OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL AS
MATHEMATICS

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Thursday 5 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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9	
10	
11	
12	
13	
TOTAL	

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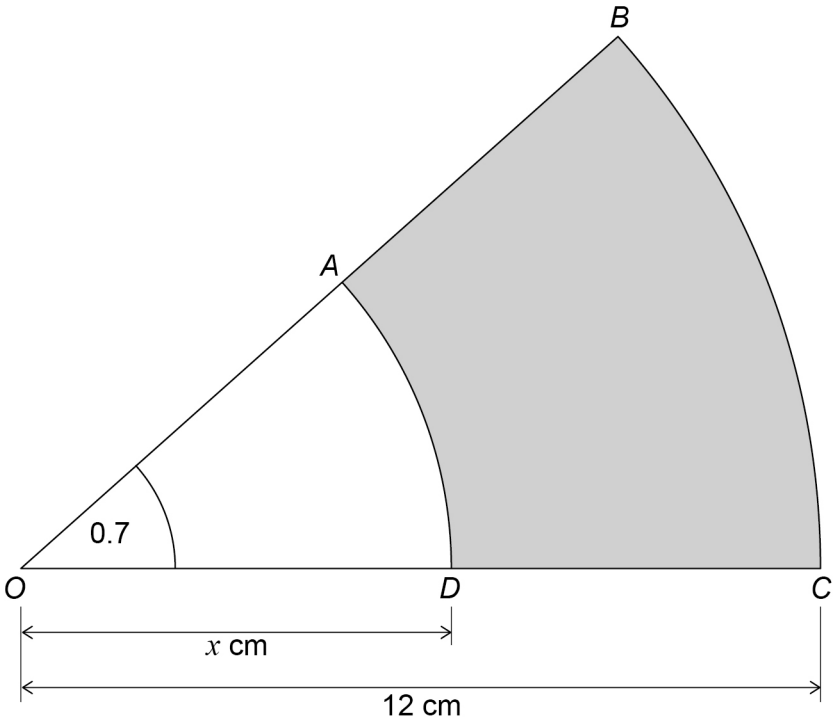
Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 The diagram shows two sectors OAD and OBC of two different circles each with centre O

The point A lies on OB and the point D lies on OC



The angle BOC is 0.7 radians

The length of OC is 12 cm

1 (a) Find the length of the arc BC

[1 mark]

Answer _____

Do not write outside the box

1 (b) The area of the shaded region *ABCD* is 35 cm^2

The length of *OD* is x cm

Find the value of x

Give your answer in the form $a\sqrt{b}$ where a and b are prime numbers.

[3 marks]

Answer _____

4

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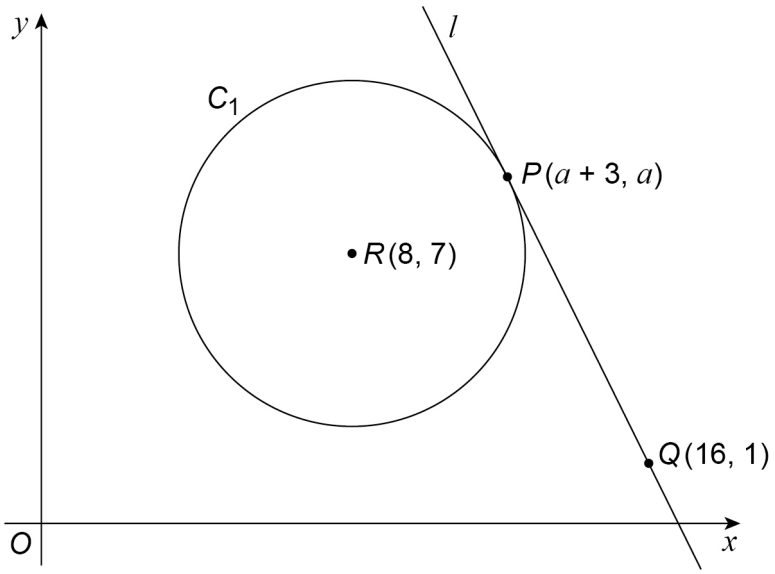
2 The diagram shows the circle C_1

The point $R(8, 7)$ is the centre of C_1



2 (a) The line l is the tangent to C_1 at the point $P(a+3,\ a)$ where $a > 7$

The point $Q(16, 1)$ lies on l

2 (a) (i) Show that a satisfies the equation $a^2 - 13a + 36 = 0$

[4 marks]

Do not write outside the box

2 (a) (ii) It is given that $a > 7$

By solving the equation given in **part (a)(i)** find the equation of l

Give your answer in the form $bx + cy = d$ where b, c and d are integers.

[3 marks]

Answer _____

2 (b) C_2 is a different circle. The translation $\begin{bmatrix} 3 \\ -10 \end{bmatrix}$ maps C_1 onto C_2

Find the equation of C_2

Give your answer in the form $(x-e)^2 + (y-f)^2 = k$ where e, f and k are integers.

[4 marks]

Answer _____

11

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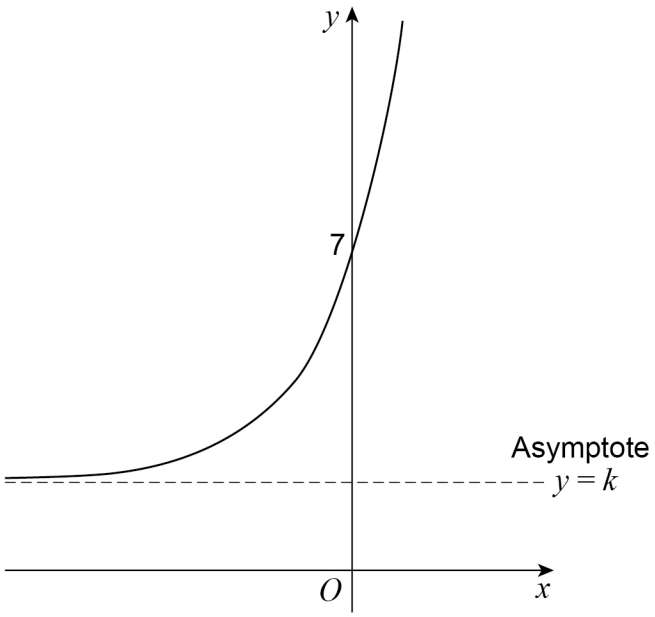
3 The diagram shows a sketch of part of a curve and its asymptote with equation $y = k$



The curve has equation $y = A \times (1.5)^{3x} + 2$ where A is a constant.

The curve intersects the y-axis at the point $(0, 7)$

3 (a) Find the value of A

[1 mark]

Answer _____

3 (b) The asymptote has equation $y = k$

State the value of k

[1 mark]

Answer _____

Do not write outside the box

3 (c) The curve passes through the point $(b,\ 4)$

Find the value of b

Give your answer to three significant figures.

[2 marks]

Answer _____

4

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4 The diagram shows the quadrilateral $EFGH$ and the line segment EG

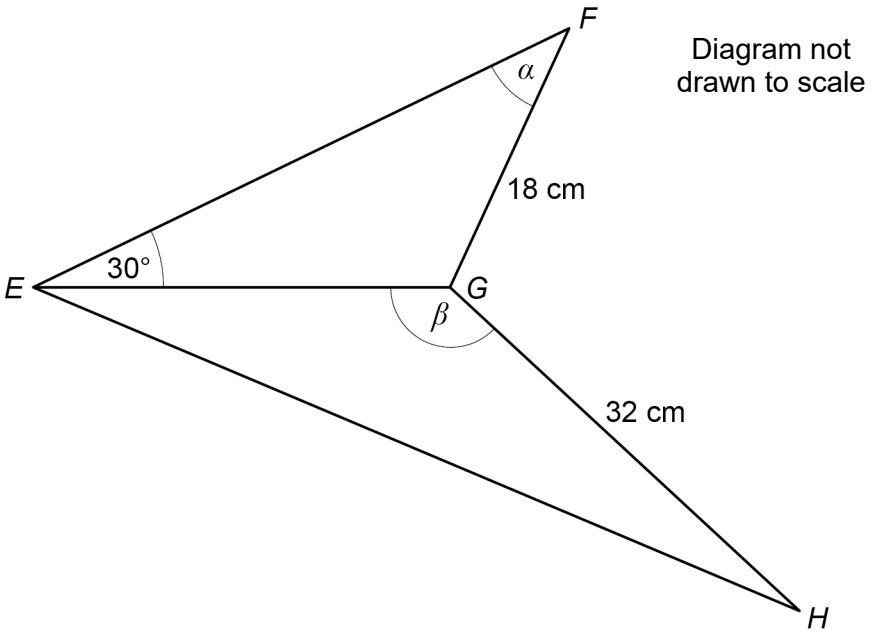

Diagram not drawn to scale

The length of FG is 18 cm

The length of GH is 32 cm

The angle FEG is 30°

The angle EFG is α where $\sin\alpha = \frac{7}{12}$

The angle EGH is β and is **obtuse**.

The area of triangle EGH is 210 cm^2

Find the value of β giving your answer in degrees to three significant figures.

[6 marks]

Do not write outside the box

Answer _____

6

Turn over for the next question

Do not write outside the box

5 It is given that x satisfies the equation

$$2\log_7(2x-5)-\log_7(2x+3)=2$$

5 (a) Show that

$$2x^2-59x-61=0$$

[4 marks]

Do not write outside the box

5 (b) Use the result given in **part (a)** to solve the equation

$$2\log_7(2x-5)-\log_7(2x+3)=2$$

[2 marks]

Answer _____

6

Turn over for the next question

Do not write outside the box

6 (a) Show that

$$\frac{33+16\cos^2 x}{7+4\sin x}$$

can be written in the form $p - q\sin x$ where p and q are positive integers to be found.

[3 marks]

6 (b) (i) Hence show that the greatest possible value of $\frac{33+16\cos^2 x}{7+4\sin x}$ is 11

[1 mark]

6 (b) (ii) Hence state the value of x in the interval $0^\circ \leq x \leq 360^\circ$ at which this greatest value occurs.

[1 mark]

$x =$ _____

Do not write outside the box

6 (c) Using your answer to **part (a)** solve the equation

$$\frac{33+16\cos^2 2\theta}{7+4\sin 2\theta}=6$$

in the interval $0^\circ \leq \theta \leq 90^\circ$ giving your answers to the nearest 0.1°

[4 marks]

Answer _____

9

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Section B

Statistics

Answer **all** questions in the spaces provided.

7 A company sells shoes to customers and labels their shoes using two different systems of sizes *A* and *B*

The sizes of all of the shoes sold by the company in the last year can be modelled by a discrete random variable.

The size of a shoe using system *B* can be calculated from its size using system *A* by applying the formula

$$B = 12A - 5$$

7 (a) For system *A* the mean size of all of the shoes sold is 7.02

Find the mean size of all of the shoes sold if system *B* is used.

[1 mark]

Answer _____

7 (b) For system *A* the standard deviation of the sizes of all of the shoes sold is 0.573

Find the variance of the sizes of all of the shoes sold if system *B* is used.

Give your answer to three significant figures.

[2 marks]

Answer _____

3

Turn over ►

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8 A survey is taken of the number of cats and fish owned by 180 households.

The results of the survey are shown in the table.

	No fish	**One fish**	**Two or more fish**
No cats	56	48	8
One cat	39	22	1
Two or more cats	1	5	0

A household is chosen at random.

8 (a) Find the probability that the household owns exactly one cat and one fish.

[1 mark]

Answer _____

8 (b) Find the probability that the household owns two or more fish.

[1 mark]

Answer _____

8 (c) Find the probability that the household owns two or more fish given that the household owns exactly one cat.

[1 mark]

Answer _____

Do not write outside the box

8 (d) *A* represents the event that the household owns no cats.

B represents the event that the household owns no fish.

Explain whether or not *A* and *B* are independent.

[4 marks]

7

Turn over ►

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9 The random variable X has probability distribution

X	0	1
$P(X = x)$	$1 - p$	p

where $0 < p < 1$

9 (a) Prove that $\text{Var}(X) = p(1 - p)$

[3 marks]

9 (b) State the distribution of X giving any parameters.

[1 mark]

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9 (c) A coin has 'heads' on one face and 'tails' on the other face.

The coin is tossed.

The random variable Y takes the value 1 if the coin lands with 'heads' facing upwards and 0 if the coin lands with 'tails' facing upwards.

The variance of Y is 0.2176

The coin is biased so that when it is tossed it is more likely to land with 'heads' facing upwards.

Find the probability that the coin lands with 'heads' facing upwards.

[2 marks]

Answer _____

6

Turn over for the next question

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10 The discrete random variables X_1, X_2 and X_3 are independent.

The random variable X_1 has variance 0.51

The random variable X_2 has standard deviation 0.87

The random variable X_3 has mean 2.56 and $\mathrm{E}\left(\left(X_3\right)^2\right) = 7.0336$

Find the value of $\mathrm{Var}\left(\sum_{i=1}^{3} X_i\right)$

[4 marks]

Answer _____

4

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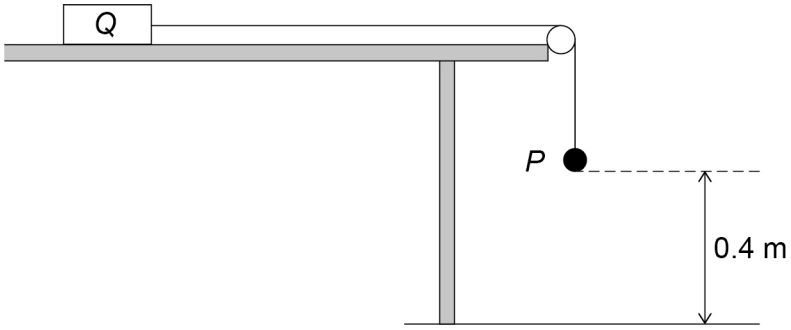
Section C

Mechanics

Answer **all** questions in the spaces provided.

11 **The acceleration due to gravity, g, should be taken as 9.8 m s^{-2}**

The diagram shows a particle P and a block Q attached by a light inextensible string.



The string is taut and passes over a smooth peg.

P has mass 0.2 kg and hangs freely at a height of 0.4 metres above horizontal ground.

Q has mass 1 kg and is held at rest on a rough horizontal surface.

The coefficient of friction between Q and the surface is 0.15

11 (a) (i) Q is released.

In the subsequent motion, P reaches the ground before Q reaches the peg.

Find the magnitude of the acceleration of P before P reaches the ground.

Give your answer to two significant figures.

[4 marks]

Do not write outside the box

Answer _____

11 (a) (ii) Find the greatest speed of P

[2 marks]

Answer _____

11 (b) Explain how you have used the fact that the peg is smooth.

[1 mark]

7

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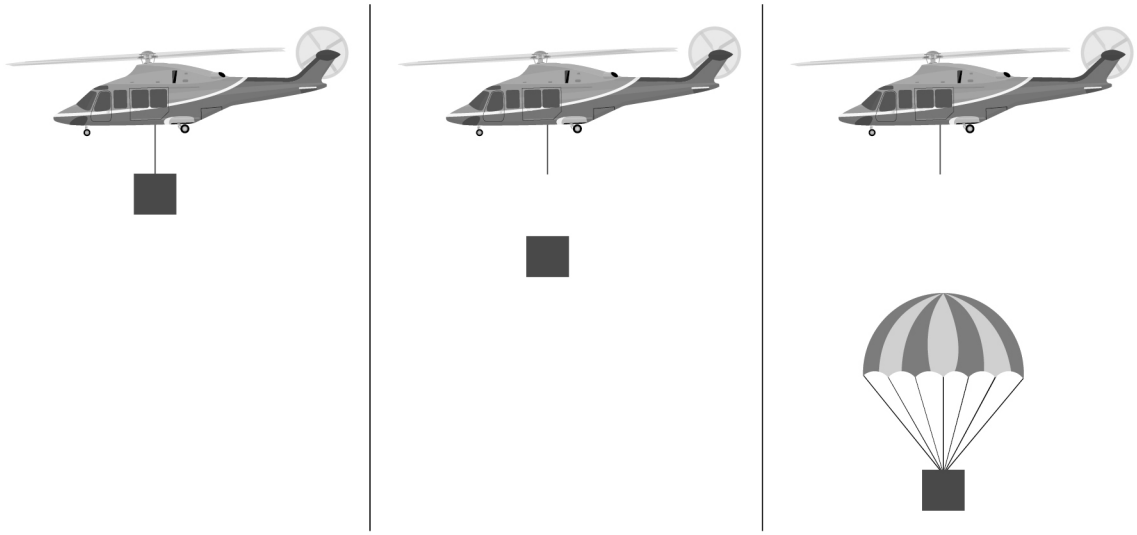
12 A helicopter is hovering in a stationary position above a fixed point on the ground.

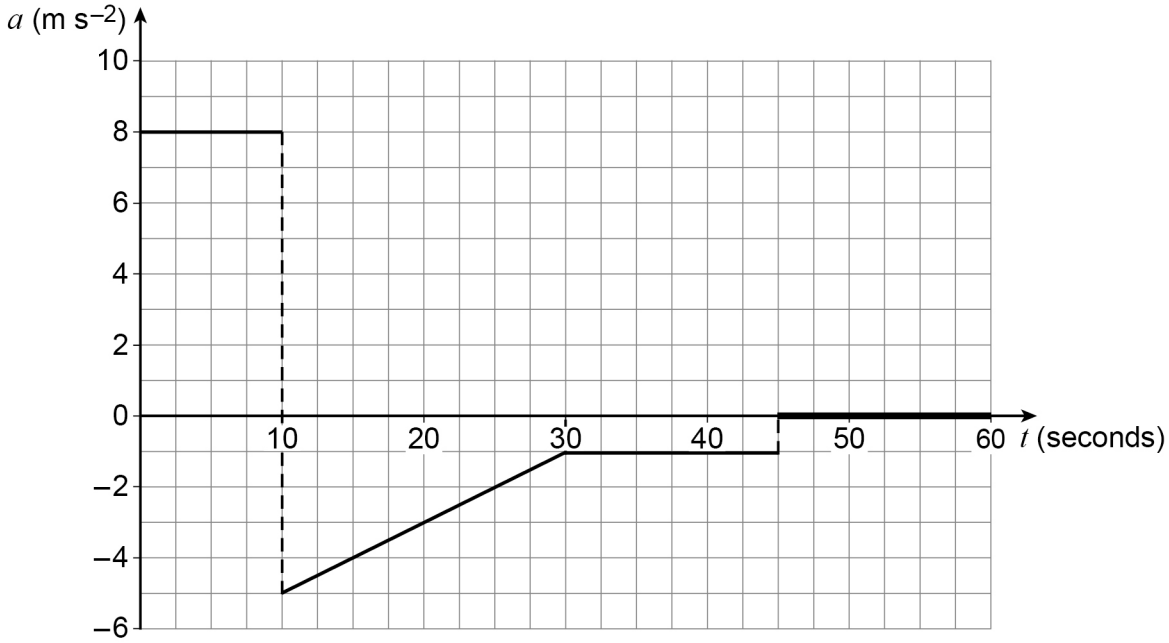
A package hangs below the helicopter.

The package is released from rest.

10 seconds after the package is released a parachute is used to reduce the speed, as shown in the diagram.



The acceleration–time graph below models the acceleration a m s^{-2} of the package at time t seconds after it is released.



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12 (a) Find the distance travelled by the package in the first 10 seconds after it is released.

[2 marks]

Answer _____

12 (b) The acceleration of the package between $t = 10$ and $t = 30$ is modelled by

$$a = pt - 7$$

where p is a positive constant.

Use the acceleration–time graph to find the value of p

[1 mark]

$p =$ _____

12 (c) (i) Find an expression in terms of t for the velocity of the package between $t = 10$ and $t = 30$

[3 marks]

Answer _____

Question 12 continues on the next page

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12 (c) (ii) Using your answer to **part (c)(i)** show that the speed of the package 30 seconds after it is released is 20 m s^{-1}

[1 mark]

12 (d) The package reaches the ground 60 seconds after being released.

For it to reach the ground safely it must land with a speed of less than 2 m s^{-1}

Determine whether or not the package reaches the ground safely based on the model.

[2 marks]

9

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13 Two particles A and B are at rest on a smooth horizontal surface.

Particle A has mass m kg where m is a constant.

Particle B has mass km kg where k and m are constants.

Particle A is set in motion with velocity 6 m s^{-1} directly towards B and they collide.

After the collision both particles have speed $3v$ m s^{-1} where v is a constant and the direction of motion of A has been reversed.

13 (a) Show that $k = \frac{2}{v} + 1$

[2 marks]

13 (b) In the subsequent motion particle B collides with a fixed wall which is perpendicular to the path of B

During the collision the wall exerts an impulse of magnitude 12 N s on the particle.

After the collision the direction of motion of B is reversed and the speed of B is v m s^{-1}

Show that $m = \frac{3}{2+v}$

[2 marks]

4

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