OXFORD

INTERNATIONAL
AQA EXAMINATIONS

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA04) Unit S2 Statistics

Friday 13 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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ANSWER IN THE SPACES PROVIDED

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Answer **all** questions in the spaces provided.

1 The continuous random variable X has mean 12 and variance 5

The continuous random variable Y has mean 15 and variance 2.5

The random variables X and Y are independent.

1 (a) Find the value of $\mathrm{E}(3Y-2X)$

[2 marks]

Answer _____

1 (b) Find the value of $\mathrm{Var}(3Y-2X)$

[2 marks]

Answer _____

4

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2 A hypothesis test is carried out by a student to determine whether there is evidence that the mean μ of a normally distributed random variable X has changed from 18.3

A random sample of size 16 has a sample mean of 19

The standard deviation of X is known to be 2.5

A 10% level of significance is used.

There are errors in **three** stages of the student's attempt at the hypothesis test below.

	Statement
Stage 1	$H_0 : \overline{x} = 18.3$ $H_1 : \overline{x} \neq 18.3$
Stage 2	Under H_0 $X \sim N\left(18.3, 2.5^2\right)$
Stage 3	$n = 16$ $\overline{X} \sim N\left(18.3, \frac{2.5^2}{16}\right)$
Stage 4	10% level of significance gives $z_{critical} = \pm 1.2816$
Stage 5	Using sample mean $= 19$ $z = \frac{19 - 18.3}{\left(\frac{2.5}{4}\right)}$
Stage 6	Test statistic $z = 1.12$
Stage 7	$z < z_{critical}$
Stage 8	As we have evidence to suggest a change in mean occurred at the 10% level of significance, we reject H_0

2 (a) Identify the **three** stages with errors and write down the correct statements for each of the stages.

[4 marks]

Error 1 Stage _____

Correct statement _____

_____

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Error 2 Stage _____

Correct statement _____

_____

Error 3 Stage _____

Correct statement _____

_____

2 (b) Explain why the Central Limit Theorem is not needed in the student's hypothesis test.

[1 mark]

_____

_____

_____

2 (c) Which of the five options below correctly completes the following sentence?

_____ is the set of values for the test statistic for which we reject the null hypothesis.

Tick (✓) **one** box.

[1 mark]

	A Confidence Interval
	A Critical Region
	An Unbiased Estimator
	A Critical Value
	A Significance Test

6

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3 The continuous random variable T has the cumulative distribution function

$$\mathrm{F}(t) = \begin{cases} 0 & t < 0 \\ \dfrac{t^3}{64} & 0 \le t \le 4 \\ 1 & t > 4 \end{cases}$$

3 (a) Find the probability density function of T for all values of t

[3 marks]

Answer _____

3 (b) The mean of T is μ and the standard deviation of T is σ

3 (b) (i) Find the value of μ

[2 marks]

Answer _____

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3 (b) (ii) Find the value of σ

Give your answer to three significant figures.

[4 marks]

Answer _____

3 (c) Find $\mathrm{P}(\mu - 2\sigma \leq T \leq \mu + 2\sigma)$

Give your answer to three significant figures.

[4 marks]

Answer _____

13

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4 Hand-drying machines are designed to switch off automatically after a set drying time.

A manufacturer makes hand-drying machines which have a mean drying time of 25 seconds.

An engineer redesigns the hand-drying machine and a random sample of 500 redesigned machines are tested.

The drying time, X seconds, is measured for each of the redesigned machines and the results are summarised as

$$\sum x = 12430 \quad \text{and} \quad \sum x^2 = 310000$$

The engineer now claims that the mean drying time has been reduced.

Test the engineer's claim at the 1% level of significance.

[10 marks]

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10

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5 (a) (i) State **two** conditions required for using a Poisson distribution as an approximation to a binomial distribution B(n, p)

[1 mark]

Condition 1 _____

Condition 2 _____

5 (a) (ii) It is given that $X \sim \text{B}(500, 0.02)$

Use a Poisson approximation to find the parameter λ for this distribution.

[1 mark]

Answer _____

5 (a) (iii) Using your Poisson approximation for X find $\text{P}(X < 10)$

Give your answer to three significant figures.

[2 marks]

Answer _____

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5 (b) Yellow jacket wasps live in ground nests found on farmland.

The number of ground nests found on farmland is known to fit a Poisson distribution with a mean of 8 per hectare.

Sally has a farm which has trialled a new crop.

Sally believes that the new crop has caused a change in the Poisson distribution mean number of ground nests for yellow jacket wasps.

Over 30 hectares Sally finds 150 ground nests for yellow jacket wasps.

5 (b) (i) Construct a hypothesis test for Sally's claim at the 10% level of significance.

[6 marks]

Question 5 continues on the next page

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5 (b) (ii) Find the critical region for the hypothesis test in **part (b)(i)** at the 10% level of significance.

[3 marks]

Answer _____

5 (c) (i) Describe in the context of the test in **part (b)(i)** a Type I error.

[1 mark]

5 (c) (ii) Calculate the probability of a Type I error for the test in **part (b)(i)**.

[1 mark]

Answer _____

15

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6 A theme park has five rides. The queue times in minutes for each ride are modelled as independent normal distributions as shown below.

Vertical Slide $X_1 \sim \mathrm{N}\left(12, 1.5^2\right)$

High Swings $X_2 \sim \mathrm{N}\left(8, 0.8^2\right)$

Gravity Wheel $X_3 \sim \mathrm{N}\left(7, 1^2\right)$

Roller Coaster $X_4 \sim \mathrm{N}\left(14, 2^2\right)$

Jump Drop $X_5 \sim \mathrm{N}\left(\mu, \sigma^2\right)$

6 (a) Find the probability that the queue time for the Vertical Slide is less than 10 minutes.

Give your answer to three significant figures.

[2 marks]

Answer _____

6 (b) Find the queue time exceeded by 10% of people for the High Swings.

Give your answer to three significant figures.

[3 marks]

Answer _____

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6 (c) For the Jump Drop it is given that

$$P(X_5 < 16) = 0.82 \quad \text{and} \quad P(X_5 > 10) = 0.53$$

Find the value of μ and the value of σ

Give your answers to three significant figures.

[6 marks]

$\mu =$ _____ $\sigma =$ _____

Question 6 continues on the next page

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6 (d) Use your answers to **part (c)** to suggest why modelling the Jump Drop queue time as this normal distribution might not be appropriate in this context.

[1 mark]

6 (e) A person queues once for each of the Vertical Slide, High Swings, Gravity Wheel and Roller Coaster.

The total queue time is T minutes.

Find $\mathrm{P}(T < 35)$

Give your answer to three significant figures.

[3 marks]

Answer

15

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7 Don runs a business where people make appointments to ride ponies.

The number of appointments made by customers for rides each week can be modelled by a Poisson distribution with $\lambda = 2.5$

7 (a) Find the probability that there are exactly four appointments in a given week.

Give your answer to four significant figures.

[2 marks]

Answer _____

7 (b) The summer holiday period lasts 6 weeks.

Find the probability that there are more than 18 appointments over the summer holiday period.

Give your answer to four significant figures.

[3 marks]

Answer _____

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7 (c) The time, T weeks, between appointments can be modelled by an exponential distribution.

7 (c) (i) Find the mean of T

[1 mark]

Answer _____

7 (c) (ii) Find the variance of T

[1 mark]

Answer _____

7 (d) A week lasts 7 days.

Find the value of c such that $\mathrm{P}(T < c) = 0.9$

Give your answer to the nearest day.

[3 marks]

Answer _____

10

Turn over ►

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8 The probability density function f for a continuous random variable X is given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{96}{(5x+k)^2} & 2 \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

where k and d are constants, $k > 0$ and $d > 2$

8 (a) Show that the cumulative distribution function F for X is given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2 \\ \dfrac{96(x-2)}{(10+k)(5x+k)} & 2 \le x \le d \\ 1 & x > d \end{cases}$$

[4 marks]

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8 (b) It is given that $\mathrm{F}(4.4) = 0.8$

Find the value of k

[3 marks]

Answer _____

7

END OF QUESTIONS

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