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FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	<table><tr><td>$M_{10} - M_1$</td><td>$\sum_{i=1}^{10} M_i$</td><td>$\frac{1}{10}\sum_{1}^{10} M_i$</td></tr><tr><td>N $(0, 2\sigma^2)$</td><td>N $(10\mu, 10\sigma^2)$</td><td>$N\left(\mu, \frac{\sigma^2}{10}\right)$</td></tr></table>	**B1** **B1** **B1**	N $(0, 2\sigma^2)$ N $(10\mu, 10\sigma^2)$ $N\left(\mu, \frac{\sigma^2}{10}\right)$
		3	

Q	Answer	Marks	Comments
1(b)(i)	N$(0, 1)$	**B1**	Identifies standard normal distribution
		1	

Q	Answer	Marks	Comments
1(b)(ii)	The expression includes population parameters	**B1**	**oe or** mention of μ **or** σ not allowed in the expression Condone unknown parameters
		1	

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2	H_0: $\sigma_A^2 = \sigma_B^2$ H_1: $\sigma_A^2 \neq \sigma_B^2$	**B1**	Both hypotheses needed **oe**
	$s_A^2 = \frac{1}{11-1}(556.3)$ [= 55.63] $s_B^2 = \frac{1}{7-1}(341.4)$ [= 56.9]	**M1**	Correct calculation for one of s_A^2 or s_B^2
	$F = \frac{s_B^2}{s_A^2} = \frac{56.9}{55.63}$	**M1**	Calculates F test statistic with their values **oe** $\frac{5690}{5563}$ **or** $\frac{s_A^2}{s_B^2} = \frac{55.63}{56.9}$
	$= 1.02$	**A1**	**AWRT** 1.02 or $\frac{s_A^2}{s_B^2}$ = **AWRT** 0.98
	dof $\nu_A = 10$, $\nu_B = 6$	**M1**	**PI**
	$F_{6,10}(0.975) = 4.072$	**A1**	**AWRT** 4.1 or p-value of **AWRT** 0.46 or $\frac{1}{F_{6,10}(0.975)}$ = **AWRT** 0.25
	$1.02 < 4.072$, Do not reject H_0	**A1ft**	Dependent on second M mark Correct comparison of their test statistic and their critical value Critical value must be from the lower tail if F < 1 or from the upper tail if F > 1 or their p-value and 0.025 and concludes that H_0 is not rejected
	There is insufficient evidence to suggest that the company select one machine over the other.	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion
		8	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3(a)	$P(X \geq 5) = 0.03641 < 0.05$ $P(X \geq 4) = 0.1087 > 0.05$ [Critical region is] $X \geq 5$	M1 A1	$P(X \geq 5) = 0.03641$ or $P(X \geq 4) = 0.1087$ **oe** eg $P(X \leq 4) = 0.96359$ or $P(X \leq 3) = 0.8913$ Both probabilities must be seen Either conclusion statement or both probabilities compared with 0.05 must be seen
		2	

Q	Answer	Marks	Comments
3(b)	$P(X \leq 4 \mid \lambda = 3.4) = 0.744$ P(Type II Error) = 0.744	M1 A1	Finds $P(X \leq 4 \mid \lambda = 3.4) =$ **AWRT** 0.74 or $P(X \leq 5 \mid \lambda = 3.4) =$ **AWRT** 0.87 **AWRT** 0.744 Do not ignore subsequent working
		2	

Q	Answer	Marks	Comments
3(c)	[Power = 1 – P(Type II Error)] = 0.256	B1	**AWRT** 0.256
		1	

	Question 3 Total	**5**	

Q	Answer	Marks	Comments
4(a)	$\left[\mathrm{E}\left(\overline{X}-\overline{Y}\right)=\right]\mu_X-\mu_Y$	B1	
		1	

Q	Answer	Marks	Comments
4(b)	$\mathrm{Var}\left(\overline{X}-\overline{Y}\right)=\frac{5^2}{50}+\frac{7^2}{14}$	M1	**oe** calculation
	$=4$	A1	**AG** Must see either $\frac{5^2}{50}$, $\frac{7^2}{14}$, $\frac{25}{50}$ or $\frac{49}{14}$ in a correct calculation before final answer
		2	

Q	Answer	Marks	Comments
4(c)	A linear combination of [independent] normally distributed random variables is also normally distributed	E1	Condone just stating that X and Y are normally distributed or $\overline{X}$ and $\overline{Y}$ are normally distributed
		1	

Q	Answer	Marks	Comments
4(d)	H_0: $\mu_X = \mu_Y$ H_1: $\mu_X \neq \mu_Y$ $\left[\left(\overline{X}-\overline{Y}\right)\sim \mathrm{N}(0,4)\right]$	B1	Both hypotheses **oe**
	$z=\pm\frac{28.5-24.3}{\sqrt{4}}$	M1	**PI**
	$=\pm 2.1$	A1	**PI** by $p = 0.0179$ **oe**
	$z_{\text{crit}}=\pm 1.8808$	B1	**AWRT** 1.88 or $p = 0.0179$ **oe**
	$2.1 > 1.8808$, Reject H_0	A1ft	Correct comparison of their test statistic and their critical value with consistent signs or their p-value and 0.03 **oe** and concludes that H_0 is rejected **ft** their comparison
	Sufficient evidence to suggest that the two populations have different means	E1ft	Conclusion in context, consistent with conclusion on H_0 or test statistic and critical value if not explicitly stated Must not be definite
		6	

	Question 4 Total	10	

Q	Answer	Marks	Comments
5	H_0: $\mu_{\text{New}} = \mu_{\text{Original}}$ H_1: $\mu_{\text{New}} > \mu_{\text{Original}}$	**B1**	Both hypotheses needed **oe**
	$s_p^2 = \frac{(9-1)\times 0.09 + (5-1)\times 0.12}{9+5-2}$ [= 0.1]	**M1**	Apply formula for pooled variance.
	$t = \pm \frac{3.1-2.6}{\sqrt{0.1\times\left(\frac{1}{9}+\frac{1}{5}\right)}}$	**M1** **m1**	**M1**: Correct numerator or correct denominator, **ft** their pooled variance **PI** **m1**: Both numerator and denominator correct, **ft** their pooled variance **PI**
	$= \pm 2.83$	**A1**	**AWRT** 2.83 or 2.835
	dof $= \nu = 9 + 5 - 2 = 12$	**B1**	**PI** by correct critical value
	$t_{\text{crit}} = t_{12} = \pm 2.681$	**B1**	**AWRT** 2.68 or p-value of 0.0075
	$2.83 > 2.681$, Reject H_0	**A1ft**	Dependent on second M mark Correct comparison of their test statistic and their critical value with consistent signs or their p-value and 0.01 and concludes that H_0 is rejected **ft** their comparison
	There is sufficient evidence to suggest that the yield for the new variety has increased	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion
		9	

	Question 5 Total	**9**	

Q	Answer	Marks	Comments
6(a)	E($\bar{X}$) = 3μ **and** E($\bar{Y}$) = 4$a\mu$	B1	**PI**
	E(S) = μ = 3μ – 4$a\mu$	M1	μ = their E($\bar{X}$) – their E($\bar{Y}$)
	a = 0.5	A1	**oe CSO**
		3	

Q	Answer	Marks	Comments
6(b)	Var ($\bar{X}$) = $\frac{\sigma^2}{n}$, Var ($\bar{Y}$) = $\frac{b\sigma^2}{n}$	M1	Finds Var ($\bar{X}$) or Var ($\bar{Y}$)
	Var(S) = $3^2\frac{\sigma^2}{n} + (-4)^2\frac{b\sigma^2}{n}$ $\left[= (9+16b)\frac{\sigma^2}{n}\right]$	M1	Finds Var(S) with their Var ($\bar{X}$) and Var ($\bar{Y}$)
	Var(S) $\to$ 0 as $n \to \infty$ so S is a consistent estimator	A1	**CSO**, Statement needed
		3	

Q	Answer	Marks	Comments
6(c)(i)	$\text{Var}(T) = \left(\frac{3}{2}\right)^2 \frac{\sigma^2}{n} + (-1)^2 \frac{b\sigma^2}{n}$ $\left[= \left(\frac{9}{4} + b\right)\frac{\sigma^2}{n}\right]$ Relative Efficiency $= \frac{\frac{1}{\text{Var}(T)}}{\frac{1}{\text{Var}(S)}} = \frac{9+16b}{\frac{9}{4}+b}$ $\frac{36+64b}{9+4b}$	**M1** **A1**	Finds correct $\text{Var}(T)$ **AG** Must see a correct unsimplified expression for relative efficiency before the final answer
		2	

Q	Answer	Marks	Comments
6(c)(ii)	As $b \to 0$, $\frac{36+64b}{9+4b} \to 4$ As $b \to \infty$, $\frac{36+64b}{9+4b}\left[= \frac{\frac{36}{b}+64}{\frac{9}{b}+4}\right] \to 16$ $\frac{36+64b}{9+4b}$ is an increasing function for $b > 0$ (and lies between the limits)	**B1** **B1** **E1**	Condone b = 0, $\frac{36+64b}{9+4b} = 4$ or $\frac{36+64b}{9+4b} = 4 + \frac{48b}{9+4b} > 4$ Condone $b = \infty$, $\frac{36+64b}{9+4b} = 16$ or $\frac{36+64b}{9+4b} = 16 - \frac{108}{9+4b} < 16$ Correct reference to increasing function, such as a sketch of the graph of $\frac{36+64b}{9+4b}$ or conclusion following both correct inequalities
		3	

Q	Answer	Marks	Comments
6(c)(iii)	Efficiency of T relative to $S > 1$ so T is the more efficient estimator of μ	**E1**	
		1	

	Question 6 Total	**12**	

Q	Answer	Marks	Comments
7(a)	$\overline{x} = 94$	**B1**	**PI**
	$s^2 = \frac{1}{11-1}\left(97846 - \frac{1034^2}{11}\right)$	**M1**	**PI**
	$s^2 = 65$	**A1**	**oe** eg s = **AWRT** 8.06
	$t_{\text{crit}} = t_{10} = 2.228$	**B1**	**AWRT** 2.2
	$94.0 \pm 2.228\sqrt{\frac{65}{11}}$	**M1**	**ft** their values of $\overline{x}$, s^2 and critical value
	(88.6, 99.4)	**A1**	**CSO**
		6	

Q	Answer	Marks	Comments
7(b)	The emission values are taken from a normal distribution	**B1**	Context (CO_2 emission values) not needed
		1	

Q	Answer	Marks	Comments
7(c)(i)	100 lies above the confidence interval	**E1**	
	The car model may qualify for the partial government refund	**E1**	Condone definite statement
		2	

Q	Answer	Marks	Comments
7(c)(ii)	As 100 is in the confidence interval, car model may not qualify for the partial government refund	**E1**	Condone definite statement.
		1	

	Question 7 Total	**10**	

Q	Answer	Marks	Comments
8(a)	$1394\int_{1500}^{2000}\frac{1}{1500}e^{\frac{-x}{1500}}dx$	M1	Correct integral with correct limits. Condone λ for 1/1500 Condone 1501 and/or 1999 Condone 1394 may be implied by later working
	$\left[=1394\left[-e^{-\frac{x}{1500}}\right]_{1500}^{2000}\right]$		
	$1394\left(e^{-1}-e^{-\frac{4}{3}}\right)=145[.37..]$	A1	**AG** Must see either a correct exact answer or a more accurate answer before the final answer
		2	

Q	Answer	Marks	Comments
8(b)	$\sum\frac{(M-E)^2}{E}=\frac{144}{395}+\frac{676}{283}+\frac{64}{203}+\frac{400}{145}+\frac{25}{104}+\frac{25}{75}+\frac{100}{189}$	M1	**oe PI** Condone copying errors
	$=6.93$	A1	
		2	

Q	Answer	Marks	Comments
8(c)	H_0: Distances between mutations have an exponential distribution with $\lambda=\frac{1}{1500}$	B1	**oe** Both hypotheses referring to exponential distribution.
	H_1: Distances between mutations do not have an exponential distribution with $\lambda=\frac{1}{1500}$	B1	$\lambda=\frac{1}{1500}$ seen in hypotheses
	$\chi_6^2(0.90)=10.645$	B1	**AWRT** 10.6
	$6.93<10.645$, Do not reject H_0	B1ft	Compares 6.93 with their critical value and concludes that H_0 is not rejected
	Insufficient evidence not to support the researcher's belief	E1ft	Gives a conclusion in context based on a comparison of their test statistic and the correct critical value Condone definite conclusion
		5	

	Question 8 Total	9	

Q	Answer	Marks	Comments
9(a)(i)	$\mathrm{M}'_X(t) = \lambda \mathrm{e}^t \mathrm{e}^{\lambda(\mathrm{e}^t - 1)}$	**M1**	Correctly differentiated function
	$\mathrm{E}(X) = \mathrm{M}'_X(0) = \lambda \mathrm{e}^0 \mathrm{e}^{\lambda(\mathrm{e}^0 - 1)} = \lambda$	**A1**	**AG**, substitutes $t = 0$ into correct expression
		2	

Q	Answer	Marks	Comments
9(a)(ii)	$\mathrm{M}''_X(t) = \lambda \mathrm{e}^t \mathrm{e}^{\lambda(e^t - 1)} + \lambda^2 \mathrm{e}^{2t} \mathrm{e}^{\lambda(e^t - 1)}$	**B1**	Correctly differentiated function
	$\mathrm{M}''_X(0) = \lambda \mathrm{e}^0 \mathrm{e}^{\lambda(e^0 - 1)} + \lambda^2 \mathrm{e}^{2\times 0} \mathrm{e}^{\lambda(e^0 - 1)} \left[= \lambda + \lambda^2 \right]$	**M1**	Substitutes $t = 0$ into their $\mathrm{M}''_X(t)$
	$\mathrm{Var}(X) = \lambda + \lambda^2 - \lambda^2 = \lambda$	**A1**	**CSO**
		3	

Q	Answer	Marks	Comments
9(b)(i)	$\mathrm{M}_Z(t) = \mathrm{M}_X(t) \times \mathrm{M}_Y(t)$	**M1**	
	$= \mathrm{e}^{\lambda(e^t - 1)} \times \mathrm{e}^{\mu(e^t - 1)}$ $= \mathrm{e}^{(\lambda + \mu)(e^t - 1)}$	**A1**	Allow unsimplified form
		2	

Q	Answer	Marks	Comments
9(b)(ii)	$v = \lambda + \mu$	**B1ft**	Must be consistent with answer for **(b)(i)**
		1	

Q	Answer	Marks	Comments
9(c)	$\mathrm{M}_W(t)=\left(1+\dfrac{\lambda\left(\mathrm{e}^t-1\right)}{n}\right)^n$	M1	
	$\lim_{n\to\infty}\mathrm{M}_W(t)=\mathrm{e}^{\lambda\left(\mathrm{e}^t-1\right)}$	M1	Use of $\lim_{n\to\infty}\left(1+\dfrac{\alpha}{n}\right)^n=\mathrm{e}^{\alpha}$
	This is the MGF for the Poisson Distribution with population parameter $\lambda=np$	A1	Identifies MGF as the same as the MGF for the Poisson distribution with $\lambda=np$
		3	

Q	Answer	Marks	Comments
9(d)	Binomial distribution can be **approximated** by Poisson distribution as $n\to\infty$	E1	**oe**
		1	

	Question 9 Total	**12**	