos\left(\frac{3\pi}{4}\right)\mathbf{i} - 333\sin\left(\frac{3\pi}{4}\right)\mathbf{j}$	M1	Substituting $t = \frac{\pi}{4}$ into their **a**
	$\mathbf{a} = \frac{45}{\sqrt{2}}\,\mathbf{i} - \frac{333}{\sqrt{2}}\,\mathbf{j}$	A1	Any correct form involving surds **CSO**, do not **ISW**
		5	

	Question 4 Total	**10**	

Q	Answer	Marks	Comments
5(a)	The entire weight of the rod appears to act through the midpoint of *XY*	E1	Any correct explanation
		1	

Q	Answer	Marks	Comments
5(b)	**See artwork below**	M1 A1	At least two of the forces drawn on a diagram in the correct direction and named. All forces correctly drawn and named. Do not condone 'gravity' in place of 'weight'
	 		
		2	

Q	Answer	Marks	Comments
5(c)	Taking moments about X		
	$4\times15\times9.8\cos(35°)=6R_{\text{peg}}$	M1	At least one side correct $R_{\text{peg}}=$ reaction force on rod due to peg
	$R_{\text{peg}}=80.2769...$ [N]	A1	$R_{\text{peg}}=10g\cos(35°)$
	Forces in equilibrium (vertical) $R_{\text{ground}}+R_{\text{peg}}\cos(35°)=15\times9.8$	M1	Both sides correct $R_{\text{ground}}=$ reaction force on rod due to ground
	$R_{\text{ground}}=15\times9.8-80.2769...\times\cos(35°)$		
	$R_{\text{ground}}=81.2410...$ [N]	A1	$R_{\text{ground}}=15g-10g\cos^2(35°)$
	Forces in equilibrium (horizontal) $F=R_{\text{peg}}\sin(35°)$	M1	Both sides correct **PI** $F=$ friction on rod due to rough ground
	$F=46.0449...$ [N]	A1	$F=10g\cos(35°)\sin(35°)$
	$F\le\mu R_{\text{ground}}$	M1	Condone $F=\mu R_{\text{ground}}$
	$46.0449...\le\mu\times81.2410...$		
	$0.5667...\le\mu$		$\dfrac{10g\cos(35°)\sin(35°)}{15g-10g\cos^2(35°)}\le\mu$
	The minimum value of μ is 0.567 to 3 sf	A1	
		8	

	Question 5 Total	**11**	

Q	Answer	Marks	Comments
6(a)	$\begin{bmatrix} t^2+4t+1 \\ 2t^2-2t \end{bmatrix}+\begin{bmatrix} 2t^2-1 \\ -t^2-2t+1 \end{bmatrix}+\begin{bmatrix} t^2-12t+3 \\ t^2-t+1 \end{bmatrix}$	M1	Summing the three forces
	$=\begin{bmatrix} 4t^2-8t+3 \\ 2t^2-5t+2 \end{bmatrix}$	A1	Both components correct
		2	

Q	Answer	Marks	Comments
6(b)	$\begin{bmatrix} 4\times 2^2-8\times 2+3 \\ 2\times 2^2-5\times 2+2 \end{bmatrix}$	M1	Substituting $t=2$ into their resultant force vector
	$[0.2\mathbf{a}=]\ \begin{bmatrix} 3 \\ 0 \end{bmatrix}$	A1	**PI** by correct magnitude of acceleration.
	Magnitude of acceleration is 15 [m s^{-2}]	A1	**CAO** Must be a positive scalar and not a vector.
		3	

Q	Answer	Marks	Comments
6(c)	$\begin{bmatrix} 4t^2-8t+3 \\ 2t^2-5t+2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	M1	Sets their resultant force vector from **part (a)** equal to the zero vector. **PI**
	$(2t-1)(2t-3)=0$ $(2t-1)(t-2)=0$	M1 A1	**M1**: At least one quadratic correctly factorised **PI** by correct pair of roots **A1**: Both quadratics correctly factorised **PI** by both correct pairs of roots
	$t=\frac{1}{2}$	A1	**oe** and no other values of t stated as the answer
		4	

	Question 6 Total	9	

Q	Answer	Marks	Comments
7(a)	Energy at *A* (GPE) $5\times9.8\times10=490$ [J]	B1	**PI** by later working May be seen in a calculation
	$F=0.3\times5\times9.8\times\cos(40°)$	B1	Friction, 11.26…
	Energy at *C* (GPE & WD against friction) $5\times9.8\times h$ $+\ 0.3\times5\times9.8\times\cos(40°)\times\dfrac{h-2}{\sin(40°)}$	B1	**PI** Correct energy expression, GPE + work done against friction, in terms of h or $x=\dfrac{h-2}{\sin(40°)}$
	Conservation of Energy (between *A* and *C*) $490=49h+\dfrac{14.7h-29.4}{\tan(40°)}$	M1	Setting their energy at *A* (or *B*, 392 J) equal to their energy at *C*
	$h=\dfrac{490+\dfrac{29.4}{\tan(40°)}}{49+\dfrac{14.7}{\tan(40°)}}\left[=\dfrac{490+35.037...}{49+15.518...}\right]$	m1	Attempt to rearrange their CoE equation for h or for finding $x=9.16799...$ or $h-2=5.89...$
	$h=7.89$ [m]	A1	**CAO** to 3 sf
		6	

Q	Answer	Marks	Comments
7(b)	Magnitude of force down the slope $[5\times9.8\times\sin(40°)=]$ 31.4965... [N]	M1	**M1** At least one correct magnitude of force
	Magnitude of force up the slope $[0.3\times5\times9.8\times\cos(40°)=]$ 11.2608...[N]	A1	**A1** Both magnitudes correct **M1 A1 PI** by resultant force of 20[.235] N
	[As the magnitude of the force down the slope is greater than the magnitude of the force up the slope] the particle slides back down [the rough track from *C* towards *B*]	E1ft	Correct conclusion based on their magnitudes of the forces up and down the slope.
		3	

	Question 7 Total	9	

Q	Answer	Marks	Comments
8(a)	When the ball collides with the wall, the vertical component of velocity is zero.	B1	Use of $v = 0$ in the vertical direction when ball collides with the wall May be seen in a calculation
	$v^2 = u^2 + 2as$		
	$0 = (15\sin\theta)^2 + 2\times(-9.8)\times 1.9$	M1	Use of $u = 15\sin\theta$ and $g = [\pm]\ 9.8$ with $v^2 = u^2 + 2as$
	$\sin^2\theta = \frac{931}{5625}\ \left[= \frac{37.24}{225} = 0.1655...\right]$	m1	This line of working or better
	$\sin\theta = [\pm]\frac{7\sqrt{19}}{75}\ \ [= 0.4068...]$		
	$\theta = 24.00589...^\circ \Rightarrow \theta = 24^\circ$	A1	**AG** Must be convincingly shown
		4	

Q	Answer	Marks	Comments
8(b)(i)	KE of ball before collision with wall $0.5\times0.4\times(15\cos24°)^2=37.55543...$ [J]	B1	Correct KE before collision **PI** by correct answer
	KE of ball after collision with wall $0.5\times37.55543...=18.77771...$ [J]	M1	0.5 multiplied by their KE before collision **PI** by correct answer
	$v=\sqrt{\frac{2KE}{m}}=\sqrt{\frac{2\times18.77771...}{0.4}}$ $=9.69$ [m s^{-1}]	A1	**CAO** [Unrounded answer is 9.68961…]
		3	

Q	Answer	Marks	Comments
8(b)(ii)	Time for ball to fall 1.9 metres back to ground level $s=\frac{1}{2}at^2 \Rightarrow t=\sqrt{\frac{2s}{a}}$ $t=\sqrt{\frac{2\times(-1.9)}{-9.8}}=0.62269...$ [s]	B1	**oe**, such as $\frac{\sqrt{19}}{7}$ **PI** by correct horizontal displacement
	Horizontal displacement of ball when it is at ground level $0.62269...\times9.69=6.03$ [m]	M1 A1	**M1**: Multiplies their time to fall by their answer from **(b)(i)** **A1**: Correct horizontal displacement. Allow values in the range 6.00 to 6.043
	As 6.03 [m] is less than 6.25 [m] [but greater than 5.75 m] so the ball does land in the hole.	E1ft	Correct conclusion based on comparing their horizontal displacement with the location of the hole, e.g. [5.75, 6.25]
		4	

	Question 8 Total	**11**	

Q	Answer	Marks	Comments
9(a)	Forces acting on *Y* $T = m_Y g = 8 \times 9.8$ $T = 78.4$ [N]	B1	**AG** Correct calculation leading to given result
		1	

Q	Answer	Marks	Comments
9(b)(i)	Forces in vertical direction acting on *X*		
	$T\cos\alpha = m_X g$	M1	Consideration of vertical forces on *X*
	$\cos\alpha = \dfrac{6 \times 9.8}{78.4} = 0.75$	A1	Correct fraction or division shown leading to printed result
		2	

Q	Answer	Marks	Comments
9(b)(ii)	Vertical component of force [from string acting on smooth ring] $T + T\cos\alpha$ or $m_X g + m_Y g$ $= 137.2$ [N]	M1	
	Horizontal component of force [from string acting on smooth ring] $T\sin\alpha = 78.4 \times \sqrt{1 - 0.75^2}$ $= 51.85672...$ [N]	M1	Note $\sin\alpha = \dfrac{\sqrt{7}}{4}$ $[= 0.6614...]$
	Magnitude of force from string acting on smooth ring $\sqrt{51.85672...^2 + 137.2^2} = 147$ [N]	A1	**AWRT** 147
		3	

Q	Answer	Marks	Comments
9(c)	Resultant force acting on *X* $T\sin\alpha = m_X\omega^2 r$ and $r = l\sin\alpha$ $l = \frac{78.4}{6\times 7^2}$ or $r = \frac{\sqrt{7}}{15}$ [= 0.176...] $l = \frac{4}{15}$ [m]	M1 m1 A1	Use of $F = m\omega^2 r$ **PI** by $\frac{49\sqrt{7}}{15}$ [= 8.642...], the acceleration, or correct working leading to a value for l or r **AWRT** 0.27 [m] **CSO**
		3	

Q	Answer	Marks	Comments
9(d)(i)	They both accelerate vertically downwards at 9.8 m s^{-2}	E1	Any correct similarity
		1	

Q	Answer	Marks	Comments
9(d)(ii)	*X* has a horizontal component of velocity whilst *Y* only has a vertical component of velocity	E1	Any correct difference
		1	

Q	Answer	Marks	Comments
9(d)(iii)	They both fall the same vertical displacement and their initial components of velocity in the vertical direction are zero [and they both experience the same acceleration] Hence they both have reach the ground simultaneously [as they have the same time of flight]	E1 E1	Reference to same vertical displacement or $u = 0$ in vertical direction Reference to same vertical displacement and $u = 0$ in vertical direction, and concludes they both reach ground simultaneously
		2	

	Question 9 Total	**13**	